

# Tutorial Letter 204/2/2016

## Introductory Financial Mathematics DSC1630

### Semester 2

### Department of Decision Sciences

**Important Information:**

This tutorial letter contains the solutions of  
Assignment 04.

Bar code

Dear student

I hope you are doing well and that you are enjoying this module. You have completed the four assignments for the course and all that is left to pass this module is the examination. It is now time to start your revision for the examination. Try to work through all the assignments (first and second semester), evaluation exercises and the previous examination paper when you prepare for the examination. The questions in the upcoming examination paper are similar to the problems in the above mentioned. You are also welcome to try the first semester's assignments. The solutions are available on myUnisa under Additional resources. Remember, practice makes perfect! The more examples you work through the more you will be able to recognise a problem and know how to solve it. Please visit the Announcement tab on myUnisa as I regularly post some extra notes for your convenience. An announcement on the examination as well as the solutions of one of the previous examination papers, will be posted just before the exam.

You are also welcome to contact me by e-mail, fax, telephone or a personal appointment if you need help regarding the study material. My contact details and contact hours are as follows:

**Mrs Adèle Immelman:**

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**E-mail:** immelfm@unisa.ac.za

08:00 until 13:30 - Monday till Friday: **Appointments and Telephone**

13:30 until 16:00 - Monday till Thursday: **Telephone only**

Lastly, I wish you everything of the best with your preparation for the last hurdle, the examination.

Mrs Adèle Immelman

## 1 Solution Summary

The following is a summary of the correct answers:




Q 1	Option 3	Q 9	Option 4
Q 2	Option 2	Q 10	Option 1
Q 3	Option 5	Q 11	Option 4
Q 4	Option 1	Q 12	Option 1
Q 5	Option 4	Q 13	Option 3
Q 6	Option 4	Q 14	Option 1
Q 7	Option 2	Q 15	Option 4
Q 8	Option 3		

## 2 Assignment 04 – Detailed Solution

1. We need to determine the *standard deviation* of the number of houses sold. Using the statistical keys of our calculator and entering the data points, we determine that the value of  $s_x$  is:

$$\begin{aligned}s_x &= 6,60808\dots \\ &\approx 6,6.\end{aligned}$$

The standard deviation is 6,6.

EL-738	HP10BII
<p><i>Switch to STA mode with <b>two variables</b></i></p> <p>MODE 1 1</p> <p><i>Clear all the memory keys</i></p> <p>2ndF M CLR 0 0</p> <p><i>Enter the data</i></p> <p>5 (x,y) [next to the ENT keys] 500</p> <p>DATA [on the ENT key]</p> <p>15 (x,y) 900 DATA</p> <p>19 (x,y) 1 500 DATA</p> <p>7 (x,y) 2 000 DATA</p> <p><i>Calculate <math>s</math></i></p> <p>ALPHA <math>s_x</math> =</p> <p><math>s_x = 6.60808\dots</math> is displayed.</p> <p><i>Cancel STA mode:</i></p> <p>MODE 0</p>	<p><i>Clear all the keys</i>  C ALL</p> <p> CL <math>\Sigma</math> [on <math>\rightarrow M</math> key]</p> <p><i>Enter the data</i></p> <p>5 INPUT 500 <math>\Sigma</math> +</p> <p>15 INPUT 900 <math>\Sigma</math> +</p> <p>19 INPUT 1 500 <math>\Sigma</math> +</p> <p>7 INPUT 2 000 <math>\Sigma</math> +</p> <p><i>Calculate <math>s</math></i></p> <p> <math>s_x, s_y</math> [on 8 key]</p> <p>6.60808... is displayed.</p>

### [Option 3]

2. We need to determine the *correlation coefficient*  $r$  of the fitted line that represents the data. The correlation coefficient tells you how strong the linear relationship between the two sets of data is.

Using the statistical keys of our calculator and entering the data points we determine that the correlation coefficient is

$$\begin{aligned}r &= 0,16428\dots \\ &\approx 0,16\end{aligned}$$

**EL-738**

*Switch to STA mode with two variables*

MODE 1 1

*Clear all the memory keys*

2ndF M CLR 00

*Enter the data*

5 (x,y) 500 DATA [on the ENT key]

15 (x,y) 900 DATA

19 (x,y) 1 500 DATA

7 (x,y) 2 000 DATA

*Calculate r*

ALPHA r =

0.16428... is displayed.

*Cancel STA mode:*

MODE 0

**HP10BII**

 C ALL

*Clear all the memory keys*

 CL Σ [on →M key]

*Enter the data*

5 INPUT 500 Σ +

15 INPUT 900 Σ +

19 INPUT 1 500 Σ +

7 INPUT 2 000 Σ +

*To get the value of r press*

  $\hat{x}, r$   SWAP [on K key]

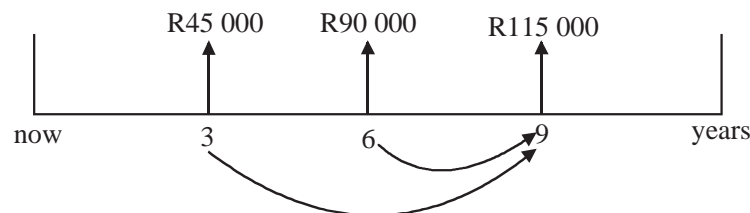
0.16428... is displayed.

