Self-evaluation exercises and solutions

Chapter 1

Self-evaluation exercises and solutions

1.1 Self-evaluation exercises

1.1.1 Self-evaluation exercise 1

Content: Chapter 2

1. At what simple interest rate must R7 000 be invested for a period of nine months to accumulate to R7 630?

2. Michael needs R1 200 urgently. Peter is prepared to lend him the money on condition that he pays him R1 295 four months from now. What simple interest rate is Peter earning on this transaction?

3. How large an amount must be invested on 3 March at a simple interest rate of 12% per year, to accumulate to R612 on 2 May of the same year?

4. Mputle Maputle borrows R1 500 on 10 March. How much interest is he paying if he has to pay back the loan on 2 July of the same year and a simple interest rate of 21,5% per year is charged on his loan?

5. How long does it take for an amount of R3 500 to accumulate to R3 755 if simple interest of 18% per year is earned?

6. When will R2 000 invested on 6 March at a simple interest rate of 15% accumulate to R2 240?

7. Sipho borrows money on 31 August and signs an agreement stating that he will pay back the loan on 2 November of the same year. If the discount rate is 18% per year, and he receives R5 000 on 31 August of the same year, what is the face value of the agreement?

8. What is the equivalent simple interest rate of the previous question?

9. John Drake needs to make the following payments against a loan on his lorry:

   R10 000 after six months

   R20 000 after one year

   R40 000 after two years. As a result of drought on his farm, John Drake could not pay the first two payments. After 18 months, John Drake has a record harvest and immediately makes a down payment of R50 000 against his loan. What single size payment should he make two years from now to settle his debt if simple interest of 17% per year is charged on all amounts?

The solutions to these questions are to be found on p 25 of this tutorial letter.

If you need extra exercises you can do the additional exercises available on myUnisa.

1.1.2 Self-evaluation exercise 2

Content: Chapter 3

1. A deceased had R1 400 in a Swiss bank at the time of his death. Twenty years later the beneficiary of his will learns about the investment and claims it. How much must be paid to the beneficiary if the investment earned 12,5% interest per year compounded half yearly, over the 20 years?

2. How long will it take to save R25 000 for a trip to Europe if you deposit R15 000 into a savings account now, earning interest of 12% per year, compounded monthly? The initial R15 000 will also be used to pay for the trip.

3. An amount of R1 000 has accumulated to R1 500 after two and a half years. Calculate the interest rate per year if interest is compounded monthly.

4. Willem Grobler invests R12 000 at an interest rate of 10,5% per year, compounded monthly. After four years and three months Willem withdraws R15 000 and invests it at 12% interest per year, compounded quarterly. What is the total accumulated amount of both accounts after six years?

5. An investment company invests its funds at an interest rate of 12% per year, compounded monthly. What is the effective interest rate that the company earns?

6. On 3 January Granddad Steyn deposited R2 500 into a savings account for his grandchild who was born on 26 July the previous year. Interest is credited at 18,75% per year on the first day of every month.

   (a) How much money does his grandchild receive on his first birthday if simple interest is used for odd periods and compound interest for the rest of the term?

   (b) How much does he receive if fractional compounding is used for the full period?

7. Joseph would like to buy a lawnmower. He has three options when it comes to borrowing the R3 750 from the bank:

   17,5% per year, compounded semi-annually

   16% per year, compounded quarterly

   16% per year, compounded monthly

   Make use of continuous compounding rates to decide which option Joseph should take.

8. A wholesaler has to pay the following amounts to a manufacturer:

   R200 000 after three months

   R300 000 after one year and three months

   R400 000 after two years. He would like to reschedule his three payments by making only two payments. The first payment will be made at the end of the first year and the second, twice the size of the first, at the end of 21 months. If interest is calculated at 18,75% compounded quarterly, what is the size of each payment? Use month 21 as the comparison date.

The solutions to these questions are to be found on p 29 of this tutorial letter.

Additional exercises are available on myUnisa.

1.1.3   Self-evaluation exercise 3

Content: Chapters 4 and 5

1. Mr White opened an annuity fund and deposited R3 000 into it. Thereafter he deposited R500 at the end of each month into this fund. Mr Jones, on the other hand, opened his annuity fund by depositing R5 000 into it. He thereafter deposited R300 at the end of each month into this fund. After 15 years of making deposits into the annuity funds, the two friends decided to compare their investments. Calculate the amounts in the two funds after the 15 years if a 12,5% interest rate compounded monthly is applicable.

2. An insurance agent offers services to clients who are concerned about their personal financial planning for retirement. To explain the advantages of an early start to investing, she points out that if the 25-year-old John starts to save R2 000 at the *beginning* of each year for 10 years (and makes no further contributions) John *will earn more* than Jane who waits 10 years and then save R2 000 at the *beginning* of each year until retirement at an age of 65 (a total of 30 contributions). Find the net earnings (compound amount minus total contributions) of John and Jane at age 65. An annual interest rate of 7% is applicable and the deposits are made at the *beginning* of each year.

3. A poor student has to repay his study loan of R80 000 which he received when he enrolled for the first time, in equal monthly payments. The repayment will start after he has finished his education in four years' time. Determine his monthly payments if he wants to repay his debt in five years (after studying for four years) and if interest is calculated at 15% per year, compounded monthly.

4. Determine approximately the accumulated value of R500 payments made every month for a period of eight years if interest is compounded semi-annually at 13,5% per year.

5. Vusi and Vivian want to purchase a new flat and feel that they can afford a mortgage payment of R2 500 a month. They are able to pay R100 000 deposit and obtained a 20-year, 14,75% per annum mortgage bond (compounded monthly).

   (a) How much can they afford to spend on a flat?
   (b) After eight years Vusi receives a huge promotion and decides to buy a much bigger flat. What equity do they have in their present flat after eight years?

6. Maitland Engineering wants to replace machinery after seven years. The company has been investing a sum of R5 000 in a sinking fund every six months for this purpose. The investment has been earning interest at the rate of 16% per year compounded semi-annually. Determine the balance of the fund after seven years.

7. A medical practitioner of Gauteng buys a holiday home on the west coast with a cash deposit of R200 000 plus monthly payments of R10 000 for a period of five years. Interest is 12% per annum compounded monthly. Calculate the cash price of the house.

The solutions to these questions are to be found on p 34 of this tutorial letter.

Additional exercises are available on myUnisa.

1.1.4   Self-evaluation exercise 4

Content: Chapter 6

1. An investor has to decide between two alternative projects: A and B. The initial investment outlays and the cash inflows of each of the projects are listed in the table below. If the capital cost is 19% per year, use the internal rate of return, the net present value and the profitability index respectively to advise him with regard to the two projects. All funds are in R1 000s.

Year	Project A INVESTMENT 800	Project B INVESTMENT 750
	Cash inflows	Cash inflows
1	400	200
2	300	500
3	350	450

2. Denise and Jan want to start a business. They can choose between two options: a shoe shop and a CD shop. The two shops require the following cash flows (in R'000):

Year	Shoe shop	CD shop
0	-100	-400
1	50	75
2	-50	100
3	75	400

What advice would you give them if you consider the MIRR criterion with an interest rate of 16,5% per year applicable for the cash outflows and an interest rate of 19% per year applicable for the cash inflows?

The solutions to these questions are to be found on p 37 of this tutorial letter.

Additional exercises are available on myUnisa.

1.1.5   Self-evaluation exercise 5

Content: Chapter 7

1. Consider the following bond

XYZ:	
Coupon rate (half yearly)	16,5% per annum
Redemption date	1 June 2029
Yield to maturity	14,2% per annum
Settlement date	14 April 2013

   Calculate the all-in price, the accrued interest and the clean price on the settlement date.

2. Calculate the all-in price, the accrued interest and the clean price for the bond in question 1 on the following settlement date: 25 May 2013.

   The solutions to these questions are to be found on p 39 of this tutorial letter.

Additional exercises are available on myUnisa.

1.1.6   Self-evaluation exercise 6

Content: Chapter 8

1. In 2011 the average hourly wage of construction workers was R28,41 . Miners made R27,50 per hour on average and production workers in manufacturing made R26,65. There were 6,2 million production workers in manufacturing, 1 million miners and 4,4 million construction workers in 2011. What was the average hourly wage for workers in all three fields?

2. Assume you are a member of a scholarship committee and are trying to decide between two students who are competing for one award. Your decision must be made on the basis of the grades the students earned in courses taken during the first semester of their third year. The grades are shown below:

	Student A	Student B
First course	81	83
Second course	88	93
Third course	83	76

   (a) If you make the award on the basis of the arithmetic mean, which student would you select?

   (b) If you select the student who is most consistent, which student would you select? Justify your choice.

3. In a regression survey of interest rates and investments made over ten years the following results were observed.

Year	Yearly investment (Thousands of rands)	Average interest (Percent)
1	1 060	13,8
2	940	14,5
3	920	13,7
4	1 110	14,7
5	1 550	14,8
6	1 850	15,5
7	2 070	16,2
8	2 030	15,9
9	1 780	14,9
10	1 420	15,1

   (a) Plot the data on a graph with average interest rate on the horizontal axis and yearly investment on the vertical axis. Comment on the graph.

   (b) Calculate the coefficient of correlation between average interest rate and yearly investment.

   (c) Calculate and interpret the coefficient of determination, r^2.

   (d) Develop an effective prediction equation for yearly investment.

   (e) Can we forecast yearly investment if the average interest rate is 16,5?

   The solutions to these questions are to be found on p 41 of this tutorial letter.

1.1.7 Self-evaluation exercise 7

Typical examination questions

1. On his ninth birthday on 21 February Little John received R420. His parents immediately invested the money in an account that earns 7,5% simple interest. The amount of money that can be withdrawn on 5 June for the same year equals

   [1]   R411,21.

   [2]   R428,89.

   [3]   R428,98.

   [4]   R429,07.

   [5]   none of the above.

2. An interest rate of 16,4% compounded quarterly is equivalent to a weekly compounded interest of

   [1]   16,073%.

   [2]   16,098%.

   [3]   16,714%.

   [4]   16,741%.

   [5]   none of the above.

3. On Dandy Darrell's 21st birthday he notices that he is going bald. He decides that he will go for a hair implant when he turns 30. He estimates that the implant will cost him R12 500. He starts saving immediately by paying an amount monthly into an account earning 9,09% interest compounded monthly. The monthly payment that Dandy Darrell makes into the account equals

   [1]   R64,27.

   [2]   R74,63.

   [3]   R75,20.

   [4]   R115,75.

   [5]   none of the above.

4. At an interest rate of 14,9% per year compounded quarterly, R1 000 invested monthly for 12 years will accumulate to

   [1]   R66 914,38.

   [2]   R385 478,48.

   [3]   R390 233,94.

   [4]   R395 600,34.

   [5]   none of the above.

Questions 5, 6 and 7 refer to the following bond:

Consider Bond XYZ.

Coupon	11,59%
Yield to maturity	9,46%
Settlement date	18 *April* 2013
Date to maturity	15 *November* 2038
Nominal value	R750 000

5. The all-in price on the settlement date equals

   [1]   R119,45625%.

   [2]   R119,55642%.

   [3]   R119,56986%.

   [4]   R125,31160%.

   [5]   none of the above.

6. The accrued interest equals

   [1]   −R1,72890%.

   [2]   −R0,86445%.

   [3]   −R0,85734%.

   [4]   R4,89003%.

   [5]   none of the above.

7. The clean price to the nearest rand equals

   [1]   R750 000.

   [2]   R896 732.

