

Tutorial Letter 202/1/2018

Introductory Financial Mathematics
DSC1630

Semester 1

Department of Decision Sciences

Important Information:

This tutorial letter contains the solutions of
Assignment 02


Bar code

Dear student

The solutions to the questions of the compulsory Assignment 02 are included in this tutorial letter. Study them when you prepare for the examination. I have added comments on how to approach each problem as well as a section on when to use which formula plus a list of formulas similar to the one you will receive in the exam.

You will also find the key operations for the SHARP EL-738/F/FB, HP10BII and the HP10BII+ calculators as well as the end of each answer. I hope it will help you to understand the workings of your calculator. **Just remember that these key operations are not the only method, these are just examples.** You can use any method as long as you get the same answer.

REMEMBER TO CLEAR ALL DATA FROM THE MEMORY BEFORE YOU ATTEMPT ANY CALCULATIONS.

- **SHARP EL-738/F/FB users:** Press 2ndF M-CLR 0 0 if you wish to clear all the memories and 2ndF CA if you just want to clear only the financial keys.
- **HP10BII and HP10BII+ users:** Press  C ALL

Remember to please contact me via email, telephone or appointment if you need help regarding the study material. Please note that only students with appointments will be assisted. My contact details and contact hours are:

Office: Theo van Wijk - Laboratory Building, Room 1-4, Preller street, Muckleneuk, Pretoria

Tel: Out of order at moment. Use e-mail but when up and running: +27 12 4334691

E-mail: immelmf@unisa.ac.za

07:00 until 13:30 - Monday till Friday: **Appointments and Telephone/email**

13:30 until 16:00 - Monday till Thursday: **Telephone or email**

Lastly everything of the best with Assignment 03. Please work through the self-evaluation exercises on myUnisa **before** you attempt to answer the assignments.

Kind regards,

Mrs Adèle Immelman

Note: the final answer displayed on your calculator depends on how many decimals your calculator's display is set to.

1 Solution Summary

The following is a summary of the correct answers:

Q 1	Option 1
Q 2	Option 2
Q 3	Option 3
Q 4	Option 3
Q 5	Option 3
Q 6	Option 4
Q 7	Option 1
Q 8	Option 2
Q 9	Option 2
Q 10	Option 5
Q 11	Option 3
Q 12	Option 4
Q 13	Option 1
Q 14	Option 3
Q 15	Option 3



2 Assignment 02 – Detailed Solution

Question 1

First we identify the type of problem. Both the words *compounded quarterly* and *continuous compounding rate* are found in the question. This is a conversion between two types of interest rates. We are asked to determine the continuous compounding rate that is equivalent to the nominal rate of 17,5% compounded quarterly. The formula for the continuous compounding interest rate, if the nominal interest rate j_m is given, is $c = m \ln(1 + \frac{j_m}{m})$. Now given are $m = 4$ and $j_m = 0,175$. Thus

$$\begin{aligned} c &= m \ln\left(1 + \frac{j_m}{m}\right) \\ &= 4 \ln\left(1 + \frac{0,175}{4}\right) \\ &= 0,171279\dots \end{aligned}$$

The equivalent continuous compounding rate is 17,128%.

EL-738 and EL-738F	HP10BII and HP10BII+
2ndF CA	 C ALL
Use normal keys	Use normal keys
4 2ndF ln [on the 2 key]	$0.175 \div 4 = +1 =$
$(1 + 0.175 \div 4) =$	 LN [on the 2 key]
0.17128... is displayed.	$\times 4 =$
Multiply with 100 to get %	0.17128... is displayed.
$\times 100 =$	Multiply with 100 to get %
17.1279... is displayed.	$\times 100 =$
	17.1279... is displayed.

[Option 1]

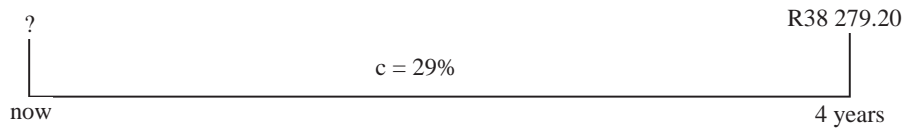
Question 2

This is a continuous compounding calculation as the term *compounded continuously* is found in the question. The formula for continuous compounding is

$$S = Pe^{ct}.$$

Note: The mathematical constant e is a unique real number. The number e is of considerable importance in mathematics. The numerical value of e truncated to 20 decimal places is 2,71828182845904523536. Some calculators have a specific key (for example e^x) to calculate the value of the power of e . You should be able to call up the specified value of e by entering e^1 into your calculator.

Let's draw the time line:



Now given are the future value or accumulated sum (S) that she needs to pay back in four years' time of R38 279,20, the continuous interest rate (c) of 29%, and the number of years of the loan (t) of four years. We need to determine the present value of the amount of money borrowed now, P . Thus

$$\begin{aligned}
 S &= Pe^{ct} \\
 38\,279,20 &= Pe^{(0,29 \times 4)} \\
 \frac{38\,279,20}{e^{(0,29 \times 4)}} &= P \\
 P &= 12\,000,00 \dots
 \end{aligned}$$

The initial amount borrowed is R12 000.

EL-738 and EL-738F	HP10BII and HP10BII+
2ndF CA	C ALL
Use normal keys	Use normal keys
$38\,279.20 \div 2\text{ndF } e^x$ [on the \pm key]	$0.29 \times 4 =$ e^x [on the 1 key]
$(0.29 \times 4) =$	$\rightarrow M$ [store in memory]
12 000.000... is displayed.	$38\,279.20 \div RM =$
	12 000.000... is displayed.