**[Option 2]**

3. The *future value of the cash inflows* is the future value of all the positive cash flows. As we need to determine the future value it means we must move each inflow to the end of the investment period, namely year nine and then add them together to determine the total future value of all the inflows. A future value is calculated using

$$S = P \left( 1 + \frac{j_m}{m} \right)^{tm}$$



The time line is:



We need to move the R45 000 from year three to year nine, thus six years forward; the R90 000 from year six to year nine, thus three years forward and the R115 000 is already at year nine. Thus the total future value is:

$$\begin{aligned} \text{FV} &= 45\,000(1 + 0,1159)^6 + 90\,000(1 + 0,1159)^3 + 115\,000 \\ &= 326\,948,85 \\ &\approx 326\,950 \end{aligned}$$

The future value of the cash inflows is approximately R326 950.

EL-738	HP10BII
<p><i>Use financial keys</i></p> <p><i>Remember we calculate the FV of each inflow then add them together in memory</i></p> <p>2ndF M-CLR 0 0</p> <p>2ndF P/Y 1 ENT ON/C</p> <p>11.59 I/Y</p> <p>45 000± PV</p> <p>6 N</p> <p>COMP FV</p> <p>86 888.876... <i>is displayed. Store in memory</i></p> <p>M+</p> <p><i>Calculate second FV. P/Y and I/Y are already stored.</i></p> <p>3 N</p> <p>90 000± PV</p> <p>COMP FV</p> <p>125 059.976... <i>is displayed. Add to memory</i></p> <p>M+</p> <p><i>Recall total memory value and add 115 000</i></p> <p>RCL M+ + 115 000 =</p> <p>326 948.85 <i>to two decimals is displayed.</i></p>	<p><i>Use financial keys</i></p> <p><i>Remember we calculate the FV of each inflow then add them together in memory</i></p> <p> C ALL</p> <p>1  P/YR</p> <p>11.59 I/YR</p> <p>45 000± PV</p> <p>6 N</p> <p>FV</p> <p>86 888.876... <i>is displayed. Store in memory</i></p> <p>→ M</p> <p><i>Calculate second FV. P/YR and I/YR are already stored.</i></p> <p>3 N</p> <p>90 000± PV</p> <p>FV</p> <p>125 059.976... <i>is displayed. Add to memory</i></p> <p>M+</p> <p><i>Recall total value and add 115 000</i></p> <p>RM +115 000 =</p> <p>326 948.85 <i>to two decimals is displayed.</i></p>

**Note:** As the I/Y or I/YR value stays the same in all the calculations we enter it only once.

[Option 5]

4. We are asked to calculate the MIRR value. The MIRR formula consists of the future value of the cash inflows and the present values of the cash outflows. Now given is the present value of the cash outflows and calculated in Question 3 is the future value of the cash inflows. Thus

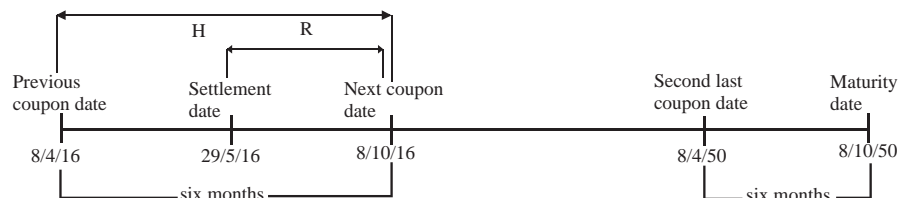
$$\begin{aligned}
 \text{MIRR} &= \left( \frac{C}{PV_{out}} \right)^{\frac{1}{n}} - 1 \\
 \text{MIRR} &= \left( \frac{326\,948.85}{95\,000} \right)^{\frac{1}{9}} - 1 \\
 &= 0.1472 \dots
 \end{aligned}$$

The MIRR is 14.72%.

EL-738	HP10BII
2ndF M CLR 00 <i>Use normal keys</i> $(326\,948.85 \div 95\,000)$ 2ndF $y^x(1 \div 9) = -1 =$ 0.1472... <i>is displayed.</i> <i>Multiply by 100 to get percentage</i> $\times 100 =$ 14.72 <i>to two decimals is displayed.</i>	<input type="checkbox"/> C ALL <i>Use normal keys</i> $326\,948.85 \div 95\,000 =$ <input type="checkbox"/> $y^x$ <input type="checkbox"/> $(1 \div 9$ <input type="checkbox"/> $) = -1 =$ 0.1472... <i>is displayed.</i> <i>Multiply by 100 to get percentage</i> $\times 100 =$ 14.72 <i>to two decimals is displayed.</i>

**[Option 1]**

5. We need to determine the *all-in price* of the Bond F234. First we draw the time line of the Bond F234:



We use the following formula to determine the price on 08/10/16:

$$P = da_{\overline{n}|z} + 100(1+z)^{-n}$$

where  $n$  is the number of outstanding coupon payments after the settlement date until the maturity date.

Now given is  $z = 0,07955 \div 2$ ; and  $d = 10,5 \div 2$ . All we need is  $n$ , the number of half years.