   [3]   R901 008

   [4]   R903 162

   [5]   none of the above.

8. If the NPV of the Smell Nice Shop is R1 255 and the profitability index is 1,083, then the initial investment in the shop equals

   [1]   R1 158,82.

   [2]   R1 255,00.

   [3]   R10 416,50.

   [4]   R15 120,48.

   [5]   none of the above.

Questions 9 and 10 refer to the following situation:

Three years ago Malcolm borrowed R7 500 from Sarah on the condition that he would pay her back in five years' time. Interest is calculated at 13,5% per year every three months. Nine months ago he also borrowed R2 500 from her at an interest rate of 15,7% per year compounded monthly payable two years from now.

9. The total amount that Malcolm owes Sarah two years from now will equal

   [1]   R10 000,00.

   [2]   R16 141,88.

   [3]   R18 353,73.

   [4]   R18 406,16.

   [5]   none of the above.

10. After seeing what he will owe Sarah two years from now, Malcolm asks Sarah if he can reschedule his debt by paying R9 000 now and the rest four years from now. Sarah agrees on condition that the new agreement will be subject to an interest of 11% per year compounded half-yearly. The amount that Malcolm must pay Sarah in four years' time will equal

    [1]   R7 642,92.

    [2]   R8 924,87.

    [3]   R8 989,83.

    [4]   R9 406,16.

    [5]   none of the above.

11. In order to settle a debt Trevor agrees to pay Jill R4 500 every six months for six years plus an additional R10 500 at the end of the six years. The present value of Trevor's debt at the beginning of the agreement period if money is worth 9,15% per annum compounded half-yearly equals

    [1]   R40 858,13.

    [2]   R46 996,52.

    [3]   R51 358,13.

    [4]   R80 389,66.

    [5]   none of the above.

12. If the MIRR for a project lasting eight years is 10,81% and the present value of the outflows equals R291 930,00, then the future value of the cash inflows will approximately equal

    [1]   R128 400,00.

    [2]   R263 450,00.

    [3]   R323 500,00.

    [4]   R663 600,00.

    [5]   none of the above.

Questions 13 and 14 refers to the following situation:

Just Water Plumbing agreed to establish the Spanner Fund from which they will pay Spanner R2 500 per month indefinitely as compensation for injuries he sustained while working at the No Water Dam. The money is worth 14% per year compounded monthly.

13. The opening balance of this fund equals

    [1]   R161 013,55.

    [2]   R214 285,71.

    [3]   R250 000,00.

    [4]   R448 026,10.

    [5]   none of the above.

14. Spanner asks whether Just Water Plumbing could reschedule the compensation into two payments: One payment five years from now when his son will go to university and the other payment exactly the same size as the first one ten years from now when his daughter will attend a beauty school. They agree to this on condition that the interest rate stays the same. The present value of the payments will equal

    [1]   R80 506,78.

    [2]   R107 142,86.

    [3]   R224 013,05.

    [4]   R286 783,06.

    [5]   none of the above.

Questions 15 and 16 refer to the following situation:

A study was undertaken at eight garages to determine how the resale value of a car is affected by its age. The following data was obtained:

Garage	Age of car (in years) (x)	Re-sale value (R) (y)
1	1	41 250
2	6	10 250
3	4	24 310
4	2	38 720
5	5	8 740
6	4	26 110
7	1	38 650
8	2	36 200

The garage manager suspects a linear relationship between the two variables. Fit a curve of the form $y = a + bx$ to the data.

15. The equation is equal to

   [1] $y = 7{,}0417 - 0{,}001x$.

   [2] $y = 0{,}001 + 7{,}0417x$.

   [3] $y = 48\,644{,}17 - 6\,596{,}93x$.

   [4] $y = 6\,596{,}93 - 48\,644{,}17x$.

   [5]   none of the above.

16. The correlation coefficient equals

   [1]   0,0000.

   [2]   −0,9601.

   [3]   0,8450.

   [4]   1,0000.

   [5]   none of the above.

The solutions to these questions are to be found on p 45 of this tutorial letter.

Additional exercises are available on myUnisa.

1.1.8   Self-evaluation exercise 8

Typical examination questions

Question 1

James borrows R2 000 at a simple interest rate of 8% per annum. The amount that he owes at the beginning of the eighth year equals

[1]   R1 120,00

[2]   R3 120,00

[3]   R3 280,00

[4]   R3 427,65

[5]   none of the above

Question 2

After making a down payment of R5 000 on a boat, Mr Clark also had to pay an additional R700 per month for it for three years. Interest was charged at 14,5% per year compounded monthly on the unpaid balance. The original price of the boat equals

[1]   R6 611,60

[2]   R20 336,44

[3]   R25 336,44

[4]   R36 337,23

[5]   none of the above

Question 3

If R100 accumulates to R115 at a simple interest rate of 8% per annum, then the length of time (in years) of the investment is given by the expression

[1] $\frac{1}{8}\left(\frac{115}{100} - 1\right)$

[2] $\frac{1}{8}\left(\frac{115}{100} + 1\right)$

[3] $\frac{100}{8}\left(\frac{115}{100} - 1\right)$

[4] $\left(\frac{115}{100} + 1\right) \times \frac{1}{0,08}$

[5]   none of the above

Question 4

Jonas needs R14 500 to buy a computer. Compunet is prepared to lend him the money on condition that he pays the money back in ten months' time. The amount that he must pay back if a discount rate of 28% is applicable will equal

[1]   R11 116,67

[2]   R11 756,76

[3]   R17 883,33

[4]   R18 913,04

[5]   none of the above

Question 5

If the continuous compounding rate for a nominal rate compounded every three months is 11,832%, then the nominal rate equals

[1]   11,66%

[2]   11,832%

[3]   12,01%

[4]   12,07%

[5]   12,56%

Question 6

If R25 000 accumulates to R32 850 after 39 months, then the continuous compounding rate equals

[1]   7,5%

[2]   7,6%

[3]   8,4%

[4]   8,8%

[5]   9,7%

Question 7

Nene is making monthly payments towards a loan of R250 000 which she borrowed for six years. An interest rate of 11,8% per year, compounded monthly, is applicable. After 33 months the interest rate changes to 15,6% per year, compounded quarterly. The amount that Nene has paid off when the interest rate changes equals

[1]   R93 151,85

[2]   R102 009,77

[3]   R147 990,23

[4]   R156 848,15

[5]   R160 432,47

Question 8

The effective rate for a continuous compounding rate of 17,5% is

[1]   16,13%

[2]   17,5%

[3]   19,12%

[4]   19,13%

[5]   none of the above

Questions 9 and 10 relate to the following situation:
Tracy deposited R25 000 into an account earning 9,75% interest per year, compounded quarterly. After five years the interest rate changed to 10% per year, compounded weekly. She then decided to deposit R500 every week into this account.

Question 9

The balance in this account after five years equals

[1]   R34 750,00

[2]   R37 187,50

[3]   R41 198,25

[4]   R48 780,49

[5]   none of the above

Question 10

After owning this account for nine years Tracy decides to close it. The amount of money that Tracy can expect to withdraw then equals

[1]   R127 725,46

[2]   R129 000,00

[3]   R167 519,80

[4]   R168 194,18

[5]   R188 074,51

Question 11

Jonathan bought a 107 cm plasma screen television set. He agrees to *immediately* start to pay R1 403 per month. The term of the agreement is 24 months and the applicable interest rate is 20,124% per year, compounding monthly. The original price of the television set equals

[1]   R27 079,22

[2]   R27 533,34

[3]   R27 995,08

[4]   R30 385,36

[5]   R33 672,00

Question 12

A simple interest rate of 9,68% is equivalent to a simple discount rate of 7,5%. The time under consideration is

[1]   2,2 years

[2]   2,4 years

[3]   2,8 years

[4]   3 years

[5]   6 years

Question 13

The net present value (NPV) of the Beautiful People Shop is R14 983 and the profitability index (PI) is 1,034. The initial investment in the shop approximately equals

[1]   R7 366

[2]   R14 490

[3]   R14 983

[4]   R15 492

[5]   none of the above

Questions 14 and 15 relate to the following situation:
Charlene intends to open a hairdressing salon and borrows the money from Aunt Amor. Charlene feels that she will only be able to start repaying her debt after five years. Charlene will then pay Aunt Amor R35 000 every six months for four years. Money is worth 17,9% per year compounded semi-annually.

Question 14

The present value of Charlene's debt at the time she starts paying back will equal

- [1] R69 484,18
- [2] R194 079,19
- [3] R225 113,21
- [4] R280 000,00
- [5] R385 298,07

Question 15

The amount of money that Aunt Amor lends Charlene equals

- [1] R82 358,16
- [2] R95 527,55
- [3] R118 818,94
- [4] R163 502,53
- [5] R163 741,31

Question 16

The equation for the present value of Bond ABC on 01/07/2012 is given by

$$P(01/07/2012) = \frac{14{,}7}{2} a \overline{29}|_{0,135 \div 2} + 100 \left(1 + \frac{0{,}135}{2} \right)^{-29}.$$

The fraction of the half year to be discounted back is

$$f = \frac{74}{181}.$$

The accrued interest equals R4,30932%. The clean price for Bond ABC equal

- [1] R100,40824%
- [2] R104,71756%
- [3] R107,56456%
- [4] R111,87388%
- [5] R114,90174%

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Question 17

Consider Bond 567

Coupon rate:	12,4% per year
Yield to maturity:	10,8% per year
Settlement date:	23 May 2013
Maturity date:	12 December 2034

The all-in-price equals

- [1]   R112,61841%
- [2]   R112,62197%
- [3]   R113,27116%
- [4]   R118,78268%
- [5]   R119,47116%

Questions 18 and 19 relate to the following situation:
Three years ago Daniel borrowed R10 000 from Sarah on condition that he would pay her back in six years' time. Interest is calculated at 14,75% per year, compounded quarterly. Six months ago he also borrowed R17 500 from her at an interest rate of 10,5% per year, compounded monthly. This loan will be paid back three years from now.

Question 18

The total amount that Daniel will owe Sarah three years from now is

- [1]   R33 881,55
- [2]   R45 656,47
- [3]   R46 362,95
- [4]   R47 778,84
- [5]   R49 078,73

Question 19

Daniel asks Sarah if he can reschedule his debt, paying R18 000 now and the rest two years from now. Sarah agrees to this on condition that the interest rate for the new agreement starting *now* changes to 13,4% per year compounded half-yearly. The amount that Daniel must pay Sarah two years from now equals

- [1]   R12 313,49
- [2]   R20 584,99
- [3]   R23 330,83
- [4]   R33 881,55
- [5]   R43 915,82

Question 20

The accumulated amount (rounded to the nearest thousand rand) of semi-annual payments of R5 500 for ten years into an account earning 8,9% interest per year compounded monthly, equals

- [1]   R72 000,00
- [2]   R83 000,00
- [3]   R110 000,00
- [4]   R172 000,00
- [5]   R173 000,00

Question 21

The following figures show the profit of a greengrocer for the past five years: R360 000, R550 000, R200 000, R80 000 and R700 000.
The standard deviation of the data equals

- [1]   R225 424
- [2]   R252 032
- [3]   R378 000
- [4]   R1 890 000
- [5]   none of the above

Question 22

Fawzia took out an endowment policy that matures in 20 years. The expected interest rate per year is 10%. Her first payment is R3 600 per year, after which the yearly payments will increase by R360 each year. The amount that she can expect to receive on the maturity date will be

- [1]   R213 030
- [2]   R340 380
- [3]   R412 380
- [4]   R484 380
- [5]   none of the above

Question 23

The following table shows the number of loans approved for different amounts during the second half of 2011.