[Option 2]


Question 3

This is a conversion between two types of interest rates as it is asked to determine the *effective rate* that is equivalent to a *continuous compounding* rate (c) of 17,5%. The effective interest rate formula if the continuous compounding rate c is given, is $j_\infty = 100(e^c - 1)$.

Thus

$$\begin{aligned}
 j_\infty &= 100(e^c - 1) \\
 &= 100(e^{0,175} - 1) \\
 &= 19,12462 \dots \\
 &\approx 19,12
 \end{aligned}$$

The effective rate is 19,12%.

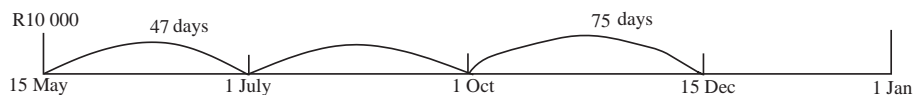
EL-738 and EL-738F	HP10BII and HP10BII+
2ndF CA	 C ALL
Use normal keys	Use normal keys
$100 \times (2\text{ndF } e^x \text{ [on the pm.eps key]})$	$0.175 \text{ } e^x \text{ [on the 1 key]}$
$0.175 - 1) =$	$-1 = \times 100 =$
19.12462... is displayed.	19.12462... is displayed.

[Option 3]

Question 4

This is an odd period calculation and the method to be used namely, *simple interest for odd periods and compound interest* for the full period, is given. To determine the odd periods it is very important to note when the interest is being paid. In this situation interest is paid quarterly starting at the beginning of the year. Interest is thus paid on 1 January, 1 April, 1 July and 1 October. The R10 000 is invested for a period of seven months, thus the amount is invested from 15 May till 15 December.

Now let's draw a time line of the situation:



There are two odd periods. One from 15 May to 1 July and one from 1 October to 15 December. If we use the days table in the Study guide, page 203, then the number of days between 15 May and 1 July is $182 - 135 = 47$ days, and the number of days between 1 October and 15 December is $349 - 274 = 75$ days.

The period between 1 July and 1 October consists of one full quarter period. If you count the interest dates you get 2, but remember ($t = \frac{3}{12}; m = 4$) the compounded period is between two dates, thus the value of tm is 1.

Now we use simple interest for odd periods and compound interest for the full periods. Thus the accumulated value at 15 Dec is:

$$\begin{aligned}
 & 10\,000 \left(1 + 0,15 \times \frac{47}{365} \right) \left(1 + \frac{0,15}{4} \right)^1 \left(1 + 0,15 \times \frac{75}{365} \right) \\
 & = 10\,901,34776 \dots
 \end{aligned}$$

The interest is equal to

$$\begin{aligned}
 I &= 10\,901,3477 \dots - 10\,000 \\
 &= 901,3477 \dots
 \end{aligned}$$

The amount of interest is equal to R901,35.

Alternatively you can split the calculation into three separate steps.

Note: do not round off during your calculations. Only round off the last answer.

Value at 1 July:

$$10\,000 \left(1 + 0,15 \times \frac{47}{365} \right) = 10\,193,15068 \dots$$

Value at 1 October:

$$10\,193,15068 \dots \left(1 + \frac{0,15}{4} \right)^1 = 10\,575,39384 \dots$$

Value at 15 December:

$$10\,575,39384 \dots \left(1 + 0,15 \times \frac{75}{365} \right) = 10\,901,34776 \dots$$

Now the interest is:

$$\begin{aligned} I &= 10\,901,34776 \dots - 10\,000 \\ &= 901,34776 \dots \end{aligned}$$

The amount of interest is R901,35.

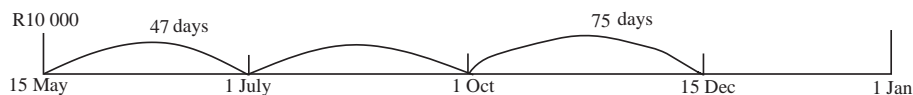
EL-738 and EL-738F	HP10BII and HP10BII+
2ndF CA	C ALL
<i>Use normal and financial keys</i>	<i>Use normal and financial keys</i>
$10\,000(1 + 0.15 \times 47 \div 365) =$	(1+ (0.15 x 47 ÷
$10\,193.15 \dots$ is displayed.	365)) = × 10 000 =
<i>Use as PV in next calculation</i>	<i>10 193.15 \dots is displayed.</i>
$\times \pm 1 = PV$	<i>Use as PV in next calculation</i>
2ndF P/Y 4 ENT ON/C	$\pm PV 4$ P/YR 15
15 I/Y 1 N	I/YR 1 N FV
COMP FV	$10\,575.39 \dots$ is displayed.
$10\,575.39 \dots$ is displayed.	<i>Use as PV in simple interest calculation</i>
<i>Use as PV in simple interest calculation</i>	\times (1+ (0.15 × 75 ÷
$\times (1 + 0.15 \times 75 \div 365) =$	365)) =
$10\,901.34775 \dots$ is displayed.	$10\,901.34775 \dots$ is displayed.
<i>Calculate interest</i>	<i>Calculate interest</i>
$-10\,000 =$	$-10\,000 =$
$901.34775 \dots$ is displayed.	$901.34775 \dots$ to two decimals is displayed.

[Option 3]

Question 5

This is an odd period calculation using the method given, namely *fractional compounding*. We make use of the ordinary compound interest formula $S = P(1 + \frac{im}{m})^{tm}$ but we express the odd periods as a fraction of a year, as t must always be in units of years.