**TIPS TO CALCULATE  $n$  IN GENERAL:**

Now  $n$  is the number of half years from the coupon date after the settlement date, until the maturity date. As a start we determine the first coupon date after the settlement date. Secondly we determine the number of half years until the maturity date. Now there are two situations that can exist when calculating  $n$ :

- (a) **If the month of the next coupon date is the same as the month of the maturity date** then subtract the year of the next coupon date from the year of the maturity date - that gives you the number of years until maturity. But you need the number of half years until maturity thus multiply the years by 2 to calculate  $n$ .

**For example:**

Settlement date is 14/9/2013

Next coupon is 4/10/2013

Maturity date is 4/10/2034

Now because the months of the next coupon date and the maturity date are the same namely month 10 we subtract the years namely  $2034 - 2013 = 21$  years thus  $21 \times 2 = 42$  half years. Thus  $n = 42$ .

- (b) **If the month of the next coupon date is the different from the month of the maturity date** then ignore the next coupon date and move to the second coupon date from the settlement date - thus you try to get the months the same.

Subtract the year of the second coupon date from the year of the maturity date as in method 1 - that gives you the number of years. But you need the number of half years thus multiply the years by 2 and then add 1 for the period you have ignored.

**For example:**

Settlement date is 14/9/2013

Next coupon is 4/10/2013

Maturity date is 4/4/2034

Now the month of the next coupon and maturity is different. Thus ignore the first coupon date 4/10/2013 and look at the next coupon date which is 4/4/2014. Now the month of the coupon date 2 and the maturity date is the same, namely month 4. We subtract  $2034 - 2014 = 20$  years, thus  $20 \times 2 = 40$  half years but we have ignored one period thus  $n = 40 + 1 = 41$ .

Now to calculate  $n$  we must first determine the number of years from the next coupon date until the maturity date.

The month of the coupon date (10) and the maturity date (10) is the same. We subtract the years  $2050 - 2016 = 34$  years.

This 34 must be multiplied by two to obtain half years.

$$\begin{aligned} n &= 34 \times 2 \\ &= 68 \end{aligned}$$

We multiply it by two because the coupon payments are made every six months. The number of coupon payments  $n$  are therefore 68.

$$\begin{aligned} P(08/10/16) &= da_{\overline{n}|z} + 100(1+z)^{-n} \\ &= \frac{10,5}{2} a_{\overline{68}|0,07955 \div 2} + 100\left(1 + \frac{0,07955}{2}\right)^{-68} \\ &= 122,68841 + 7,04892 \\ &= 129,73733 \end{aligned}$$

The present value of Bond F234 is R129,73733%.

As the settlement date is more than 10 days from the next coupon date we must add the coupon that is due on 8 October 2016 and we call it a cum interest case.

$$\begin{aligned} P(08/10/16) &= 129,73733 + 5,25 \\ &= 134,98733 \end{aligned}$$

The present value of Bond F234 on 8 October 2016 is R134,98733%.

This present value must be discounted back to the settlement date of 29 May 2016 by using the fraction  $f = \frac{R}{H}$ .

$R$  is the number of days from the settlement day until the next coupon date.  $R$  equals day number 281 (8 October) minus day number 149 (29 May).  $R = 132$ .

$H$  is the number of days in the half year (between the previous coupon date and the following coupon date) in which the settlement date falls.  $H$  equals day number 281 (8 October) minus day number 98 (8 April).  $H = 183$ .



$$\begin{aligned} \text{All-in price} &= 134,98733 \left( 1 + \frac{0,07955}{2} \right)^{-\frac{132}{183}} \\ &= 131,24248 \end{aligned}$$

The all-in price is R131,24248%.

**There are different ways to use your calculator to calculate the all-in price. In TUT101 memory keys are used as well as P/Y or P/YR = 1. In this case the interest then has to be divided by two manually as the calculator divide the interest by the P/Y or P/YR value, that is one. Here is another method where we use P/Y or P/YR = 2. In this case it is not necessary to divides the interest by 2 (P/Y or P/YR value) the calculator does that automatically. We can also skip the use of memory keys by not clearing the values (2ndF CA) that are the same though out the calculation, thus shortening the calculation in total.**





Remember to set your decimal display to five decimals for the R% format.