Amount of loan in R100 000 (x)	Number of loans (y)
2	45
3	250
4	250
5	175
6	125

The regression line equation is

[1] $y = 0,00279x + 3,528$

[2] $y = 3,528x + 0,00279$

[3] $y = 8,5x + 135$

[4] $y = 135x + 8,5$

[5] $y =$ none of the above

The solutions to these questions are to be found on p 52 of this tutorial letter.

Additional exercises are available on myUnisa.

1.2 Solutions: self-evaluation exercises

1.2.1 Solution to self-evaluation exercise 1

1. Simple interest $I = Prt$

   I = interest earned = R7 630 − R7 000 = R630
   P = present value = R7 000
   r = simple interest rate = ?
   t = term = nine months = $\frac{9}{12}$ = $\frac{3}{4}$ year

$$
\begin{aligned}
I &= Prt \\
630 &= 7\,000 \times r \times \tfrac{3}{4} \\
r &= \tfrac{630 \times 4}{7\,000 \times 3} \\
r &= 0{,}12 \\
&= 12\%
\end{aligned}
$$

   The simple interest rate is 12,0%.

2. Simple interest: $I = Prt$

   I = interest earned = R1 295 − R1 200 = R95
   P = present value = R1 200
   r = simple interest rate = ?
   t = term = four months = $\frac{4}{12}$ year

$$
\begin{aligned}
I &= \mathrm{Pr}\,t \\
95 &= 1\,200 \times r \times \tfrac{4}{12} \\
r &= \tfrac{95 \times 12}{4 \times 1\,200} \\
r &= 0{,}2375 \\
&= 23{,}75\%
\end{aligned}
$$

   The simple interest rate is 23,75%.

3. Simple interest: $S = P(1 + rt)$

   S = Accumulated amount = R612
   P = present value = ?
   r = simple interest rate = 0,12
   t = term of loan = from 3 March until 2 May.

Period	Number of days
3–31 March	29 (3rd included)
1–30 April	30
1–2 May	$\underline{1}$ (2nd excluded)
	60 days

OR

Use the number of each day of the year table. Day number 122 (2 May) minus day number 62 (3 March) equals 60.

$$
\begin{aligned}
S &= P\left(1 + rt\right) \\
P &= \frac{S}{1+rt} \\
&= \frac{612}{\left(1 + 0{,}12 \times \frac{60}{365}\right)} \\
&= 600{,}16
\end{aligned}
$$

R600,16 must be invested on 3 March.

4. Simple interest: $I = Prt$

$$
\begin{aligned}
I &= \text{interest earned} = ? \\
P &= \text{present value} = \text{R1 500} \\
r &= \text{simple interest rate} = 0{,}215 \\
t &= \text{term of loan} = \text{from 10 March until 2 July}
\end{aligned}
$$

Day number 183 (2 July) minus day number 69 (10 March) equals 114.

$$
\begin{aligned}
I &= Prt \\
&= 1\,500 \times 0{,}215 \times \tfrac{114}{365} \\
&= 100{,}73
\end{aligned}
$$

He has to pay R100,73 interest on the loan of R1 500.

5. Simple interest: $S = P(1 + rt)$

$$
\begin{aligned}
S &= \text{accumulated amount} = \text{R3\,755} \\
P &= \text{present value} = \text{R3\,500} \\
r &= \text{simple interest rate} = 0{,}18 \\
t &= \text{term of investment} = ?
\end{aligned}
$$

$$
\begin{aligned}
S &= P(1 + rt) \\
\tfrac{S}{P} &= 1 + rt \\
\tfrac{S}{P} - 1 &= rt \\
t &= \left(\tfrac{S}{P} - 1\right) \div r \\
&= \left(\tfrac{3\,755}{3\,500} - 1\right) \div 0{,}18 \\
&= 0{,}4048 \text{ years} \\
&= 0{,}4048 \times 365 \text{ days} \\
&= 147{,}7 \approx 148 \text{ days.}
\end{aligned}
$$

R3 500 must be invested for 148 days for an interest rate of 18% per year to accumulate to R3 755.

6. Simple interest: $I = Prt$

$$
\begin{aligned}
I &= \text{interest earned} = \text{R2\,240} - \text{R2\,000} = \text{R240} \\
P &= \text{present value} = \text{R2\,000} \\
r &= \text{simple interest rate} = 0{,}15 \\
t &= \text{term of investment} = ?
\end{aligned}
$$

$$
\begin{aligned}
I &= Prt \\
t &= \tfrac{I}{P \times r} \\
t &= \tfrac{240}{2\,000 \times 0{,}15} \\
&= 0{,}8 \text{ years} \\
&= 0{,}8 \times 365 \text{ days} \\
&= 292 \text{ days.}
\end{aligned}
$$

Day number 65 (6 March) plus 292 equals day number 357 that is 23 December.

If R2 000 is invested on 6 March at an interest rate of 15% per year, it will accumulate to R2 240 on 23 December of the same year.

7. Discount: $P = S(1 - dt)$

P = present value = amount that he receives = R5 000
S = face value or future value = ?
d = discount rate = 0,18
t = term of loan = 31 August until 2 November of the same year.

Day number 306 (2 November) minus day number 243 (31 August) equals 63.

$$
\begin{aligned}
P &= S(1 - dt) \\
S &= \frac{P}{(1 - dt)} \\
&= \frac{5\,000}{\left(1 - 0{,}18 \times \frac{63}{365}\right)} \\
&= 5\,160{,}32
\end{aligned}
$$

The face value is R5 160,32.

8. Interest paid in the previous question is R160,32(5 160,32 − 5 000):

The simple interest rate equivalent to the above interest can be calculated as $I = Prt$.

$$
\begin{aligned}
I &= Prt \\
160{,}32 &= 5\,000 \times r \times \frac{63}{365} \\
\frac{160{,}32}{5\,000} \times \frac{365}{63} &= r \\
r &= 0{,}18577 \\
&= 18{,}58\%.
\end{aligned}
$$

The equivalent simple interest is 18,58%.

9.
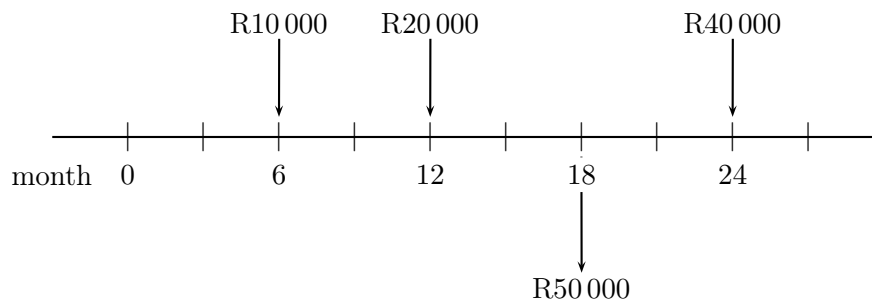
He has to pay his debt. We must calculate the value of all his payments and obligations at the same time, namely at month 24.

Obligations:

R10 000 must be moved 18 months forward:

$$
10\,000 \left(1 + 0{,}17 \times \frac{18}{12}\right) = 12\,550{,}00
$$

R20 000 must be moved 12 months forward:

$$20\ 000 \left(1 + 0{,}17 \times \frac{12}{12}\right) = 23\ 400{,}00$$

There is no need to move the R40 000.

Payments:

R50 000 must be moved six months forward:

$$50\ 000 \left(1 + 0{,}17 \times \frac{6}{12}\right) = 54\ 250{,}00$$

The amount that he has to pay at month 24 is:

$$
\begin{aligned}
\text{Obligations} - \text{payments} \ &= \ (12\ 550{,}00 + 23\ 400{,}00 + 40\ 000{,}00) - 54\ 250{,}00 \\
&= \ 21\ 700{,}00
\end{aligned}
$$

The amount to be paid is R21 700,00.

1.2.2 Solution to self-evaluation exercise 2

1. Compound interest: $S = P\left(1 + \frac{j_m}{m}\right)^{tm}$

   $$\begin{aligned}
   S &= \text{future value} = ? \\
   P &= \text{present value} = \text{R1\,400} \\
   m &= \text{number of compounded periods per year} = 2 \\
   t &= \text{the number of years for which the investment is made} = 20 \text{ years.} \\
   j_m &= \text{interest per year} = 12{,}5\%
   \end{aligned}$$

   $$\begin{aligned}
   S &= P\left(1 + \frac{j_m}{m}\right)^{tm} \\
   &= 1\,400\left(1 + \frac{0{,}125}{2}\right)^{20\times 2} \\
   &= 15\,822{,}88
   \end{aligned}$$

   After 20 years R15 822,88 must be paid to the inheritor.

2. Compound interest: $S = P\left(1 + \frac{j_m}{m}\right)^{tm}$

   $$\begin{aligned}
   S &= \text{R25\,000} \\
   P &= \text{R15\,000} \\
   m &= 12 \\
   t &= ? \\
   j_m &= 0{,}12
   \end{aligned}$$

   $$\begin{aligned}
   S &= P\left(1 + \frac{j_m}{m}\right)^{tm} \\
   25\,000 &= 15\,000\left(1 + \frac{0{,}12}{12}\right)^{12t} \\
   \frac{25\,000}{15\,000} &= \left(1 + \frac{0{,}12}{12}\right)^{12t} \\
   \ln\left(\frac{25\,000}{15\,000}\right) &= 12\,t\ln\left(1 + \frac{0{,}12}{12}\right) \\
   \frac{\ln\left(\frac{25\,000}{15\,000}\right)}{\ln\left(1 + \frac{0{,}12}{12}\right)} &= 12t \\
   12t &= 51{,}34 \\
   t &\approx 4\tfrac{1}{4}
   \end{aligned}$$

   It will take $4\frac{1}{2}$ years.

3. Compound interest: $S = P\left(1 + \frac{j_m}{m}\right)^{tm}$.

   $$\begin{aligned}
   S &= \text{R1\,500} \\
   P &= \text{R1\,000} \\
   j_m &= ? \\
   t &= 2^{1}\!/_{2} \\
   m &= 12.
   \end{aligned}$$

29

$$S = P\left(1 + \frac{j_m}{m}\right)^{tm}$$

$$1\,500 = 1\,000\left(1 + \frac{j_m}{12}\right)^{2{,}5 \times 12}$$

$$\left(\tfrac{1\,500}{1\,000}\right)^{\frac{1}{30}} = 1 + \frac{j_m}{12}$$

$$\left(\tfrac{1\,500}{1\,000}\right)^{\frac{1}{30}} - 1 = \frac{j_m}{12}$$

$$12\left[\left(\tfrac{1\,500}{1\,000}\right)^{\frac{1}{30}} - 1\right] = j_m$$

$$j_m = 0{,}1633$$

$$= 16{,}33\%.$$

The interest rate per year is 16,33%.

4. We firstly calculate the value of R12 000 after four years and three months with:

$$P = 12\,000$$
$$j_m = 0{,}105$$
$$t = 4\tfrac{3}{12}\,\text{years} = 4{,}25\,\text{years}$$
$$m = 12$$
$$S = ?$$

$$S = P\left(1 + \frac{j_m}{m}\right)^{tm}$$
$$S = 12\,000\left(1 + \frac{0{,}105}{12}\right)^{4{,}25 \times 12}$$
$$= 18\,712{,}95$$

Then R15 000 of the R18 712,95 is withdrawn and invested for the remaining one year and nine months at an interest rate of 12% per year compounded quarterly, with
$j_m = 0{,}12$ $m = 4$ and $t = 1\tfrac{9}{12}$.