To determine the odd periods it is very important to note when the interest is being paid. In this situation interest is paid quarterly starting at the beginning of the year. Interest is thus paid on 1 January, 1 April, 1 July and 1 October. The R10 000 is invested for a period of seven months, thus the amount is invested from 15 May till 15 December. Now let's draw a time line of the situation:



There are two odd periods. One from 15 May to 1 July and one from 1 October and 15 December. If we use the days table in the Study guide, page 203, then the number of days between 15 May and 1 July is $182 - 135 = 47$ days, and the number of days between 1 October and 15 December is $349 - 274 = 75$ days.

In between, the period from 1 July until 1 October consists of one full quarter period. If you count the interest dates you get 2 but remember that there is only one compounding period between two dates.

Next we express the odd periods as fraction of a year.

We divide the number of days by 365, as there are 365 days in a year, and divide the number of quarters by 4, as there are 4 quarters in a year, to change the days and quarters to fractions of a year. Thus $t = \frac{47}{365} + \frac{1}{4} + \frac{75}{365}$.

Now given are the present value of R10 000 and the interest of 15% compounded quarterly, and asked is the interest paid on 15 December. First we calculate the future value. Thus

$$\begin{aligned} S &= 10\,000 \left(1 + \frac{0,15}{4}\right)^{\left(\left(\frac{47}{365} + \frac{1}{4} + \frac{75}{365}\right) \times 4\right)} \\ &= 10\,898,4311 \dots \end{aligned}$$

Now the interest is:

$$\begin{aligned} I &= 10\,898,4311 \dots - 10\,000 \\ &= 898,4311 \dots \end{aligned}$$

The amount of interest is R898,43.

EL-738 and EL-738F	HP10BII and HP10BII+
2ndF CA	C ALL
Use financial keys	Use financial keys
2ndF P/Y 4 ENT ON/C	4 P/YR 10 000 \pm PV 15
$\pm 10\,000$ PV	I/YR 47 + 75 = $\div 365$ + (1 \div
15 I/Y 47 + 75 = $\div 365$ = +1 \div 4 =	4) = $\times 4$ = N
$\times 4$ = N COMP FV	FV
10 898.4311... is displayed.	10 898.4311... is displayed.
Calculate interest	Calculate interest
-10 000 =	-10 000 =
898.4311... is displayed.	898.4311... is displayed.

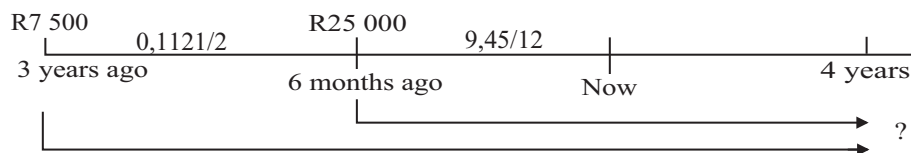
[Option 3]

Question 6

This is a compound interest calculation as the term *compounded* is found in the question. The formula for compound interest calculations is:

$$S = P\left(1 + \frac{j_m}{m}\right)^{tm}.$$

First we draw a time line of the problem:



In this question Jake wants to pay back both loans at the end of year four.

You cannot add the values of the loans at different time periods together. To determine the total payment due at the end of year four we must first move both loans to the same date, namely the one on which the payment is due, namely **year four**.

First we calculate the value of the R7 500 loan at year four:




We need to move the R7 500 from three years in the past to four years in the future, thus seven years forward in time using: $S = P(1 + j_m/m)^{tm}$. Thus $S = 7\,500(1 + 0,1121/2)^{(7 \times 2)}$

The R25 000 must be moved forward in time from six months in the past to year four, that is four and a half years forward. Thus $S = 25\,000(1 + 0,0945/12)^{(4,5 \times 12)}$.

Now the total owing at year four is:

$$\begin{aligned}
 & 7\,500 \left(1 + \frac{0,1121}{2}\right)^{(2 \times 7)} + 25\,000 \left(1 + \frac{0,0945}{12}\right)^{(12 \times 4,5)} \\
 & = 54\,278,92225 \dots
 \end{aligned}$$

Jake will have to pay Martha R54 278,92 four years from now.

EL-738 and EL-738F	HP10BII and HP10BII+
<p><i>Use financial keys</i></p> <p><u>First calculation: R7 500 FV</u></p> <p>2ndF M-CLR 00</p> <p>2ndF P/Y 2 ENT ON/C</p> <p>11.21 I/Y</p> <p>7 500 \pm PV</p> <p>2 \times 7 = N</p> <p>COMP FV</p> <p>16 093.25... <i>is displayed</i></p> <p><i>Store this in memory for later addition</i></p> <p>M+</p> <p><u>Second calculation: R25 000 FV</u></p> <p>2ndF CA</p> <p>2ndF P/Y 12 ENT ON/C</p> <p>9.45 I/Y</p> <p>25 000 \pm PV</p> <p>4.5 \times 12 = N</p> <p>COMP FV</p> <p>38 185.66... <i>is displayed</i></p> <p><u>Now add to first answer</u></p> <p>M+</p> <p><i>Recall total added</i></p> <p>RCL M+</p> <p>54 278.92... <i>is displayed.</i></p>	<p><i>Use financial keys</i></p> <p><u>First calculation: R7 500 FV</u></p> <p> C ALL</p> <p>2  P/YR</p> <p>11.21 I/YR</p> <p>7 500 \pm PV</p> <p>2 \times 7 = N</p> <p>FV</p> <p>16 093.25... <i>is displayed</i></p> <p><i>Store this in memory for later addition</i></p> <p>\rightarrowM</p> <p><u>Second calculation: R25 000 FV</u></p> <p>12  P/YR</p> <p>9.45 I/YR</p> <p>25 000 \pm PV</p> <p>4.5 \times 12 = N</p> <p>FV</p> <p>38 185.66... <i>is displayed</i></p> <p><u>Now add first answer</u></p> <p>M+</p> <p><i>Recall total added</i></p> <p>RM</p> <p>54 278.92... <i>is displayed.</i></p>