EL-738 and EL-738F	HP10BII and HP10BII+
<p>2ndF M-CLR 0 0  <i>First we calculate the PV of the 100</i>  2ndF P/Y 2 ENT ON/C  <math>\pm 100</math> FV  7.955 I/Y  68 N  COMP PV  7.04892 is displayed.  <b>DO NOT CLEAR!!!!</b>  <i>Secondly we calculate the PV of the coupon flow. The P/Y, N and I/Y are the same as above and is already entered. We just enter the PMT value</i>  <math>10.5 \div 2 = \times \pm 1 = \text{PMT}</math>  COMP PV  <i>The value at 08/10/16</i>  129.73733 is displayed.  <b>DO NOT CLEAR!!!!</b>  <i>Add a coupon as it is cum interest.</i>  <math>+5.25 =</math>  134.98733 is displayed.  <i>Now use it to get the all-in price</i>  Store this answer as FV and make PMT zero  <i>The P/Y and I/Y are the same as above</i>  <math>\times \pm 1 = \text{FV}</math>  0 PMT 132 <math>\div</math> 183 =N  COMP PV  131.24248 to five decimals is displayed.</p>	<p> C ALL  <i>First we calculate the PV of the 100</i>  2  P/YR  100<math>\pm</math> FV  7.955 I/YR  68 N  PV  7.04892 is displayed. Clear screen  C    <i>Secondly we calculate the PV of the coupon flow. The P/YR, N and I/YR are the same as above and is already entered. We just enter the PMT value</i>  <math>10.5 \div 2 = \pm \text{PMT}</math>  PV  <i>The value at 08/10/16</i>  129.73733 is displayed.    <i>Add a coupon as it is cum interest.</i>  <math>+5.25 =</math>  134.98733 is displayed.  <i>Now use it to get the all-in price</i>  Store this answer as FV and make PMT zero  <i>The P/YR and I/YR are the same as above</i>  <math>\pm \text{FV}</math>  0 PMT  132 <math>\div</math> 183 =N  PV  131.24249 to five decimals is displayed.</p>

**Note:** The answers differ in the last decimal due to rounding. Choose the answer closest.

**Or alternative method:**

EL-738	HP10BII
2ndF CA	 C ALL
2ndF P/Y	1  P/YR
1 ENT ON/C	10.5 $\div$ 2 = $\pm$ PMT
$\pm 10.5 \div 2 = \text{PMT}$	68 N
$7.955 \div 2 = \text{I/Y}$	$7.955 \div 2 = \text{I/YR}$
68 2ndF $\times$ P/Y N or 68 N	PV
COMP PV	122.68841 appears
122.68841 is displayed.	on the screen
Store in memory	C
M+	100 $\pm$ FV
2ndF CA	$7.955 \div 2 = \text{I/YR}$
$\pm 100 \text{ FV}$	68 N
$7.955 \div 2 = \text{I/Y}$	PV
68 2ndF $\times$ P/Y N or 68 N	129.737333 appears
COMP PV	on the screen
PV = 7.04892	+5.25 =
+RCL M+ =	134.98733 appears
129.73733 appears	on the screen
on the screen	 C ALL
+5.25 =	1  P/YR
134.98733 appears	134.98733 $\pm$ FV
on the screen	$7.955 \div 2 = \text{I/YR}$
2ndF CA	132 $\div$ 183 = N
2ndF P/Y 1 ENT ON/C	PV
$\pm 134.98733 \text{ FV}$	131.24248 is displayed.
$7.955 \div 2 = \text{I/Y}$	
132 $\div$ 183 = N	
COMP PV	
131.24248 is displayed.	

**[Option 4]**

6. We need to determine the *yearly coupon rate* or  $c$ . Now given is a formula which includes the half yearly coupon  $d$ . Thus if we can solve for  $d$  we can just multiply it by two to get the yearly coupon rate. To solve for  $d$  we first manipulate the formula until it looks similar to an annuity formula and then solve for  $d$  (or PMT), using our calculator.

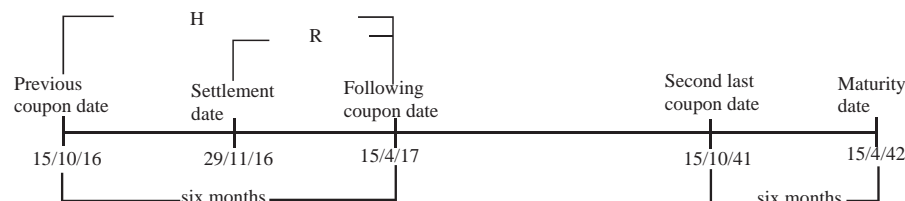
$$\begin{aligned}
 P &= da_{\overline{n}|z} + 100(1+z)^{-n} \\
 107,55174 &= da_{\overline{n}|z} + 100 \left( 1 + \frac{0,135}{2} \right)^{-29} \\
 107,55174 &= da_{\overline{29}|0,135 \div 2} + 15,04289 \\
 da_{\overline{29}|0,135 \div 2} &= 107,55174 - 15,04289 \\
 da_{\overline{29}|0,135 \div 2} &= 92,50885 \\
 d &= 7,35
 \end{aligned}$$

The yearly coupon rate is  $7,35 \times 2 = 14,70\%$ .

EL-738	HP10BII
2ndF CA	C ALL
<i>Use financial keys</i>	<i>Use financial keys</i>
2ndF P/Y	1  P/YR
1 ENT ON/C	92.50885± PV
±92.50885 PV	29 N
29 N	$13.5 \div 2 = I/YR$
$13.5 \div 2 = I/Y$	PMT
COMP PMT	7.35 <i>is displayed.</i>
7.35 <i>is displayed.</i>	$7.35 \times 2 =$
$7.35 \times 2 =$	14.70 <i>to two decimals is displayed.</i>
14.70 <i>to two decimals is displayed.</i>	

[Option 4]

7. We need to determine the *accrued interest* of Bond ABC. First we draw our time line:



As there are more than 10 days from the settlement date to the following coupon date this is a cum-interest case. The accrued interest can be calculated using the formula  $\frac{H-R}{365} \times c$ .