After one year and nine months the R15 000 will accumulate to:

$$S = P\left(1 + \frac{j_m}{m}\right)^{tm}$$
$$= 15\,000\left(1 + \frac{0{,}12}{4}\right)^{1{,}75 \times 4}$$
$$= 18\,448{,}11$$

From the R18 712,95 there is R3 712,95 (18 712,95 − 15 000,00) left that earns 10,5% interest, compounded monthly, therefore $j_m = 0{,}105$, $m = 12$ and $t = 1{,}75$ for the remaining one year and nine months (21 months).

After one year and nine months (21 months) the R3 712,95 will accumulate to:

$$S = P\left(1 + \frac{j_m}{m}\right)^{tm}$$
$$= 3\,712{,}95\left(1 + \frac{0{,}105}{12}\right)^{1{,}75 \times 12}$$
$$= 4\,458{,}34$$

The total accumulated amount for both accounts at the end of the six year period is
R22 906,45 (18 448,11 + 4 458,34).

5. Effective interest: $j_{eff} = 100\left[\left(1 + \frac{j_m}{m}\right)^m - 1\right]$
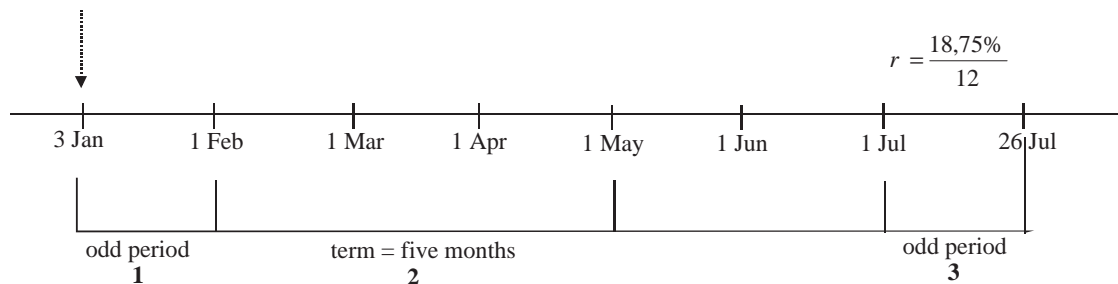
j_m = nominal rate = 0,12
m = number of times per year that the interest are calculated = 12

$$
\begin{aligned}
j_{eff} &= 100\left[\left(1 + \frac{0,12}{12}\right)^{12} - 1\right] \\
&= 12,68\%.
\end{aligned}
$$

The effective interest rate is 12,68%.

6.

(a)

Period 1:   odd period of 29 days (Day number 32 - 3 = 29.)
Period 2:   term = five months
Period 3:   odd period of 25 days
Value of R2 500 on 1 February:

$$
\begin{aligned}
S_1 &= P(1 + rt) \\
&= 2\,500\left(1 + \frac{29}{365} \times 0,1875\right) \\
&= 2\,537,24
\end{aligned}
$$

Value of R2 537,24 on 1 July: with $m = 12$ and $t = \frac{5}{12}$.

$$
\begin{aligned}
S_2 &= 2\,537,24\left(1 + \frac{j_m}{m}\right)^{tm} \\
&= (2\,537,24) \times \left(1 + \frac{0,1875}{12}\right)^{\left(\frac{5}{12} \times \frac{12}{1}\right)} \\
&= 2\,741,75
\end{aligned}
$$

Value of R2 741,75 on 26 July:

$$
\begin{aligned}
S_3 &= 2\,741,75(1 + rt) \\
&= (2\,741,75) \times \left(1 + \frac{25}{365} \times 0,1875\right) \\
&= 2\,776,96
\end{aligned}
$$

Thus the value on 26 July is

$$
\begin{aligned}
&2\,500\left(1 + \frac{29}{365} \times 0,1875\right)\left(1 + \frac{0,1875}{12}\right)^{\left(\frac{5}{12} \times \frac{12}{1}\right)}\left(1 + \frac{25}{365} \times 0,1875\right) \\
&= 2\,776,96.
\end{aligned}
$$

The grandchild will receive R2 776,96 on his first birthday.

(b) $S = P \left(1 + \frac{j_m}{m}\right)^{tm}$

$$
\begin{array}{rcl}
j_m & = & \text{interest rate per year} = 0{,}1875 \\
m & = & \text{number of compounding periods per year} = 12 \\
t & = & \text{term of investment} = \text{five compounding periods} = \frac{5}{12} \text{ years plus} \\
& & \text{the number of odd days as a fraction of a year } \left(\frac{5}{12} + \frac{29+25}{365}\right).
\end{array}
$$

$$
\begin{array}{rcl}
S & = & 2\,500 \left(1 + \frac{0{,}1875}{12}\right)^{\left(\frac{5}{12} + \frac{29+25}{365}\right) \times 12} \\
& = & 2\,776{,}90
\end{array}
$$

He will receive R2 776,90 if fractional compounding is used.

7. Continuous compounding rate: $c = m \ln\left(1 + \frac{j_m}{m}\right)$

   Option A:

   $$
   \begin{array}{rcl}
   c & = & 2\ln\left(1 + \frac{0{,}175}{2}\right) \\
   & = & 16{,}78\%
   \end{array}
   $$

   Option B:

   $$
   \begin{array}{rcl}
   c & = & 4\ln\left(1 + \frac{0{,}16}{4}\right) \\
   & = & 15{,}69\%
   \end{array}
   $$

   Option C:

   $$
   \begin{array}{rcl}
   c & = & 12\ln\left(1 + \frac{0{,}16}{12}\right) \\
   & = & 15{,}89\%
   \end{array}
   $$

   The best option for Joseph is the one with the lowest interest rate, thus option B.

8.
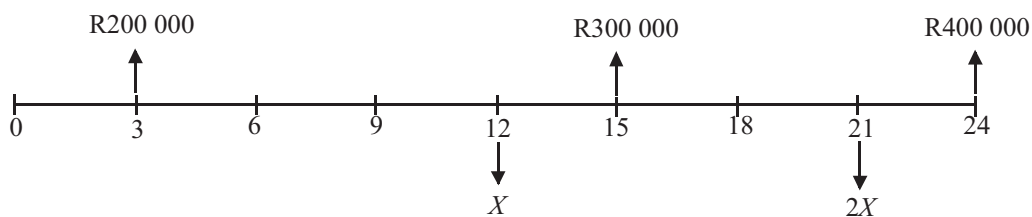
Due to the time value of money, we must move all the moneys to the same date namely the comparison date, month 21.

The value of the debt after 21 months:

For R200 000:

$$
\begin{array}{rcl}
t & = & 18 \div 12 \text{ (month 21 minus month 3)} = 1{,}5 \text{ years} \\
m & = & 4 \\
j_m & = & 0{,}1875.
\end{array}
$$

$$\begin{aligned} S &= P\left(1 + \frac{j_m}{m}\right)^{tm} \\ &= 200\,000 \times \left(1 + \frac{0,1875}{4}\right)^{(1,5 \times 4)} \end{aligned}$$

For R300 000: $300\,000 \times \left(1 + \frac{0,1875}{4}\right)^{(0,5 \times 4)}$. $t = $ six months $= 0,5$ years.

For R400 000: $400\,000 \left(1 + \frac{0,1875}{4}\right)^{-(0,25 \times 4)}$. $t = $ three months $= 0,25$ years.

The total debt after 21 months is thus:

$$200\,000 \left(1 + \frac{0,1875}{4}\right)^{(1,5 \times 4)} + 300\,000 \left(1 + \frac{0,1875}{4}\right)^{(0,5 \times 4)} + 400\,000 \left(1 + \frac{0,1875}{4}\right)^{-(0,25 \times 4)}$$

$$= 263\,268,54 + 328\,784,18 + 382\,089,55$$

$$= 974\,142,27$$

The value of the payments at the end of the 21 months:

First payment: $X\left(1 + \frac{0,1875}{4}\right)^{(0,75 \times 4)}$. $t = $ nine months $= 0,75$ years.

Second payment: $2X$. (no interest is applicable)

$$\begin{aligned} \text{Payment} &= \text{Obligations} \\ X\left(1 + \frac{0,1875}{4}\right)^{(0,75 \times 4)} + 2X &= 974\,142,27. \\ X\left[\left(1 + \frac{0,1875}{4}\right)^{(0,75 \times 4)} + 2\right] &= 974\,142,27 \,(\text{Take the common factor } X, \text{ out.}) \\ X &= \frac{974\,142,27}{\left[\left(1 + \frac{0,1875}{4}\right)^{(0,75 \times 4)} + 2\right]} \\ X &= 309\,514,87 \end{aligned}$$

The size of the first payment at the end of the first year is thus R309 514,87.

The payment after 21 months is thus:
$$2 \times 309\,514,87$$
$$= 619\,029,74$$

The payment is R619 029,74.

1.2.3   Solution to self-evaluation exercise 3

1. **Mr White**

   The initial payment accumulated to:

   $$
   \begin{aligned}
   S &= P\left(1+\tfrac{j_m}{m}\right)^{tm} \\
     &= 3\,000\left(1+\tfrac{0,125}{12}\right)^{15\times12} \\
     &= 19\,373,65
   \end{aligned}
   $$

   The monthly payments from an ordinary annuity and accumulated to:

   $$
   \begin{aligned}
   S &= Rs_{\overline{n}|i} \\
     &= 500s_{\overline{15\times12}|0,125\div12} \\
     &= 261\,978,42
   \end{aligned}
   $$

   Mr White's fund accumulated to R281 352,07 $(261\,978,42 + 19\,373,65)$:

   Mr Jones

   The initial deposit accumulated to:

   $$
   \begin{aligned}
   S &= P\left(1+\tfrac{j_m}{m}\right)^{tm} \\
     &= 5\,000\left(1+\tfrac{0,125}{12}\right)^{15\times12} \\
     &= 32\,289,42
   \end{aligned}
   $$

   The monthly payments accumulated to:

   $$
   \begin{aligned}
   S &= Rs_{\overline{n}|i} \\
     &= 300s_{\overline{15\times12}|0,125\div12} \\
     &= 157\,187,05
   \end{aligned}
   $$

   Mr Jones's fund accumulated to R189 476,47 $(157\,187,05 + 32\,289,42)$

   Mr White has R91 875,60 $(281\,352,07 - 189476,47)$ more than Mr Jones in his fund.

2. John invests R2 000 for 10 years:

Time:	10 years
Payments:	R2 000
Interest:	7% per year

   $$
   \begin{aligned}
   S &= (1+i)Rs_{\overline{n}|i} \\
     &= (1+0,07)2\,000s_{\overline{10}|0,07} \\
     &= 29\,567,20
   \end{aligned}
   $$

   This amount now accumulates compound interest for 30 years:

   $$
   \begin{aligned}
   \text{where} \quad j_m &= 0,07 \\
   m &= 1 \\
   t &= 30.
   \end{aligned}
   $$

   $$
   \begin{aligned}
   S &= P\left(1+\tfrac{j_m}{m}\right)^{tm} \\
     &= 29\,567,20\left(1+0,07\right)^{30} \\
     &= 225\,073,07
   \end{aligned}
   $$

Net earnings of John:
$$\begin{aligned} S &= 225\,073,07 - (2\,000 \times 10) \\ &= 225\,073,07 - 20\,000 \\ &= 205\,073,07 \end{aligned}$$

John's earnings are R205 073,07.