[Option 4]

Question 7

Now Jake of Question 6 decided to *reschedule* his payment towards his two loans again by splitting it into two payments of equal size, one now and one at year three. Now we know that what he owes he must pay back, thus

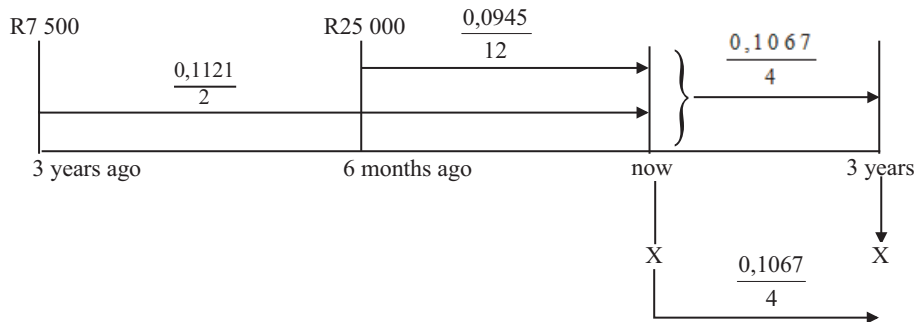
the total of the two loans = total of all the payments.

We cannot add values at different time periods together, thus we move them to the same date, namely the one that is asked, in this case, three years from now.

This is a compound interest calculation as the word *compounded* is found in the question. The formula for compound interest calculations is:

$$S = P\left(1 + \frac{j_m}{m}\right)^{tm}$$

Drawing the time line:



Now to complicate things further the interest rate changes from time now to 10,67% compounded quarterly. Which interest rate do we use to move the loans forward in time to year three? We split our calculations in two. First we move the loans from their original dates to the date on which the interest rates change (now) using the old interest rates. Then secondly we move their values at time now to year three using the new interest rate.

The total value of the two loans at time now using the old interest rates is:

$$7\,500 \left(1 + \frac{0,1121}{2}\right)^{(3 \times 2)} + 25\,000 \left(1 + \frac{0,0945}{12}\right)^{(\frac{6}{12} \times 12)} \\ = 36\,607,98026 \dots$$

The value of the loans at year three using the new interest rate is thus:

$$36\,607,98 \dots (1 + 0,1067/4)^{(3 \times 4)} = 50\,207,84847 \dots$$

Now the payments must also be moved forward in time to year three. The first payment must be moved from time now to year three and the other payment is due at year three. Thus the total of the payments is $X(1 + 0,1067/4)^{3 \times 4} + X$.





Now what you owe you must pay back, thus

$$\text{the total of all the loans} = \text{total of the payments.}$$

The value at year three at an interest rate of 10,67% compounded quarterly is:

$$\begin{aligned} 36\,607,98026 \dots \left(1 + \frac{0,1067}{4}\right)^{(3 \times 4)} &= X \left(1 + \frac{0,1067}{4}\right)^{(3 \times 4)} + X \\ 50\,207,84847 \dots &= X(1,37150 \dots + 1) \\ 50\,207,84847 \dots &= X(2,37150 \dots) \\ X &= \frac{50\,207,84847 \dots}{2,37150 \dots} \\ X &= 21\,171,34660 \dots \end{aligned}$$

The amount is R21 171,35.

EL-738 and EL-738F	HP10BII and HP10BII+
<p>2ndF M-CLR 0 0 <i>Use financial keys</i> <u>Calculate the FV of 7 500 now</u> 2ndF P/Y 2 ENT ON/C ±7 500 PV 11.21 I/Y $3 \times 2 = N$ COMP FV 10 403.22877...<i>is displayed.</i> <i>Store it for later use</i> M+ <u>Calculate the FV of 25 000</u> 2ndF P/Y 12 ENT ON/C ±25 000 PV 9.45 I/Y $6 \div 12 \times 12 = N$ COMP FV 26 204.751... <i>is displayed.</i> <i>Add to memory</i> M+ <u>Move the loan amount now</u> <u>to year 3 at interest of 10.67%</u> 2ndF P/Y 4 ENT ON/C ± RCL M+ = PV 10.67 I/Y $3 \times 4 = N$ COMP FV 50 207.848... <i>is displayed</i> <i>which is the total of the</i> <i>loans at year 3. Store</i> <i>in ALPHA keys memory A for</i> <i>later use</i> STO A</p>	<p> C ALL <i>Use financial keys</i> <u>Calculate the FV of 7 500 now</u> 2  P/YR 7 500± PV 11.21 I/YR $3 \times 2 = N$ FV 10 403.22877...<i>is displayed.</i> <i>Store it for later use</i> →M <u>Calculate the FV of 25 000</u> 12  P/YR 25 000± PV 9.45 I/YR $6 \div 12 \times 12 = N$ FV 26 204.751...<i>is displayed.</i> <i>Add to memory</i> M+ <u>Move the loan amount now</u> <u>to year 3 at interest of 10.67%</u> 4  P/YR RM ± PV 10.67 I/YR $3 \times 4 = N$ FV 50 207.848... <i>is displayed</i> <i>which is the total of the</i> <i>loans at year 3. Store</i> <i>in memory for later use</i> →M</p>