Now given is the yearly coupon rate ( $c$ ) of 9,75%. We need to determine  $H$  and  $R$ .

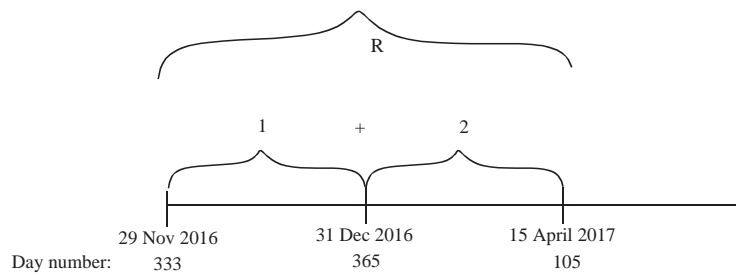
As the  $R$  and  $H$  periods stretch over more than one year we can't just subtract the day numbers. We make use of a different method to calculate  $R$  and  $H$  in this situation.

## Calculating R:

$R$  is the number of days from the settlement date 29/11/2016 to the next coupon date 15/04/2017.

### Method 1:

As the days are over different years, namely 2016 and 2017 we first count all the days in one year (2016) and then add the other days in the next year (2017). Thus we add all the days in 2016 from 29 November until 31 December including 31 December, plus all the days in 2017 from 1 January 2017 until 15 April (not included). We include 31 December since it forms part of the total time from 29/11/2016 to the next coupon date 15/04/2017. We can't leave it out.



1. **Count all the days starting from 29/11/2016 until 31/12/2016.** Thus, by using the date table, the number of days between 29 November and 31 December is  $365 - 333 = 32$ . But this doesn't include 31 December. Thus we need to add one day as 31 December is still part of the whole time period. Thus there are 33 days from 29 November 2016 until and including 31 December.
2. **Count the days from 01/01/2017 until 15/04/2017** (not included as the rule says always include the first day but not the last day of your total time period). The number of days from 1 January till 15 April, using date numbers in the date table, is  $105 - 1 = 104$ .
3. **Add the two date periods together.** The total number of days between 29/11/2016 and the next coupon date 15/04/2017 is  $R = 104 + 33 = 137$ .

### Method 2:

Ignore the years and first subtract the date value in the date table of 15 April from 29 November which gives you  $333 - 105 = 288$  days. But as the two dates are in different years (2016 and 2017) you subtract 288 from 365 to get  $R = 365 - 288 = 137$ .

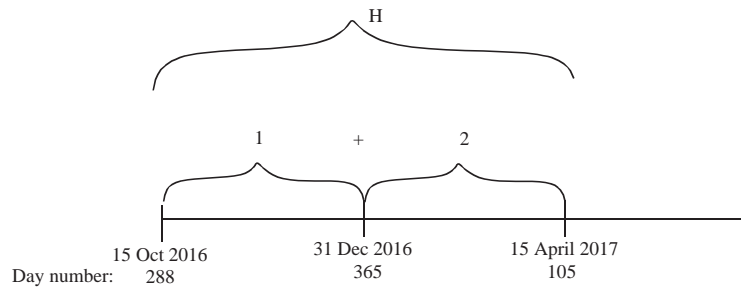
Or day number 365 minus day number 333 (29 November) plus day number 105 (15 April) gives you  $R = 137$ . Please note we don't use a leap year in this module.

## Calculating H

$H$  is the number of days in the half year in which the settlement date falls, i.e. from the coupon date before the settlement date (15/10/2016), to the coupon date after the settlement date (15/04/2017). Again there are two ways of calculating it.

### Method 1:

As the days are over different years namely 2016 and 2017 we first count all the days in one year (2016) and then add the other days in the next year (2017). Thus we add all the days in 2016 from 15 October 2016 until 31 December 2016, including 31 December plus all the days in 2017 from 1 January 2017 until 15 April (not included). We include 31 December since it forms part of the total time from 15/10/2016 to the next coupon date 15/04/2017. We can't leave it out.



1. **Count all the days starting from 15/10/2016 until 31/12/2016.** Thus by using the date table, the number of days between 15 October 2016 and 31 December 2016, is  $365 - 288 = 77$ . But this doesn't include 31 December. Thus we need to add one day as 31 December is still part of the whole time period. Thus there are 78 days from 15/10/2016 until 31 December 2016, including 31 December.
2. **Count the days from 01/01/2017 till 15/04/2017** (not included as the rule says always include the first day but not the last day of your total time period). The number of days from 1 January till 15 April, using date numbers in the date table, is  $105 - 1 = 104$ .
3. **Add the two date periods together.** Thus total number of days between 15/10/2016 and the next coupon date 15/04/2017 is  $H = 104 + 78 = 182$  days.

### Method 2:

Ignore the years and first subtract the date value in the date table of 15 April from 15 October which gives you  $288 - 105 = 183$  days. But as the two dates are in different years (2016 and 2017) you subtract 183 from 365 which gives you  $H = 365 - 183 = 182$  days.

Or day number 365 minus day number 288 plus day number 105 gives you,  $H = 182$ .

Please note we don't use a leap year in this module.