Jane investing R2 000 for 30 years:
$$\begin{aligned} S &= (1+i)Rs_{\overline{n}|i} \\ &= (1+0,07)2\,000s_{\overline{30}|0,07} \\ &= 202\,146,08 \end{aligned}$$

Net earnings of Jane:
$$\begin{aligned} S &= 202\,146,08 - (2\,000 \times 30) \\ &= 142\,146,08 \end{aligned}$$

Jane's earnings are R142 146,08.

3. The R80 000 accumulates interest in the four years' time:
$$\begin{aligned} S &= P\left(1 + \frac{j_m}{m}\right)^{tm} \\ &= 80\,000\left(1 + \frac{0,15}{12}\right)^{4 \times 12} \\ &= 145\,228,39 \end{aligned}$$

This R145 228,39 is the amount money that he has to repay in equal monthly payments:
$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ 145\,228,39 &= Ra_{\overline{5 \times 12}|0,15 \div 12} \\ R &= 3\,454,97 \end{aligned}$$

If he wants to repay the loan in five years' time he must pay R3 454,97 per month.

4. As the payments made and the interest dates don't correspond we must first convert the semi-annually compounding interest to monthly compounding.
$$\begin{aligned} i &= n\left[\left(1 + \frac{j_m}{m}\right)^{m \div n} - 1\right] \\ \text{with} \quad n &= 12 \\ j_m &= 0,135 \\ m &= 2 \end{aligned}$$

$$\begin{aligned} i &= 12\left[\left(1 + \frac{0,135}{2}\right)^{(2 \div 12)} - 1\right] \\ &= 0,13135... \\ &= 13,135...\% \end{aligned}$$

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ \text{with} \quad R &= 500 \\ n &= 8 \times 12 \\ i &= 0,13135... \\ S &= 500s_{\overline{8 \times 12}|0,13135... \div 12} \\ &= 84\,218,28 \end{aligned}$$

5. (a) Value of the flat:

$$
\begin{aligned}
P &= Ra_{\overline{n}|i} \\
&= 2\,500a_{\overline{20\times12}|0,1475\div12} \\
&= 192\,551,30
\end{aligned}
$$

They can afford a flat for R192 551,30 plus R100 000 deposit that is R292 551,30.

(b) After eight years they have made $8 \times 12 = 96$ payments. The present value of the loan at that stage is:

$$
\begin{aligned}
P &= Ra_{\overline{n}|i} \\
&= 2\,500a_{\overline{(20-8)\times12}|0,1475\div12} \\
&= R168\,370,01
\end{aligned}
$$

Their equity in the flat is R124 181,29 $(192\,551,30 - 168\,370,01 + 100\,000)$

6. Sinking fund:

Semi-annually payments:	R5 000
Interest:	16% per year
Time:	7 years

Accumulated value will be:

$$
\begin{aligned}
S &= Rs_{\overline{n}|i} \\
&= 5\,000s_{\overline{7\times2}|0,16\div2} \\
&= 121\,074,60
\end{aligned}
$$

The balance will be R121 074,60.

7.

$$
\begin{aligned}
\text{Price} &= \text{Deposit} + Ra_{\overline{n}|i} \\
&= 200\,000 + 10\,000a_{\overline{5\times12}|0,12\div12} \\
&= 200\,000 + 449\,550,38 \\
&= 649\,550,38
\end{aligned}
$$

The cash price of the house is R649 550,38.

1.2.4   Solution to self-evaluation exercise 4

1. **Internal rate of return:**

   A: $f(I) = \frac{400}{1+I} + \frac{300}{(1+I)^2} + \frac{350}{(1+I)^3} - 800 = 0.$

   The IRR = 15,37%.

   B: $f(I) = \frac{200}{1+I} + \frac{500}{(1+I)^2} + \frac{450}{(1+I)^3} - 750 = 0.$

   The IRR = 21,82%.

   As the internal rate of return $> K$ (cost of capital) for project B, invest in B.

   Net Present Value:

   A: $N = \frac{400}{(1+0,19)} + \frac{300}{(1+0,19)^2} + \frac{350}{(1+0,19)^3}, - 800.$

   The NPV $= -44$.

   B: $N = \frac{200}{(1+0,19)} + \frac{500}{(1+0,19)^2} + \frac{450}{(1+0,19)^3} - 750.$

   The $NPV = 38$.

   Advise B over A since it has a greater NPV.

   Profitability index:

   A: $PI_A = \dfrac{\text{NPV} + \text{Outlay}}{\text{Outlay}} = \frac{-44 + 800}{800} = 0{,}945.$

   B: $PI_B = \dfrac{\text{NPV} + \text{Outlay}}{\text{Outlay}} = \frac{38 + 750}{750} = 1{,}051.$

   $PI_B > 1$ therefore select B.

2. (a) Calculate the present value of the cash outlays:

   Shoe:
   $$\begin{aligned} I &= 100 + \frac{50}{(1+0,165)^2} \\ &= 100 + 36{,}84 \\ &= 136{,}84 \end{aligned}$$

   The present value is R136,84.

   CD:
   $$I = 400$$

   (b) Calculate the future value of the cash inflows at the end of the project.

   Shoe:
   $$\begin{aligned} C &= 50\,(1+0,19)^2 + 75 \\ &= 70{,}81 + 75 \\ &= 145{,}81 \end{aligned}$$

   The future value is R145,81.

   CD:
   $$\begin{aligned} C &= 75\,(1+0,19)^2 + 100\,(1+0,19) \\ &= 106{,}21 + 119{,}00 + 400 \\ &= 625{,}21 \end{aligned}$$

   The future value is R625,21.

(c) Calculate the MIRR

$$\text{MIRR} = \left[\left(\frac{C}{PV_{out}}\right)^{\frac{1}{n}} - 1\right]$$

Shoe:

$$\begin{aligned}
\text{MIRR} &= \left(\frac{145{,}81}{136{,}84}\right)^{\left(\frac{1}{3}\right)} - 1 \\
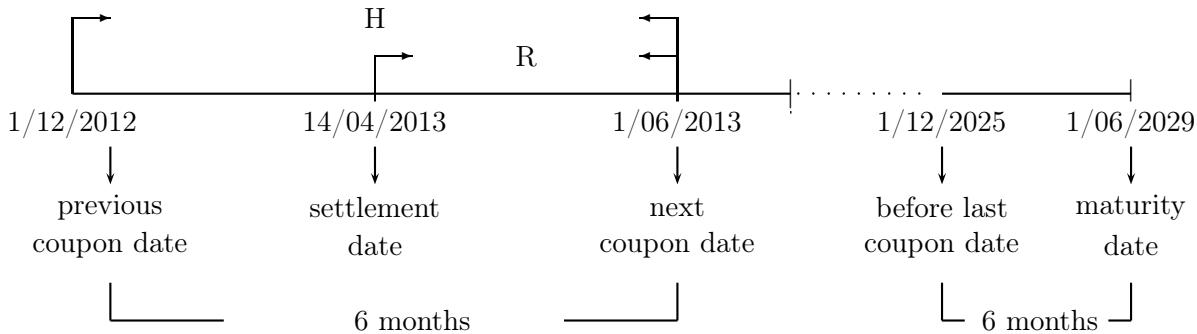&= 2{,}13\% < 19\%.
\end{aligned}$$

CD:

$$\begin{aligned}
\text{MIRR} &= \left(\frac{625{,}21}{400}\right)^{\left(\frac{1}{3}\right)} - 1 \\
&= 16{,}05\% < 19\%.
\end{aligned}$$

Since both options MIRR values are smaller than 19%, not one of the two options is advisable because he can earn more interest if he invests his money at 19%.

1.2.5   Solution to self-evaluation exercise 5

1.

The number of half yearly coupon periods is

$$
\begin{aligned}
\text{Years} \;&=\; 1/06/2013 \text{ to } 1/06/2029 \\
&=\; (2029 - 2013) \\
&=\; 16
\end{aligned}
$$

We multiply by 2 to get the number of half yearly coupons – thus 32 (16×2).

The number of days from the settlement date until the next coupon (interest) date is R:

The day number 152 (1 June) minus day number 104 (14 April) equals 48. Thus $R = 48$.

The number of days in the half year in which the settlement date falls ($1/12/2012$ to $1/06/2013$) is H.

Day number 365 (31 December) minus 335 (1 December) plus 152 (1 June) equals 182. Thus $H = 182$.

The present value of the bond on $1/06/2013$ is:

$$
\begin{aligned}
P \;&=\; da\,\overline{_{n|}}_z + 100(1 + z)^{-n} \\
&=\; \tfrac{16,5}{2}a\,\overline{_{32|}}_{0,142 \div 2} + 100\left(1 + \tfrac{0,142}{2}\right)^{-32} \\
&=\; 114{,}39343.
\end{aligned}
$$

Since the settlement date is more than ten days from the next coupon (interest) date it is a cum interest case and we must add the coupon.

$$
\begin{aligned}
P(1/06/2013) \;&=\; 114{,}39343 + 8{,}25 \\
&=\; 122{,}64343\%
\end{aligned}
$$

We must now discount this present value of the bond back to the settlement date to obtain the all-in-price.

$$
\begin{aligned}
\text{All-in-price} \;&=\; 122{,}64343 \times \left(1 + \tfrac{0,142}{2}\right)^{-\left(\frac{48}{182}\right)} \\
&=\; 120{,}44471\%
\end{aligned}
$$

The all-in price is R120,44471%.

The accrued interest:

$$
\begin{aligned}
&= \quad \frac{H-R}{365} \times c \\
&= \quad \frac{182-48}{365} \times 16,5 \\
&= \quad 6,05753
\end{aligned}
$$

The accrued interest is R6,05753%.

$$
\begin{aligned}
\text{Clean price} \quad &= \quad \text{All-in price – accrued interest} \\
&= \quad 120,44471 - 6,05753 \\
&= \quad 114,38718
\end{aligned}
$$

The clean price is 114,38718%.

2. The settlement date is 25 May 2013.

   The price on the next interest date (1 June 2013):

   $P(1 \text{ June } 2013) = \text{R}114,39343\%$ - see solution to question 1.

   Since this is an ex interest case no coupon must be added.

   The remaining number of days from 25 May 2013 to 1 June 2013 $(152 - 145)$:

   $$R = 7$$

   The number of days in the half year 1 December 2012 to 1 June 2013:

   $$H = 182$$

   Thus the fraction of the half year for discounting:

   $$f = \frac{7}{182}$$

   The all-in price is:

   $$
   \begin{aligned}
   P \quad &= \quad 114,39343 \times \left(1 + \frac{0,142}{2}\right)^{-\left(\frac{7}{182}\right)} \\
   &= \quad 114,09204
   \end{aligned}
   $$

   The all-in price is R114,09204%.

   The accrued interest:

   $$
   \begin{aligned}
   &= \quad \frac{-R}{365} \times c \\
   &= \quad \frac{-7}{365} \times 16,5 \\
   &= \quad -0,31644
   \end{aligned}
   $$

   The accrued interest is $-$R0,31644%

   $$
   \begin{aligned}
   \text{Clean price} \quad &= \quad \text{All-in price} - \text{Accrued interest} \\
   &= \quad 114,09204 - (-0,31644) \\
   &= \quad 114,40848
   \end{aligned}
   $$

   The clean price is R114,40848%.