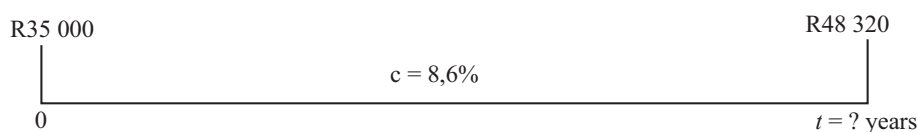
EL-738 and EL-738F	HP10BII and HP10BII+
<u>Calculate the FV of the payments at year 3</u> Now the P/Y , N and I/Y are still the same as the last calculation just enter the value of PV of X $1 \pm$ PV COMP FV 1.37150... is displayed. Add the 1 of the other X payment $+1 =$ 2.37150... is displayed and is the total of the payments. Store in ALPHA B for later use STO B <u>Determine the value of X.</u> Divide the total loan by 2.37150... ALPHA A \div ALPHA B = 21 171.345... to two decimals is displayed 2ndF M-CLR	<u>Calculate the FV of the payments at year 3</u> Now the P/YR , N and I/YR are still the same as the last calculation just enter the value of PV of X $1 \pm$ PV FV 1.37150... is displayed. Add the 1 of the other X payment $+1 =$ 2.37150... is displayed and is the total of the payments. Write down for later use <u>Determine the value of X.</u> Divide the total loan by 2.37150... RM \div 2.37150... = 21 171.345... is displayed

[Option 1]

Question 8

This is a continuous compounding calculation as the words *continuous compounded rate* is found in the question. The formula for the calculation of the accumulated sum if continuous compounding is used, is $S = Pe^{ct}$.

First we draw the time line:



Now given are the values of the accumulated sum (S) of R48 320, the present value of R35 000, and the continuous interest rate (c) of 8,6%. We need to determine the number of years t . We need to re-write the formula so that we can solve for t . Thus

$$\begin{aligned}
 S &= Pe^{ct} \\
 48\,320 &= 35\,000e^{0,086t} \\
 \frac{48\,320}{35\,000} &= e^{0,086t}
 \end{aligned}$$

We need to solve for t but t is in the power. We make use of the logarithm rule $\ln a^x = x \ln a$ to bring the t down. Now take \ln on both sides of the equation:

$$\ln \left(\frac{48\,320}{35\,000} \right) = \ln e^{0,086t}$$



$$\ln \left(\frac{48\,320}{35\,000} \right) = 0,086t \ln e$$

But $\ln e = 1$, thus

$$\frac{\ln \left(\frac{48\,320}{35\,000} \right)}{0,086} = t$$

$$t = 3,74997 \dots$$

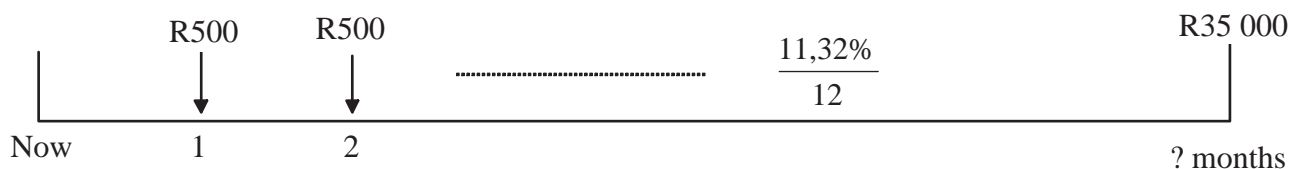
The term under consideration is 3,75 years.

EL-738 and EL-738F	HP10BII and HP10BII+
2ndF CA <i>Use normal keys</i> 2ndF ln [<i>on the 2 key</i>] (48 320 ÷ 35 000) ÷ 0.086 = 3.74997... <i>is displayed.</i>	 C ALL <i>Use normal keys</i> 48 320 ÷ 35 000 =  LN [<i>on the 2 key</i>] ÷ 0.086 = 3.74997... <i>is displayed.</i>

[Option 2]

Question 9

In this problem we have *equal deposits in equal time periods* plus the interest rate that is specified is *compounded*. Thus we are working with *annuities*. As the deposits are not specified as being paid at the beginning of each period, we take it as being paid at the end of each time period, thus we have an ordinary annuity. The time line is:





Now which annuity formula must be used? As the future value is given we make use of the future value formula of an annuity, namely $S = Rs \frac{1+i^n}{i}$, to calculate n . Now given are $S = 35\,000$; $j_m = 0,1132$; $m = 12$ and the deposits (R) of R500 each.

Now $i = j_m/m = 0,1132/12$, thus:

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ 35\,000 &= 500s_{\overline{n}|0,1132 \div 12} \end{aligned}$$

Using your calculator, we get $n = 54$

It will take Nicolet 54 months, to have R35 000 available.