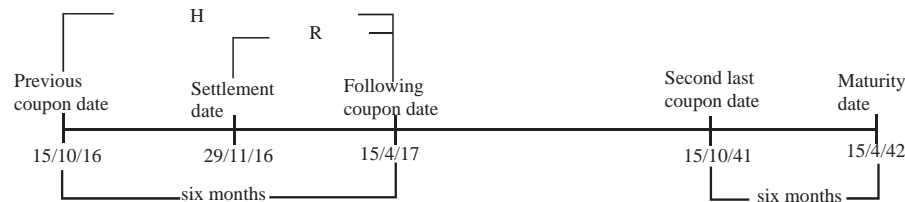
Now to calculate the accrued interest:

$$\begin{aligned}
 \text{accrued interest} &= \frac{H - R}{365} \times c \\
 &= \frac{182 - 137}{365} \times 9,75 \\
 &= 1,20205
 \end{aligned}$$

The accrued interest is R1,20205%.

**[Option 2]**

8. We need to determine the *clean price* of Bond ABC. First we draw our timeline:



We must first determine the number of years from the following coupon date until the maturity date and then multiply it by two to get the number of half yearly coupons – see notes at question 5. As the maturity date's month and the month of the following coupon date is the same, namely April we subtract the two years.

$$\begin{aligned}\text{Years} &= 42 - 17 \\ &= 25\end{aligned}$$

We now multiply 25 by two to get the number of half yearly coupons ( $n$ .) Our ( $n$ ) in the formula will be  $2 \times 25 = 50$ .

$$\begin{aligned}P(15/04/2017) &= da_{\overline{n}|z} + 100(1+z)^{-n} \\ &= \frac{9,75}{2} a_{\overline{50}|0,114\div 2} + 100 \left(1 + \frac{0,114}{2}\right)^{-50} \\ &= 86,43169\end{aligned}$$

The present value on 15 April 2017 is R86,43169%. This is a cum-interest case and we must add the coupon that we have previously ignored. This present value is thus  $R86,43169\% + R4,875\% = R91,30669\%$  and must now be discounted back to the settlement date, by using the discount factor of  $f = \frac{R}{H}$ , to determine the all-in-price.

$R$  is the number of days from the settlement date to the next coupon date and was calculated in question 7 as  $R = 137$ .



$H$  is the number of days in the halfyear in which the settlement date falls and was calculated in question 7 as  $H = 182$ .

$$\begin{aligned}\text{All-in-price} &= 91,30669 \left(1 + \frac{0,114}{2}\right)^{-137/182} \\ &= 87,57501\end{aligned}$$

The all-in-price on 29 November 2016 is 87,57501%.





$$\begin{aligned}\text{The clean price} &= \text{all-in-price} - \text{accrued interest (question 7)} \\ &= 87,57501 - 1,20205 \\ &= 86,37296\end{aligned}$$

The clean price is 86,37296%.

EL-738	HP10BII
2ndF M-CLR 0 0	 C ALL
2ndF P/Y 2 ENT ON/C	2  P/YR
$\pm 9.75 \div 2 = \text{PMT}$	$9.75 \div 2 = \pm \text{PMT}$
11.4 I/Y	11.4 I/YR
50 N	50 N
COMP PV	PV
80.17636 is displayed. Store for later use.	80.17636 is displayed
M+	$\rightarrow \text{M}$
Calculate the PV of the 100. All the values are the same except we need to make the PMT zero	Calculate the PV of the 100. All the values are the same except we need to make the PMT zero
$\pm 100 \text{ FV}$	$100 \pm \text{FV}$
0 PMT	0 PMT
COMP PV	PV
6.25534 is displayed.	6.25534 is displayed.
Add to memory + RCL M+ =	Add in memory $+\text{RM} =$
86.43169 is displayed	86.43169 is displayed
Add coupon +4.875 =	Add coupon $+4.875 =$
91.30669 is displayed. Store it as FV	91.30669 is displayed. Store it as FV
$\times \pm 1 = \text{FV}$	$\pm \text{FV}$
$137 \div 182 = \text{N}$	$137 \div 182 = \text{N}$
COMP PV	PV
87.57501 is displayed. Calculate the clean price.	87.57501 is displayed. Calculate the clean price.
$-1.20205 =$	$-1.20205 =$
86.37296 is displayed.	86.37297 is displayed.

**Note:** The answers differ in the last decimal due to rounding. Choose the answer closest.

**Alternative method:**

EL-738	HP10BII
2ndF M-CLR 0 0 2ndF P/Y I ENT ON/C $\pm 9.75 \div 2 = \text{PMT}$ $11.4 \div 2 = \text{I/Y}$ 50 N COMP PV 80.17636 <i>is displayed. Store for later use.</i> M+ 2ndF CA $\pm 100 \text{ FV}$ $11.4 \div 2 = \text{I/Y}$ 50 N COMP PV 6.25534 <i>is displayed.</i> + RCL M+ = 86.43169 <i>is displayed</i> +4.875 = 91.30669 <i>is displayed</i> 2ndF CA 2ndF P/Y 1 ENT ON/C $\pm 91.30669 \text{ FV}$ $11.4 \div 2 = \text{I/Y}$ $137 \div 182 = \text{N}$ COMP PV 87.57501 <i>is displayed.</i> -1.20205 = 86.37296 <i>is displayed.</i>	 C ALL 1  P/YR $9.75 \div 2 = \pm \text{PMT}$ $11.4 \div 2 = \text{I/YR}$ 50 N PV 80.17636 <i>is displayed</i> →M 100± FV $11.4 \div 2 = \text{I/YR}$ 50 N PV 6.25534 <i>is displayed.</i> +RM= 86.43169 <i>is displayed</i> +4.875 = 91.30669 <i>is displayed</i>  C ALL 1  P/YR 91.30669± FV $11.4 \div 2 = \text{I/YR}$ $137 \div 182 = \text{N}$ PV 87.57501 <i>is displayed.</i> -1.20205 = 86.37296 <i>is displayed.</i>