1.2.6 Solution to self-evaluation exercise 6

1. This is an example of a weighted mean calculation where the wages are the data values and the number of workers in each field is the weights. The weighted mean is

$$
\begin{aligned}
\bar{x}_w &= \frac{\sum_{i=1}^{3} x_i w_i}{\sum_{i=1}^{3} w_i} \\
&= \frac{x_1 w_1 + x_2 w_2 + x_3 w_3}{w_1 + w_2 + w_3} \\
&= \frac{(28{,}41 \times 4{,}4) + (27{,}50 \times 1) + (26{,}65 \times 6{,}2)}{4{,}4 + 1 + 6{,}2} \\
&= 27{,}39.
\end{aligned}
$$

2. (a) The arithmetic mean for student A is

$$
\begin{aligned}
\bar{x} &= \frac{\sum_{i=1}^{3} x_i}{3} \\
&= \frac{x_1 + x_2 + x_3}{3} \\
&= \frac{81 + 88 + 83}{3} \\
&= \frac{252}{3} \\
&= 84.
\end{aligned}
$$

The arithmetic mean for student B is

$$
\begin{aligned}
\bar{x} &= \frac{252}{3} \\
&= 84.
\end{aligned}
$$

There is no choice between student A and B because they have the same arithmetic mean.

(b) The standard deviation for student A is

41

$$S = \sqrt{\frac{\sum_{i=1}^{3}(x_i - \bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2}{3-1}}$$

$$= \sqrt{\frac{(81-84)^2 + (88-84)^2 + (83-84)^2}{2}}$$

$$= \sqrt{\frac{9+16+1}{2}}$$

$$= \sqrt{13}$$

$$= 3{,}61.$$

The standard deviation for student B is

$$S = \sqrt{\frac{1+81+64}{2}}$$
$$= \sqrt{73}$$
$$= 8{,}54.$$

These calculations can be done directly on your calculator. See Notes on the calculator for the key operations.

Student A will be selected because he has a smaller standard deviation than student B. His performance is more stable than that of student B.

3. (a) It looks as if there exists a positive linear correlation between average interest rate and yearly investment. This means that if the average interest rate increases, then yearly investment will also increase.
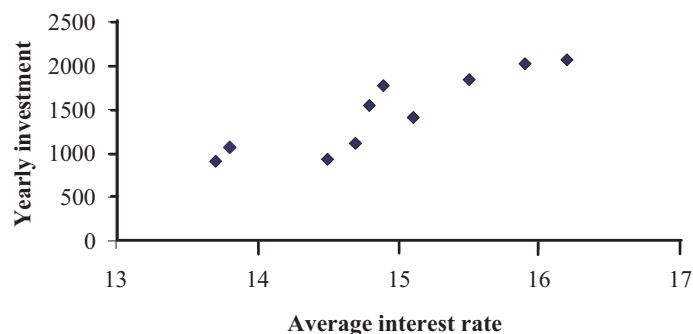
(b) **You must do these calculations on your calculator using the statistical functions directly. You do not need to do the following in-between steps for the calculations.**

Year i	Average interest x_i	Yearly investment y_i	x_i^2	$x_i y_i$	y_i^2
1	13,8	1 060	190,44	14 628	1 123 600
2	14,5	940	210,25	13 630	883 600
3	13,7	920	187,69	12 604	846 400
4	14,7	1 110	216,09	16 317	1 232 100
5	14,8	1 550	219,04	22 940	2 402 500
6	15,5	1 850	240,25	28 675	3 422 500
7	16,2	2 070	262,44	33 534	4 284 900
8	15,9	2 030	252,81	32 277	4 120 900
9	14,9	1 780	222,01	26 522	3 168 400
10	15,1	1 420	228,01	21 442	2 016 400
$n = 10$	149,1	14 730	2 229,03	222 569	23 501 300

The coefficient of correlation is

$$
\begin{aligned}
r &= \frac{n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \sqrt{n \sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2}} \\
&= \frac{10 \, (222\,569) - (149{,}1)\,(14\,730)}{\sqrt{10 \, (2\,229{,}03) - (149{,}1)^2} \sqrt{10 \, (23\,501\,300) - (14\,730)^2}} \\
&= \frac{29\,447}{32\,759{,}8161} \\
&= 0{,}8989.
\end{aligned}
$$

(c) The coefficient of determination is $r^2 = 0{,}8989^2 = 0{,}8080$. This means that almost 81% of the variation in yearly investments can be declared by the average interest rate.

(d) The equation of the straight line is $y = a + bx$ where

$$
\begin{aligned}
b &= \frac{\sum_{i=1}^{10} x_i y_i - \sum_{i=1}^{10} x_i \sum_{i=1}^{10} y_i}{n \sum_{i=1}^{10} x_i^2 - \left(\sum_{i=1}^{10} x_i\right)^2} \\
&= \frac{10(222\,569) - (149{,}1)(14\,730)}{10(2\,229{,}03) - (149{,}1)^2} \\
&= \frac{29\,447}{59{,}49} \\
&= 494{,}99
\end{aligned}
$$

and

$$
\begin{aligned}
a &= \frac{\sum_{i=1}^{10} y_i}{n} - \frac{b \sum_{i=1}^{10} x_i}{n} \\
&= \frac{14\ 730}{10} - \frac{(494{,}99)(149{,}1)}{10} \\
&= -5\ 907{,}30.
\end{aligned}
$$

Thus $\qquad y = -5\ 907{,}30 + 494{,}99x.$

NOTE: Use the statistical functions on your calculator for these calculations. It is much easier.

(e) Although an interest rate of 16,5% is not in the span of x-values, it is not too far from the rest of the x-values, and because the coefficient of correlation is large enough, we can forecast the corresponding yearly investment (extrapolate). The yearly investment for an interest rate of 16,5% is

$$
\begin{aligned}
y &= -5\,907{,}30 + 494{,}99 \times 16{,}5 \\
&= 2\,260{,}04.
\end{aligned}
$$

1.2.7   Solution to self-evaluation exercise 7

<div align="center">TYPICAL EXAM QUESTIONS</div>

1.

$$
\begin{aligned}
S &= P(1+rt) \\
\text{with } P &= 420 \\
r &= 0{,}075 \\
t &= (156-52) \\
&= \frac{104}{365}
\end{aligned}
$$

$$
\begin{aligned}
S &= 420\left(1+0{,}075 \times \frac{104}{365}\right) \\
&= 428{,}98
\end{aligned}
$$

Little John can withdraw R428,98.

2.

$$
\begin{aligned}
i &= n\left(\left(1+\frac{j_m}{m}\right)^{\frac{m}{n}} -1\right) \\
\text{with } j_m &= 0{,}164 \\
m &= 4 \\
n &= 52.
\end{aligned}
$$

$$
\begin{aligned}
i &= 52\left(\left(1+\frac{0{,}164}{4}\right)^{\frac{4}{52}} -1\right) \\
&= 0{,}16098 \\
&= 16{,}098\%.
\end{aligned}
$$

The equivalent interest rate is 16,098%.

3. This is an annuity due problem due to the fact that the word immediately is in the sentence.

$$
\begin{aligned}
S &= (1+i)\,Rs_{\overline{n}|i} \\
\text{with } I &= 0{,}0909 \div 12 \\
N &= 9 \times 12 \\
S &= 12\,500 \div \left(1+\frac{0{,}0909}{12}\right) \\
12\,500 &= \left(1+\frac{0{,}0909}{12}\right) Rs_{\overline{9\times12}|0{,}0909\div12} \\
R &= 74{,}63
\end{aligned}
$$

The monthly payments are R74,63.

4. The quarterly interest rate must first be converted to monthly compounded.

$$i = n\left(\left(1 + \frac{j_m}{m}\right)^{\frac{m}{n}} - 1\right)$$

$$\text{with } j_m = 0{,}149$$
$$m = 4$$
$$n = 12$$

$$i = 12\left(\left(1 + \frac{0{,}149}{4}\right)^{\frac{4}{12}} - 1\right)$$
$$= 0{,}14718...$$

$$S = Rs_{\overline{n}|i}$$
$$\text{with } I = 0{,}14718 \div 12$$
$$N = 12 \times 12$$
$$R = 1\,000$$
$$S = 1\,000s_{\overline{12 \times 12}|0{,}14718 \div 12}$$
$$= 390\,225{,}94$$

The accumulated amount is R390 225,94.

5.
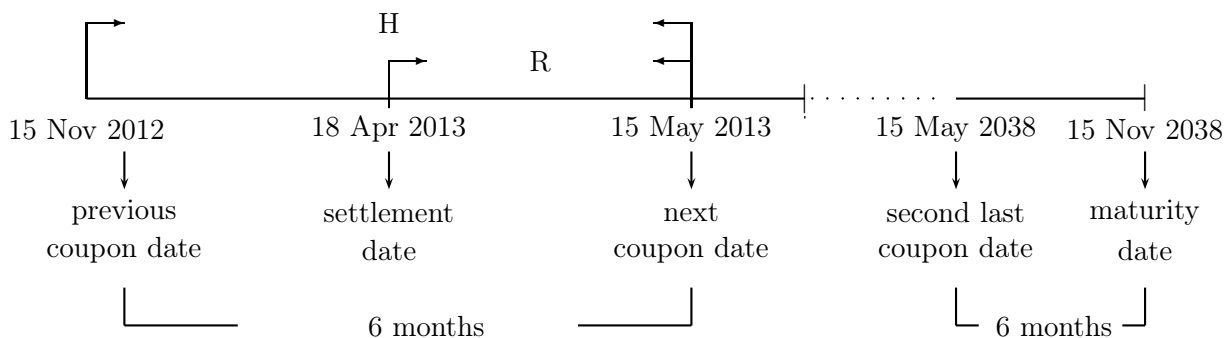
In order to calculate the number of coupons still outstanding we first determine the number of years – from the next coupon date to the maturity date – and then multiply it by two to get the number of half years. As the months May and November differ and we want to calculate the number of years we move the next coupon date six months on to 15/11/2013.

$$
\begin{aligned}
\text{Years} &= 15/11/2038 - 15/11/2012 \\
&= 25.
\end{aligned}
$$

This 25 is the number of years in which the coupon payments will be made. We must multiply this now by (2).

Thus

$$
\begin{aligned}
n &= 25 \times 2 \\
&= 50.
\end{aligned}
$$

Our calculations were done from 15/11/2012 to 15/11/2038 but the next coupon date that follows the settlement date is 15/05/2013. We must therefore add one (1) to the n.

$$
\begin{aligned}
n &= 50 + 1 \\
&= 51.
\end{aligned}
$$

The number of days from the settlement date 18/04/2013 to the next coupon date 15/05/2013 is R: Day number 135 (15 May) minus day number 108 (18 April) equals 27, thus $R = 27$.

The number of days in the half year in which the settlement date falls, (15/11/2012 to 15/05/2013) is H. Day number 365 (31 December) minus day number 319 (15 November) plus day number 135(15 May) equals 181, thus $H = 181$.

The present value of the bond on 15/05/2013 is:

$$\begin{aligned} P &= da\,\overline{\lceil n \rceil}z + 100(1+z)^{-n} \\ &= \frac{11{,}59}{2}a\,\overline{\lceil 51 \rceil}0{,}0946\div 2 + 100\left(1+\frac{0{,}0946}{2}\right)^{-51} \\ &= 120{,}38349 \end{aligned}$$

As the settlement date is more than ten days from the next coupon date, we add a coupon $-$ cum interest case.

$$\begin{aligned} P &= 120{,}38349 + 5{,}795 \\ &= 126{,}17849 \end{aligned}$$

We must now discount the present value of the bond back to the settlement date to obtain the all-in price.

$$\begin{aligned} \text{All-in price} &= 126{,}17849\left(1+\frac{0{,}0946}{2}\right)^{-\frac{27}{181}} \\ &= 125{,}31160 \end{aligned}$$

The all-in price is R125,31160%.