EL-738 and EL-738F	HP10BII and HP10BII+
2ndF CA <i>Use financial keys</i> 2ndF P/Y 12 ENT ON/C 11.32 I/Y 35 000 FV 500± PMT COMP N 54.00... <i>is displayed.</i>	 C ALL <i>Use financial keys</i> 12  P/YR 11.32 I/YR 35 000 FV 500± PMT N 54.00... <i>is displayed.</i>

[Option 2]

Question 10

This is a compound interest calculation as the term *compounded monthly* is found in the question, and only one principle value is mentioned in the question. The formula for compound interest calculations is

$$S = P\left(1 + \frac{j_m}{m}\right)^{tm}.$$

Now the principle value is given as P and that it will double over time. Because the present value P will double in the future, the future value or S is equal to $2P$. We need to determine the time period t in years. Now

$$\begin{aligned} S &= P\left(1 + \frac{j_m}{m}\right)^{tm} \\ 2P &= P\left(1 + \frac{0,12}{12}\right)^{t \times 12} \\ \frac{2P}{P} &= \left(1 + \frac{0,12}{12}\right)^{12t} \\ 2 &= 1\left(1 + \frac{0,12}{12}\right)^{12t} \end{aligned}$$

Using your calculator with PV=1; FV=2; P/Y=12 (or P/YR=12) and I/Y=12 (or I/YR=12), we solve for N or tm . Now to calculate the years t we must therefore divide N by m to get t . Remember

your calculator, calculates N or tm . Use your calculator to get $N = 69.660\dots$

$$\begin{aligned} N &= tm \\ 69,660\dots &= 12t \\ t &= \frac{69,660\dots}{12} \\ t &= 5,80506\dots \end{aligned}$$

It will take 5,81 years to double.

OR alternatively,



$$\begin{aligned} S &= P \left(1 + \frac{j_m}{m} \right)^{tm} \\ 2P &= P \left(1 + \frac{0,12}{12} \right)^{t \times 12} \\ \frac{2P}{P} &= \left(1 + \frac{0,12}{12} \right)^{12t} \\ 2 &= 1 \left(1 + \frac{0,12}{12} \right)^{12t} \end{aligned}$$

Taking \ln on both sides of the equation and using the law of logarithms $\ln a^x = x \ln a$:




$$\begin{aligned} \ln 2 &= 12t \ln \left(1 + \frac{0,12}{12} \right) \\ t &= \frac{\ln 2}{12 \ln \left(1 + \frac{0,12}{12} \right)} \\ t &= 5,80506\dots \end{aligned}$$

The time under consideration is 5,81 years.

Using financial keys:

EL-738 and EL-738F	HP10BII and HP10BII+
2ndF CA 2ndF P/Y [on the I/Y key] 12 ENT ON/C \pm 1 PV 2 FV 12 I/Y COMP N \div 12 = 5.80506... <i>is displayed.</i>	 C ALL 12  P/YR 2 \pm FV 1 PV 12 I/YR N \div 12 = 5.80506... <i>is displayed.</i>

Using normal keys:

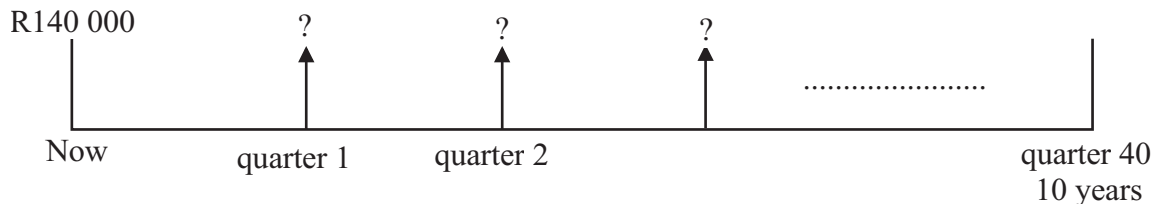
EL-738 and EL-738F	HP10BII and HP10BII+
2ndF CA 2ndF \ln [on 2 key] $2 \div (12 \text{ 2ndF } \ln$ $(1 + 0.12 \div 12)) =$ 5.80506... <i>is displayed.</i>	 C ALL $0.12 \div 12 + 1 =$  LN [on the 2 key] $\times 12 = \rightarrow M 2$  LN \div RM = 5.80506... <i>is displayed.</i>

[Option 5]

Question 11

This is an annuity calculation because of the *equal withdrawals in equal time intervals* and compound interest. The present value of the deposit P , the interest rate (compounded) and the time period are given. You need to calculate the size of the withdrawals or payments R .

The time line is:



We use the present value formula of an annuity as the present value of the annuity is given, namely $P = Ra_{\overline{n}|i}$, with $P = \text{R}140\,000$; $i = 0,135 \div 4$ and $n = 10 \times 4$. Thus

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ 140\,000 &= Ra_{\overline{10 \times 4}|0,135 \div 4} \\ R &= 6\,429,2826 \dots \end{aligned}$$

Paul can withdraw R6 429,28 every quarter.

EL-738 and EL-738F	HP10BII and HP10BII+
2ndF CA	C ALL
Use financial keys	Use financial keys
2ndF P/Y 4 ENT ON/C	4 P/YR
140 000± PV	140 000± PV
10 × 4 =N	10 × 4 =N
13.5 I/Y	13.5 I/YR
COMP PMT	PMT
6 429.2826... is displayed.	6 429.2826... is displayed.

[Option 3]

Question 12

This is a compound interest problem as the words *interest is compounded* is found in the question. Secondly it is a conversion between two types of compound interest rates.

To change the compounding period and thus the interest rate, we can use the conversion formula in the Study guide – chapter 3 – see example 3.5, namely

$$j_n = n \left(\left(1 + \frac{j_m}{m} \right)^{m \div n} - 1 \right).$$

Note: Remember n is the new compounding period.