[Option 3]

9. The terms *NPV* and *profitability index* are given in the question. The formula for NPV is

$$NPV = PV_{in} - \text{initial investment}$$

and the profitability index is

$$PI = \frac{NPV + \text{initial investment}}{\text{initial investment}}.$$

Now given are the NPV and PI. Thus if we need to determine the initial investment we can only make use of the *PI* formula as the  $PV_{in}$  term in the NPV formula is not given. Now let the initial investment be  $x$ . Then:



$$\begin{aligned}
 PI &= \frac{NPV + \text{initial investment}}{\text{initial investment}} \\
 1,24375 &= \frac{195\,000 + x}{x} \\
 1,24375x &= 195\,000 + x \\
 1,24375x - x &= 195\,000 \\
 x(1,24375 - 1) &= 195\,000 \\
 x(0,24375) &= 195\,000 \\
 x &= \frac{195\,000}{0,24375} \\
 x &= 800\,000
 \end{aligned}$$




The initial investment was R800 000.

[Option 4]

10. We need to determine the equation of the regression line. Using the statistical keys of our calculator we determine that

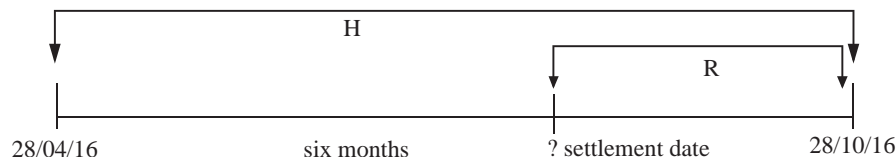
$$y = -0,016x + 17,45.$$

EL-738
<p>Use STAT mode with <b>two variable input</b> MODE 1 1</p> <p>Clear the memory keys 2ndF M CLR 00</p> <p>Enter the data 160 (x,y) 3.7 DATA [on the ENT key] 250 (x,y) 5.6 DATA 800 (x,y) 7.5 DATA 450 (x,y) 11.3 DATA 120 (x,y) 18.9 DATA 50 (x,y) 28.4 DATA ALPHA a = a = 17.45 is displayed. ALPHA b = b = -0.016 is displayed. Cancel STA mode: MODE 0</p>

HP10BII
<p>Clear the memory keys  CL <math>\Sigma</math> [on <math>\rightarrow M</math> key]</p> <p>Enter the data 160 INPUT 3.7 <math>\Sigma</math> + 250 INPUT 5.6 <math>\Sigma</math> + 800 INPUT 7.5 <math>\Sigma</math> + 450 INPUT 11.3 <math>\Sigma</math> + 120 INPUT 18.9 <math>\Sigma</math> + 50 INPUT 28.4 <math>\Sigma</math> + 0  <math>\hat{y}, m</math> [on 5 key] 17.45 is displayed  SWAP [on K key] -0.016 is displayed.</p>

[Option 1]

11. We need to determine the *settlement date* of a bond. Now given are the accrued interest, the half-yearly coupon rate and the next coupon date after the settlement date. Drawing a time line:



Now given is the accrued interest and the fact that this is a cum interest case. We use the following formula to determine  $R$ :

$$\text{Accrued interest} = \frac{H - R}{365} \times c$$

Now the given coupon date is 28 October each year. Thus the second coupon date is six months before or after (coupon payable every six months) the given date, thus 28 April each year.

$H$  is the number of days in the six months where the settlement date falls in, i.e. between 28/04/16 and 28/10/16.  $H$  equals day number 301 (28 October) minus day number 118 (28 April), namely 183. We also know that the half-yearly coupon rate is 7,375%, thus the yearly coupon rate  $c$  is  $7,375 \times 2\%$ . Now

$$\begin{aligned} 5,49589 &= \frac{183 - R}{365} \times (7,375 \times 2) \\ \left( \frac{5,49589 \times 365}{7,375 \times 2} \right) &= 183 - R \\ \left( \frac{5,49589 \times 365}{7,375 \times 2} \right) - 183 &= -R \\ R &= 183 - \left( \frac{5,49589 \times 365}{7,375 \times 2} \right) \\ R &= 47 \end{aligned}$$

Now  $R$  is the number of days between the settlement date and the next coupon date. We must now move 47 days back from 28 October 2016 to obtain the settlement date. Now 28 October is date number 301. Thus  $301 - 47$  will give us day number 254. Using the date table in the MO001 we determine that date number 254 is 11 September 2016.