6.

$$\begin{aligned} \text{The accrued interest} &= \frac{H-R}{365} \times c \\ &= \frac{181-27}{365} \times 11{,}59 \\ &= 4{,}89003 \end{aligned}$$

The accrued interest is R4,89003%.

7.

$$\begin{aligned} \text{Clean price} &= \text{All-in price} - \text{accrued interest} \\ &= 125{,}31160 - 4{,}89003 \\ &= 120{,}42157 \end{aligned}$$

The clean price for one bond is R120,42157%. The given nominal value is R750 000 therefore 7 500 bonds were bought. The clean price is R903 162 ($7\,500 \times 120{,}42157$).

8.

$$\begin{aligned} \text{PI} &= \frac{\text{NPV} + \text{initial investment}}{\text{initial investment}} \\ 1{,}083 &= \frac{1\,255 + \text{initial investment } (x)}{\text{initial investment } (x)} \\ 1{,}083x &= 1\,255 + x \\ 1{,}083x - x &= 1\,255 \\ 0{,}083x &= 1\,255 \\ x &= \frac{1\,255}{0{,}083} \\ &= 15\,120{,}48 \end{aligned}$$
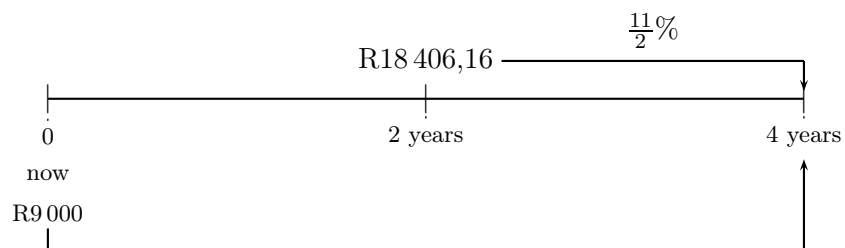
The initial investment is R15 120,48.

9.
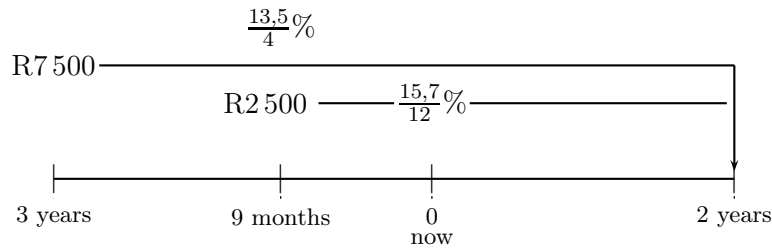
$$
\begin{aligned}
\text{The amount due} \;&=\; 7\,500\left(1+\frac{0{,}135}{4}\right)^{5\times4}+2\,500\left(1+\frac{0{,}157}{12}\right)^{2{,}75\times12} \\
&=\; 14\,567{,}05 + 3\,839{,}11 \\
&=\; 18\,406{,}16
\end{aligned}
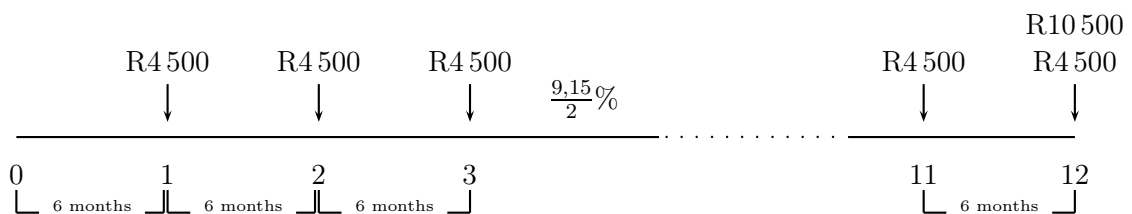$$

The amount due is R18 406,16.

10.

$$
\begin{aligned}
\text{Payments} \;&=\; \text{Obligations} \\
9\,000\left(1+\frac{0{,}11}{2}\right)^{4\times2}+X \;&=\; 18\,406{,}16\left(1+\frac{0{,}11}{2}\right)^{2\times2} \\
X \;&=\; 22\,802{,}01 - 13\,812{,}18 \\
&=\; 8\,989{,}83.
\end{aligned}
$$

The amount is R8 989,83.

11.

49

$$
\begin{aligned}
\text{PV} &= Ra_{\overline{n}|i} \\
&= 4\,500\,a_{\,\overline{6\times2}|0,0915\div2} \\
&= 40\,858{,}13
\end{aligned}
$$

We must discount the R10 000 back to now.

$$
\begin{aligned}
S &= P\left(1 + \frac{j_m}{m}\right)^{tm} \\
10\,500 &= P\left(1 + \frac{0{,}0915}{2}\right)^{6\times2} \\
P &= 6\,138{,}39
\end{aligned}
$$

$$
\begin{aligned}
\text{Thus total amount} &= PV + P \\
&= 40\,858{,}13 + 6\,138{,}39 \\
&= 46\,996{,}52
\end{aligned}
$$

The total amount is R46 996,52.

12.

$$
\begin{aligned}
\text{MIRR} &= \left(\frac{C}{PV_{\text{out}}}\right)^{\frac{1}{n}} - 1 \\
\text{with} \quad M &= 10{,}81\% \\
P &= 291\,930 \\
N &= 8
\end{aligned}
$$

$$
\begin{aligned}
0{,}1081 &= \left(\frac{C}{291\,930}\right)^{\frac{1}{8}} - 1 \\
1{,}1081 &= \left(\frac{C}{291\,930}\right)^{\frac{1}{8}} \\
C &= (1{,}1081)^8 \times 291\,930 \\
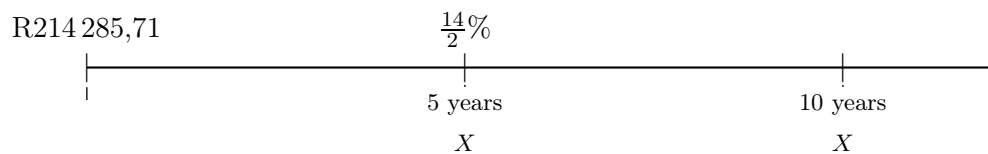&= 663\,606{,}09 \\
&\approx 663\,600{,}00
\end{aligned}
$$

The future value of the cash inflows is R663 600,00.

13.

$$P = \frac{R}{i}$$

$$\text{with } R = 2\,500$$

$$i = 0{,}14 \div 12$$

$$P = \frac{2\,500}{0{,}14 \div 12}$$

$$= 214\,285{,}71$$

The opening balance is R214 285,71.

14.

R214 285,71 $\frac{14}{2}\%$ 5 years   10 years   X   X

$$\text{Payments} = \text{Obligations}$$

$$214\,285{,}71 = X\left(1 + \frac{0{,}14}{12}\right)^{-5\times 12} + X\left(1 + \frac{0{,}14}{12}\right)^{-10\times 12}$$

$$= X\left[\left(1 + \frac{0{,}14}{12}\right)^{-60} + \left(1 + \frac{0{,}14}{12}\right)^{-120}\right]$$

$$X = \frac{214\,285{,}71}{\left[\left(1 + \frac{0{,}14}{12}\right)^{-60} + \left(1 + \frac{0{,}14}{12}\right)^{-120}\right]}$$

$$= 286\,783{,}06.$$

The present value is R286 738,06.

15. Using your calculator directly the equation for the regression line is

$$y = 48\,644{,}17 - 6\,596{,}93x$$

16. The correlation coefficient is $r = -0{,}9601$

1.2.8 Solution to self-evaluation exercise 8

TYPICAL EXAM QUESTIONS

1.

$$
\begin{aligned}
S &= P(1 + rt) \\
P &= 2\,000 \\
r &= 8\% \\
t &= 7
\end{aligned}
$$

$$
\begin{aligned}
S &= 2\,000(1 + 0{,}08 \times 7) \\
P &= 3\,120{,}00
\end{aligned}
$$

James owes R3 120,00.

2.

$$
\begin{aligned}
P &= R a_{\overline{n}|i} \\
R &= 700 \\
n &= 3 \times 12 \\
i &= 14{,}5\% \div 12
\end{aligned}
$$

$$
\begin{aligned}
P &= 700 a_{\overline{3 \times 12}|0,145 \div 12} \\
&= 20\,336{,}44
\end{aligned}
$$

The original price was R25 336,44 (20 336,44 + 5 000).

3.

$$
\begin{aligned}
S &= P(1 + rt) \\
115 &= 100(1 + 0{,}08 \times t) \\
\frac{115}{100} &= 1 + \frac{8}{100}t \\
\frac{8}{100}t &= \frac{115}{100} - 1 \\
t &= \frac{100}{8}\left(\frac{115}{100} - 1\right)
\end{aligned}
$$

4.

$$
\begin{aligned}
P &= S(1 - dt) \\
P &= 14\,500 \\
d &= 28\% \\
t &= \frac{10}{12} \\
14\,500 &= S\left(1 - 0{,}28 \times \frac{10}{12}\right) \\
S &= \frac{14\,500}{1 - 0{,}28 \times \frac{10}{12}} \\
&= 18\,913{,}04
\end{aligned}
$$

Jonas must pay R18 913,04.

5.

$$
\begin{aligned}
c &= m\ln\left(1 + \frac{j_m}{m}\right) \\
0{,}11832 &= 4\ln\left(1 + \frac{j_m}{4}\right) \\
\frac{0{,}11832}{4} &= \ln\left(1 + \frac{j_m}{4}\right) \\
e^{\frac{0{,}11832}{4}} &= 1 + \frac{j_m}{4} \\
j_m &= 4\left[\left(e^{\frac{0{,}11832}{4}}\right) - 1\right] \\
\\
&= 12{,}01\%
\end{aligned}
$$

The nominal rate is 12,01%.

6.

$$
\begin{aligned}
S &= Pe^{ct} \\
32\,850 &= 25\,000 e^{c \times \frac{39}{12}} \\
e^{c \times \frac{39}{12}} &= \frac{32\,850}{25\,000} \\
\frac{39}{12} c\ln e &= \ln\left(\frac{32\,850}{25\,000}\right) \\
c &= \ln\left(\frac{32\,850}{25\,000}\right) \times \frac{12}{39} \\
&= 8{,}4\%
\end{aligned}
$$

7.

$$
\begin{aligned}
P &= Ra_{\overline{n}|i} \\
250\,000 &= Ra_{\overline{6\times12}|0{,}118 \div 12} \\
R &= 4\,861{,}59
\end{aligned}
$$

Amount outstanding:

$$
\begin{aligned}
P &= 4\,861{,}59 a\,\overline{_{72-33}}|_{0,118\div12} \\
&= 156\,848{,}15
\end{aligned}
$$

The amount paid off is R93 151,85 $(250\,000 - 156\,848{,}15)$.

Please note: If you enter the value for PMT as $4\,861{,}59$ your answer will be R156 848,15. If you however continue with the calculations without re-entering the value for the payment your answer will be R156 848,01.

8.

$$
\begin{aligned}
J_\alpha &= 100(e^c - 1) \\
&= 100(e^{0,175} - 1) \\
&= 19{,}12\%
\end{aligned}
$$

9.

$$
\begin{aligned}
S &= P\left(1 + \frac{j_m}{m}\right)^{t_m} \\
&= 25\,000\left(1 + \frac{0{,}0975}{4}\right)^{5\times4} \\
&= 40\,468{,}72
\end{aligned}
$$

The balance in the account is R40 468,72.