Now the given interest rate is 15% ($j_m = 0,15$), compounded every second month, thus $m = 6$. The new compounded interest rate is weekly, thus $n = 52$.

$$\begin{aligned}
 j_n &= n \left(\left(1 + \frac{j_m}{m} \right)^{m \div n} - 1 \right) \\
 &= 52 \left(\left(1 + \frac{0,15}{6} \right)^{6 \div 52} - 1 \right) \\
 &= 0,14837 \dots
 \end{aligned}$$

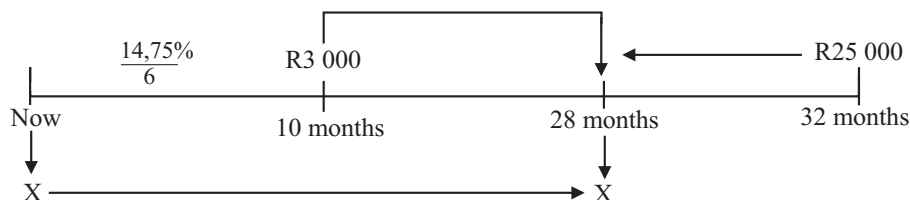
The equivalent weekly compounded rate is 14,837%.

EL-738 and EL-738F	HP10BII and HP10BII+
2ndF CA	$\boxed{\text{C}}$ ALL
Use normal keys	Use normal keys
$52((1 + 0.15 \div 6)$	$52 \times \boxed{(} \boxed{1} +$
2ndF $y^x(6 \div 52)$	$\boxed{(} 0.15 \div 6 \boxed{)} \boxed{)} \boxed{y^x} \boxed{(} 6 \div 52$
$-1) =$	$\boxed{)} - 1 \boxed{)} =$
0.14837... is displayed.	0.14837... is displayed.
$\times 100$ to calculate %	$\times 100$ to calculate %
$\times 100 =$	$\times 100 =$
14.837... is displayed.	14.837... is displayed.

[Option 4]

Question 13

This is a compound interest calculation as the word *compounded* is found in the question. Secondly we find the word payments specified in the question as well, thus it is an annuity? NO! An annuity is *equal* payments in *equal* time intervals. Here we have two payments that are same in size, but are at different time periods. Thus we do not have annuities. In this example Nkosi is *rescheduling* the way his loan must be paid back, thus we have an *equations of value* problem. Now draw the time line:



The formula used for compound interest calculations is:

$$S = P \left(1 + \frac{j_m}{m} \right)^{tm}$$

In this question the person is *rescheduling* his payments towards his loans, thus we make use of the *equation of value* principle namely, what you owe, you must pay back. Thus

the total of all the loans = total of all the payments.

But you cannot add money values due at different time periods, together. You must first move them to the same date. We use the date that the question specifies, namely month 28 as the comparison date.

First we calculate the value of each loan at month 28. We move the R3 000 from month 10 to month 28, thus 18 months forward in time. We thus calculate a future value. Now $S = P(1 + j_m/m)^{tm}$ where t is the number of years and m the number of compounding periods. Thus

$$3\,000 \left(1 + \frac{0,1475}{6}\right)^{\frac{18}{12} \times \frac{6}{1}}.$$

The R25 000 must be moved backwards in time from month 32 to month 28, thus four months in total back. We thus calculate a present value,

$$25\,000 \left(1 + \frac{0,1475}{6}\right)^{\frac{-4}{12} \times \frac{6}{1}}.$$

Remember that $25\,000 \left(1 + \frac{0,1475}{6}\right)^{\frac{-4}{12} \times \frac{6}{1}}$ is the same as $25\,000 / \left(1 + \frac{0,1475}{6}\right)^{\frac{4}{12} \times \frac{6}{1}}$.

Secondly we calculate the value of the total payments at month 28.

Now you are paying X now and X at month 28. Thus the first payment X must be moved forward from now to month 28, thus 28 months in total forward: $X(1 + \frac{0,1475}{6})^{\frac{28}{12} \times \frac{6}{1}}$. The second X is already at month 28, thus the value of X stays X .






Now

$$\begin{array}{lcl} \text{Sum of payments} & = & \text{Sum of obligations} \\ X \left(1 + \frac{0,1475}{6}\right)^{\frac{28}{12} \times \frac{6}{1}} + X & = & 3\,000 \left(1 + \frac{0,1475}{6}\right)^{\frac{18}{12} \times \frac{6}{1}} + 25\,000 \left(1 + \frac{0,1475}{6}\right)^{\frac{-4}{12} \times \frac{6}{1}} \end{array}$$

Take X out as a common factor:

$$\begin{aligned} X \left[\left(1 + \frac{0,1475}{6}\right)^{\frac{28}{12} \times \frac{6}{1}} + 1 \right] &= 3\,732,90 \dots + 23\,814,72 \dots \\ X[1,40495 \dots + 1] &= 27\,547,62 \dots \\ X &= \frac{27\,547,62 \dots}{2,40495 \dots} \\ X &= 11\,454,532 \dots \end{aligned}$$

Nkosi will pay Peter approximately R11 455 at month 28.