<b>EL-738</b>	<b>HP10BII</b>
2ndF CA	C ALL
Use normal keys	Use normal keys
$183 - (5.49589 \times 365) \div$	$5.49589 \times 365 =$
$(7.375 \times 2) =$	$\div$ $(7.375 \times 2$ $) =$
47 is displayed.	$\rightarrow M$
	$183 - RM =$
	47 is displayed.

[Option 4]

12. We are asked to determine the IRR for five successive annual cash inflows. Thus using the formula for the IRR we can write

$$0 = \frac{75\,000}{1+R} + \frac{190\,000}{(1+R)^2} + \frac{40\,000}{(1+R)^3} + \frac{150\,000}{(1+R)^4} + \frac{180\,000}{(1+R)^5} - 500\,000$$

Using your calculator we determine that the IRR is 7,78%.



EL-738	HP10BII
2ndF M CLR 00	 C ALL
$\pm 500\,000$ DATA [ <i>last key, fifth row</i> ]	1  P/YR
75 000 DATA	500 000 $\pm$ CF <sub>j</sub> [ <i>third key, third row</i> ]
190 000 DATA	75 000 CF <sub>j</sub>
40 000 DATA	190 000 CF <sub>j</sub>
150 000 DATA	40 000 CF <sub>j</sub>
180 000 DATA	150 000 CF <sub>j</sub>
ON/C	180 000 CF <sub>j</sub>
2ndF CASH [ <i>sixth key, fourth row</i> ]	 IRR/YR
2ndF CA COMP	7.78... <i>is displayed.</i>
7.78... <i>is displayed.</i>	

[Option 1]

13. We are asked is to determine the *arithmetic mean* of the given data . The arithmetic mean ( $\bar{x}$ ) is calculated as:

$$\begin{aligned}
 \bar{x} &= \frac{\sum_{i=1}^n x_i}{n} \\
 &= \frac{360\,000 + 550\,000 + 200\,000 + 80\,000 + 700\,000}{5} \\
 &= \frac{1\,890\,000}{5} \\
 &= 378\,000.
 \end{aligned}$$

The arithmetic mean is R378 000.

<b>EL-738</b> <i>Clear the memory</i> 2ndF M CLR 0 0  <i>Change to STAT mode with <b>one</b> variable input</i> MODE 1 0  <i>Enter the data</i> 360 000 DATA 550 000 DATA 200 000 DATA 80 000 DATA 700 000 DATA  <i>Calculate the mean</i> ON/C RCL [first key, fifth row] $\bar{x}$ [first key, seventh row] 378 000 is displayed.	<b>HP10BII</b> <i>Clear the memory</i>  CL $\Sigma$ <i>Enter the data</i> 360 000 $\Sigma$ + 550 000 $\Sigma$ + 200 000 $\Sigma$ + 80 000 $\Sigma$ + 700 000 $\Sigma$ +  <i>Calculate the mean</i>  $\bar{x}, \bar{y}$ [second key, fifth row] 378 000 is displayed.
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[Option 3]

14. Given the NPV, PI and IRR we need to determine if we want to invest in Investment A or B.  
Now

Criteria	Investment A	Investment B
<b>NPV</b>	44 000 > 0 Accept	−22 000 < 0 Decline
<b>PI</b>	1,945 > 1 Accept	0,071 < 1 Decline
<b>IRR</b>	16,00 > 12,00 Accept	8,04 < 12,00 Decline


We accept Investment A as the NPV is positive, the PI is greater than 1 and the IRR is greater than the cost of capital.

[Option 1]

15. This is an *average rate of return* problem as the *average rate of return* is given. The income or cash inflows are given. First we calculate the average of all the inflows. That means add them all together and divide by the number of inflows. Secondly you express the average calculated in the first step as a ratio to the outflow, and solve the investment level:

$$\begin{aligned}
 \text{ARR} &= \frac{\text{Average after-tax income}}{\text{Investment level}} \\
 0,08421 &= \frac{(2 + 5 + 3 + 4 + 7 + 3) \text{ in hundred thousand} \div 6}{\text{Investment level}} \\
 \text{Investment level} &= \frac{400\,000}{0,08421} \\
 &= 4\,750\,029,69
 \end{aligned}$$

The original investment rounded to the nearest thousand of rands is R4 750 000.

EL-738	HP10BII
<p>2ndF M-CLR 00</p> <p><i>Using normal keys</i></p> <p><i>Calculate the average</i></p> <p>200 000 + 500 000 + 300 000</p> <p>+400 000 + 700 000 + 300 000 =</p> <p>÷6 =</p> <p>400 000 <i>is displayed. Now</i></p> <p><i>divide by ARR</i></p> <p>÷0.08421 =</p> <p>4 750 029.69 <i>to two decimals is displayed.</i></p>	<p> C ALL</p> <p><i>Using normal keys</i></p> <p><i>Calculate the average</i></p> <p>200 000 + 500 000 + 300 000</p> <p>+400 000 + 700 000 + 300 000 =</p> <p>÷6 =</p> <p>400 000 <i>is displayed. Now</i></p> <p><i>divide by ARR</i></p> <p>÷0.08421 =</p> <p>4 750 029.69 <i>to two decimals is displayed.</i></p>

[Option 4]