10.

$$
\begin{aligned}
S &= P\left(1 + \frac{j_m}{m}\right)^{t_m} + R s\,\overline{_n}|_i \\
&= 40\,468{,}72\left(1 + \frac{0{,}10}{52}\right)^{4\times52} + 500 s\,\overline{_{4\times52}}|_{0,10\div52} \\
&= 60\,349{,}05 + 127\,725{,}46 \\
&= 188\,074{,}51
\end{aligned}
$$

Tracy can expect to withdraw R188 074,51.

11.

$$
\begin{aligned}
P &= (1+i)R a\,\overline{_n}|_i \\
&= (1+i)1\,403 a\,\overline{_{24}}|_{0,20124\div12} \\
&= (1+i)27\,533{,}34 \\
&= 27\,995{,}08.
\end{aligned}
$$

The original price of the television set was R27 995,08.

12.

$$
\begin{aligned}
S &= P(1+rt) \\
P &= S(1-dt) \\
S &= S(1-dt)(1+rt) \\
\frac{S}{S} &= (1-dt)(1+rt) \\
1+rt &= \frac{1}{(1-dt)} \\
rt &= \frac{1}{1-dt} - 1 \\
&= \frac{1-1(1-dt)}{(1-dt)} \\
&= \frac{1-1+dt}{1-dt} \\
rt &= \frac{dt}{1-dt} \\
r &= \frac{dt}{1-dt} \times \frac{1}{t} \\
r &= \frac{d}{1-dt} \\
1-dt &= \frac{d}{r} \\
dt &= 1 - \frac{d}{r} \\
t &= \left(1 - \frac{d}{r}\right)/d \\
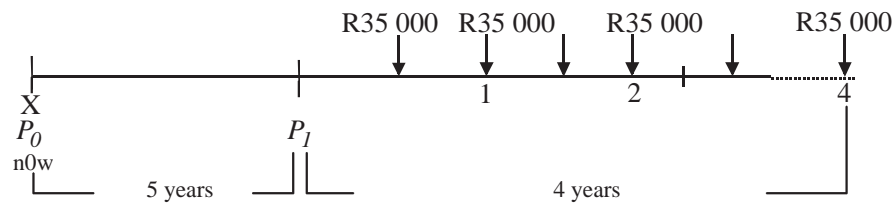t &= \left(1 - \frac{0{,}075}{0{,}0968}\right) \div 0{,}075 \\
t &= 3
\end{aligned}
$$

The time under consideration is 3 years.

13.

$$
\begin{aligned}
PI &= \frac{NPV + \text{original outlay}}{\text{original outlay}} \\
1{,}034 &= \frac{14\,983 + \text{outlay}}{\text{outlay}} \\
1{,}034 \text{ outlay} &= 14\,983 + \text{outlay} \\
1{,}034 \text{ outlay} - \text{outlay} &= 14\,983 \\
0{,}034 \text{ outlay} &= 14\,983 \\
\text{outlay} &= \frac{14\,983}{0{,}034} \\
&= 440\,676{,}47
\end{aligned}
$$

The original outlay was R440 676,47.

14.

$$
\begin{aligned}
P_1 &= Ra_{\overline{n}|i} \\
i &= 0{,}179 \div 2 \\
n &= 4 \times 2 \\
R &= 35\,000 \\
P_1 &= 35\,000\,a_{\overline{4\times2}|0{,}179\div2} \\
&= 194\,079{,}19
\end{aligned}
$$

Charlene owes Aunt Amor R194 079,19.

15. This amount must be discounted back to the time that aunt Amor gave Charlene the money.

$$
\begin{aligned}
S &= P\left(1 + \frac{j_m}{m}\right)^{tm} \\
j_m &= 0{,}179 \\
m &= 2 \\
S &= 194\,079{,}19 \\
t &= 5 \\
P_0 &= 194\,079{,}19\left(1 + \frac{0{,}179}{2}\right)^{-5\times2} \\
&= 82\,358{,}16
\end{aligned}
$$

Aunt Amor lent Charlene R82 358,16.

16.

$$
\begin{aligned}
P &= \frac{14{,}7}{2}a_{\overline{29}|0{,}135\div2} + 100\left(1 + \frac{0{,}135}{2}\right)^{-29} \\
&= 107{,}55174
\end{aligned}
$$

This is a cum-interest case due to the fact that $R = 74$.

$$
\begin{aligned}
\text{Price}(01/07/2012) &= 107{,}55174 + 7{,}35 \\
&= 114{,}90170
\end{aligned}
$$
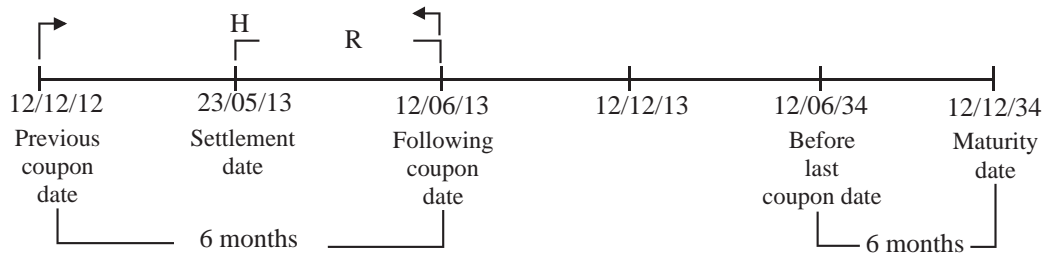
This amount must be discounted back to the settlement date.

$$P = 114{,}90170 \left(1 + \frac{0{,}135}{2}\right)^{-74/181}$$
$$= 111{,}87388$$

$$\text{Clean price} = 111{,}87388 - 4{,}30932$$
$$= 107{,}56456$$

The clean price is R107,56456%.

17.

$$\text{Years} = 12/12/34 - 12/12/13$$
$$= 21.$$

We must multiply by two to get the number of half yearly coupons and add one to get the n.

R is the number of days from the settlement date until the next coupon date.

$$R = 163(12 \text{ June}) - 143(23 \text{ May}) = 20$$

H is the number of days in the half year in which the settlement date falls. (The number of days from the coupon date just before the settlement date until the coupon date that follows the settlement date.)
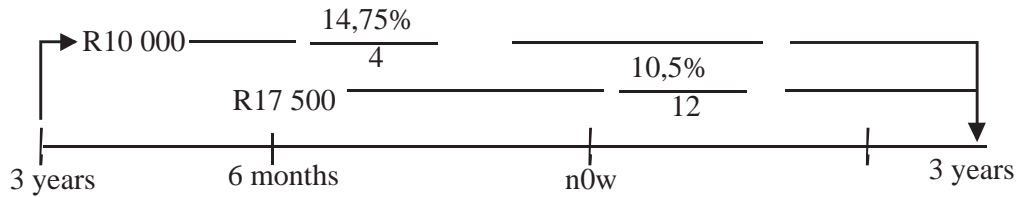
$$H = 365 - 346 + 163 = 182$$

$$P = da\,\overline{_{n|}}_z + 100(1 + z)^{-n}$$
$$= \frac{12{,}4}{2}a\,\overline{_{43|}}_{0{,}108 \div 2} + 100\left(1 + \frac{0{,}108}{2}\right)^{-43}$$
$$= 113{,}27116$$

The present value on 12 June 2013 is R113,27116% and it is cum-interest. Add the coupon $-113{,}27116+ 6{,}2 = 119{,}47116$. This amount must be discounted back to the settlement date 23 May 2013.

$$P(23/05/13) = 119{,}47116\left(1 + \frac{0{,}108}{2}\right)^{-\frac{20}{182}}$$
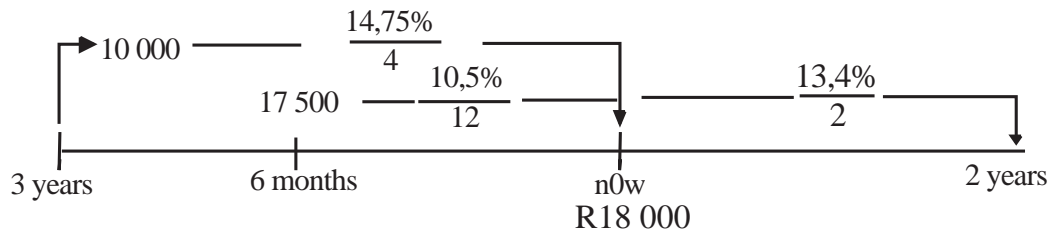$$= 118{,}78268$$

The present value is R118,78268%.

18.

$$
\begin{aligned}
S &= P_1\left(1+\frac{j_m}{m}\right)^{tm} + P_2\left(1+\frac{j_m}{m}\right)^{tm} \\
&= 10\,000\left(1+\frac{0{,}1475}{4}\right)^{6\times4} + 17\,500\left(1+\frac{0{,}105}{12}\right)^{3{,}5\times12} \\
&= 23\,847{,}00 + 25\,231{,}73 \\
&= 49\,078{,}73
\end{aligned}
$$

Daniel owes Sarah R49 078,73 three years' from now.

19.

$$
\begin{aligned}
S &= 10\,000\left(1+\frac{0{,}1475}{4}\right)^{3\times4} + 17\,500\left(1+\frac{0{,}105}{12}\right)^{0{,}5\times12} \\
&= 15\,442{,}47 + 18\,439{,}08 \\
&= 33\,881{,}55
\end{aligned}
$$

$$
\begin{aligned}
\text{Amount due} &= 33\,881{,}55 - 18\,000 \\
&= 15\,881{,}55
\end{aligned}
$$

$$
\begin{aligned}
S &= P\left(1+\frac{j_m}{m}\right)^{tm} \\
&= 15\,881{,}55\left(1+\frac{0{,}134}{2}\right)^{2\times2} \\
&= 20\,584{,}99
\end{aligned}
$$

Amount payable two years from now is R20 584,99.

20. We must first convert the monthly interest rate to semi-annually.

$$
\begin{aligned}
i &= n\left(\left(1+\frac{j_m}{m}\right)^{m\div n}-1\right) \\
&= 2\left(\left(1+\frac{0{,}089}{12}\right)^{12\div 2}-1\right) \\
&= 0{,}09067...
\end{aligned}
$$

$$
\begin{aligned}
S &= Rs_{\overline{n}\rceil i} \\
&= 5\,500s_{\overline{10\times 2}\rceil 0{,}09067\div 2} \\
&= 173\,149{,}47
\end{aligned}
$$

The accumulated amount is R173 149,47.

21.
$$
S = \sqrt{\frac{\sum_{i=1}^{n}(x_1-\bar{x})^2}{n}}
$$

We use our calculator to do this question.

The standard deviation is R252 032.

22.
$$
\begin{aligned}
S &= \left(R+\frac{Q}{i}\right)s_{\overline{n}\rceil i}-\frac{nQ}{i} \\
R &= 3\,600 \\
Q &= 360 \\
i &= 0{,}10 \\
n &= 20
\end{aligned}
$$

$$
\begin{aligned}
S &= \left(3\,600+\frac{360}{0{,}10}\right)s_{\overline{20}\rceil 0{,}10}-\frac{20\times 360}{0{,}10} \\
&= 412\,380-72\,000 \\
&= 340\,380
\end{aligned}
$$

Fawzia can expect to receive R340 380.

23. The regression line equation is
$$
y = 8{,}5x+135
$$

We enter the data directly into our calculator.