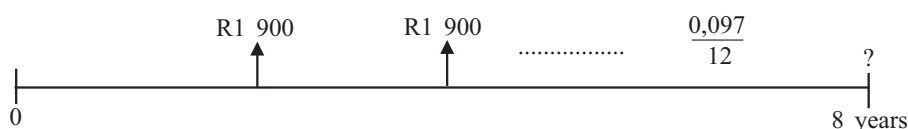
EL-738 and EL-738F	HP10BII and HP10BII+
<p>2ndF CA</p> <p><i>Use financial keys</i></p> <p><i>Calculate the FV of the first payment</i></p> <p>2ndF P/Y</p> <p>6 ENT ON/C</p> <p>± 1 PV</p> <p>14.75 I/Y</p> <p>$28 \div 12 = 2\text{ndF}$</p> <p>$\times P/Y$ N</p> <p>COMP FV</p> <p>1.40495... <i>is displayed.</i></p> <p><i>Add the second payment and store total of payments in memory</i></p> <p>+1 = M+</p> <p><i>Calculate the FV of 3 000</i></p> <p>3 000 \pm PV</p> <p>$18 \div 12 = 2\text{ndF}$</p> <p>$\times P/Y$ N</p> <p>COMP FV</p> <p>3 732.90414... <i>is displayed.</i></p> <p><i>Store for later use</i></p> <p>STO A</p> <p><i>Calculate the PV of 25 000</i></p> <p>25 000 \pm FV</p> <p>$4 \div 12 = 2\text{ndF}$ $\times P/Y$ N</p> <p>COMP PV</p> <p>23 814.7175... <i>is displayed</i></p> <p><i>Add the FV of 3 000</i></p> <p>+ ALPHA A =</p> <p>27 547.62167... <i>is displayed</i></p> <p><i>Calculate the value of X</i></p> <p><i>Divide by total of payments</i></p> <p>\div RCL M+ =</p> <p>11 454.532... <i>is displayed.</i></p>	<p> C ALL</p> <p><i>Use financial keys</i></p> <p><i>Calculate the FV of the first payment</i></p> <p>6  P/YR</p> <p>1 \pm PV</p> <p>14.75 I/YR</p> <p>$28 \div 12$  $\times P/YR$</p> <p>FV</p> <p>1.40495... <i>is displayed. Add the second payment and store total of payments in memory</i></p> <p>+1 = M+</p> <p><i>Calculate the FV of 3 000</i></p> <p>3 000 \pm PV</p> <p>$18 \div 12 =$  $\times P/YR$</p> <p>FV</p> <p>3 732.90414... <i>is displayed.</i></p> <p><i>Calculate the PV of 25 000</i></p> <p>25 000 \pm FV</p> <p>$4 \div 12 =$  $\times P/YR$</p> <p>PV</p> <p>23 814.7175... <i>is displayed.</i></p> <p><i>Add the FV of 3 000</i></p> <p>+3 732.90414... =</p> <p>27 547.62167... <i>is displayed.</i></p> <p><i>Calculate the value of X</i></p> <p><i>Divide by total of payments</i></p> <p>\div RM =</p> <p>11 454.532... <i>is displayed</i></p>

[Option 1]

Question 14

In this problem we have *equal payments in equal time periods*, plus the interest rate that is specified is *compounded*. Thus we are working with *annuities*. As the deposits are not specified as being paid at the beginning of the period, we take it as being paid at the end of each time period, thus an ordinary annuity. As we need to determine the future value of the annuity, we use the future value formula of an annuity, namely $S = Rs_{\overline{n}|i}$, to calculate S .

The time line is



Now given are the deposits (R) of R1 900 each, the interest rate (j_m) of 9,7% and the time period (t) as eight years and the compound periods as monthly ($m = 12$). Thus $n = t \times m$ or $n = 8 \times 12$ and $i = j_m/m$ or $0,097 \div 12$.

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ &= 1\,900s_{\overline{8 \times 12}|0,097 \div 12} \\ &= 274\,069,246 \dots \end{aligned}$$

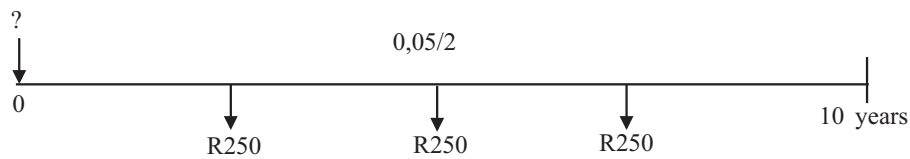
The accumulated amount is R274 069,25.

EL-738 and EL-738F	HP10BII and HP10BII+
2ndF CA <i>Use financial keys</i>	C ALL <i>Use financial keys</i>
2ndF P/Y 12	12 P/YR
ENT ON/C	1 900 \pm PMT
1 900 \pm PMT	9.7 I/YR
9.7 I/Y	8 \times P/YR
8 2ndF \times P/Y N	FV
COMP FV	274 069.246... <i>is displayed.</i>
274 069.246... <i>s displayed.</i>	

[Option 3]

Question 15

In this problem we have *equal withdrawals in equal time periods*, plus the interest rate that is specified is *compounded*. Thus we are working with annuities. As the payments are not specified as being paid at the beginning of the period, we take it as being paid at the end of each time period. As we have to calculate the present value of an ordinary annuity we use the present value formula of an annuity namely, $P = Ra_{\overline{n}|i}$. The time line is



Now given are the withdrawals (R) of R250 each, the time (t) period of 10 years, the interest rate (j_m) of 5%, the half yearly compounding periods ($m = 2$). Thus $n = tm = 10 \times 2$ and $i = j_m/m = 0,05 \div 2$:

$$\begin{aligned}
 P &= Ra_{\overline{n}|i} \\
 &= 250a_{\overline{10 \times 2}|0,05/2} \\
 &= 3\,897,29\dots
 \end{aligned}$$

The amount that should be deposited now is R3 897,29.

EL-738 and EL-738F	HP10BII and HP10BII+
2ndF CA	C ALL
Use financial keys	Use financial keys
2ndF P/Y 2 ENT ON/C	2 P/YR
± 250 PMT	250 \pm PMT
$10 \times 2 = N$	$10 \times 2 = N$
5 I/Y	5 I/YR
COMP PV	PV
3 897.29... is displayed.	3 897.29... is displayed.

[Option 3]

NOTE: YOU CAN FIND THE SUMMARY OF WHEN TO USE WHICH FORMULA AND A COPY OF THE FORMULA SHEET USED IN THE EXAMINATION ON myUNISA.