Tutorial letter 203/2/2018

Formal Logic 3

COS3761

Semester 2

School of Computing

Solutions to Assignment 3

BAR CODE

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SOLUTIONS FOR ASSIGNMENT 3 FIRST SEMESTER

Provided below are the questions, the **correct answers in bold**, and explanations, for all questions of the third assignment.

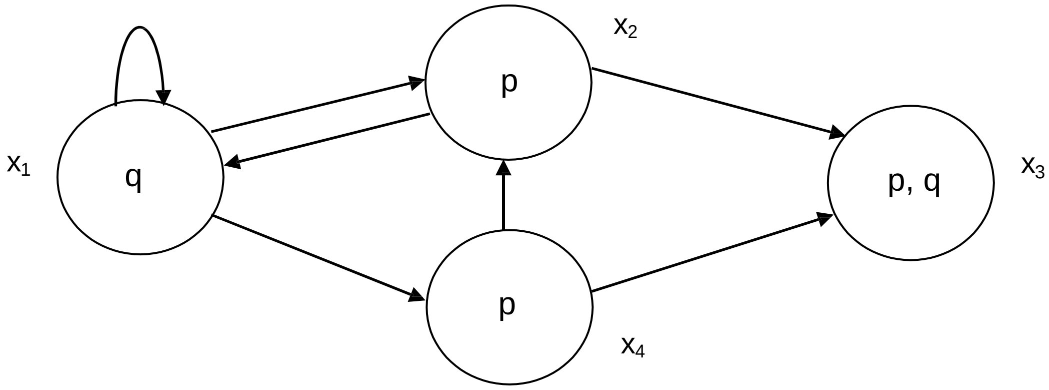


Figure 1: Kripke model used in Questions 1, 2, 3, 4 and 5

QUESTION 1

In which world of the Kripke model in Figure 1 is the formula $\Box p \land \Diamond \Box q$ true?

Option 1:	x_1	*The formula is not true in* x_1 *because* p *is not true in* x_1 *and* x_1 *is accessible from itself. Therefore* $\Box p$ *is not true in* x_1*, and therefore* $\Box p \land \Diamond \Box q$ *is not true in* x_1*.*
Option 2:	x_2	*The formula is not true in* x_2 *because* p *is not true in* x_1 *and* x_1 *is accessible from* x_2*. Therefore* $\Box p$ *is not true in* x_2*, and therefore* $\Box p \land \Diamond \Box q$ *is not true in* x_2*.*
Option 3:	x_3	*The formula is not true in* x_3 *because there is no world accessible from* x_3*. Therefore* $\Diamond \Box q$ *is not true in* x_3*, and therefore* $\Box p \land \Diamond \Box q$ *is not true in* x_3*.*
Option 4:	**x_4**	*The formula is true in* x_4 *because* p *is true in all worlds accessible from* x_4*, namely* x_2 *and* x_3*. Therefore* $\Box p$ *is true in* x_4*. Furthermore,* x_2 *is accessible from* x_4 *and* $\Box q$ *is true in* x_2 *(because* q *is true in all worlds accessible from* x_2*), so* $\Diamond \Box q$ *is true in* x_4*, and therefore* $\Box p \land \Diamond \Box q$ *is true in* x_4*.*
Option 5:	The formula is not true in any world of the Kripke model.	*See explanation of Option 4*.

QUESTION 2

Which of the following holds in the Kripke model given in Figure 1?

Option 1:	$x_1 \Vdash \Box p$	p *is not true in* x_1*, and since* x_1 *is accessible from itself,* $\Box p$ *is not true in* x_1.
Option 2:	$x_2 \Vdash \Box (p \wedge q)$	$p \wedge q$ *is not true in* x_1*, and since* x_1 *is accessible from* x_2*,* $\Box (p \wedge q)$ *is not true in* x_2.
Option 3:	$x_3 \Vdash \Box p \wedge \Diamond q$	$\Box p$ *is vacuously true in* x_3 *since there are no worlds accessible from* x_3*, but* $\Diamond q$ *is not true in* x_3 *for the same reason.*
Option 4:	$\mathbf{x_3 \Vdash \Box \Box \neg p}$	$\Box \Box \neg p$ *is vacuously true in* x_3 *since there are no worlds accessible from* x_3*.*
Option 5:	$x_4 \Vdash p \rightarrow q$	$p \rightarrow q$ *is not true in* x_4 *since* p *is true and* q *is false in* x_4.

QUESTION 3

Which of the following does not hold in the Kripke model given in Figure 1?

Option 1:	$x_1 \Vdash \Diamond q$	$\Diamond q$ *holds in* x_1 *because* q *is true in at least one world accessible from* x_1*, namely* x_1.
Option 2:	$x_1 \Vdash \Box (p \vee q)$	$\Box (p \vee q)$ *holds in* x_1 *because* $p \vee q$ is *true in all worlds accessible from* x_1*, namely* x_1, x_2 *and* x_4.
Option 3:	$x_2 \Vdash \Diamond p \wedge \Box q$	$\Diamond p \wedge \Box q$ *holds in* x_2 *because* $\Diamond p$ *is true in* x_2 (*because* p *is true in at least one world accessible from* x_2, *namely* x_2) *and* $\Box q$ *is true in* x_2 (*because* q *is true in all worlds accessible from* x_2, *namely* x_1 *and* x_3).
Option 4:	$x_3 \Vdash \Box (p \wedge \neg q)$	$\Box (p \wedge \neg q)$ *holds in* x_3 *because* $p \wedge \neg q$ *is vacuously true in all worlds accessible from* x_3, *since there aren't any such worlds*.
Option 5:	$\mathbf{x_4 \Vdash \Box (p \rightarrow q)}$	$\Box (p \rightarrow q)$ *does not hold in* x_4 *because* $p \rightarrow q$ *is not true in all worlds accessible from* x_4, *namely* x_2 *and* x_3 (*because* p *is true and* q *is false in* x_2 *and so* $p \rightarrow q$ *is false in* x_2).

QUESTION 4

Which of the following formulas is true in the Kripke model given in Figure 1?

Option 1: ◊ p — ◊ p *is not true in* x_3 *since there are no worlds accessible from* x_3. *Since* ◊ p *is not true in one of the worlds of the Kripke model, it is not true in the whole Kripke model.*

Option 2: □ q — □ q *is not true in* x_1 *because* q *is not true in all worlds accessible from* x_1. *So* □ q *is not true in the Kripke model*.

Option 3: □ ◊ q — □ ◊ q *is not true in* x_2 (*and* x_4) *because* x_3 *is one of the worlds accessible from it and* ◊ q *is not true in* x_3. *So* □ ◊ q *is not true in the Kripke model*.

Option 4: **□ (p ∨ q)** — p ∨ q *is true in every world of the Kripke model*, *so it is true in any world accessible from every world, so it is true in the Kripke model*.

Option 5: q → p — q → p *is not true in* x_1 *because* q *is true and* p *is false in* x_1. *So* q → p *is not true in the Kripke model*.

QUESTION 5

Which of the following formulas is false in the Kripke model given in Figure 1?

Option 1: p ∨ q — p ∨ q *is true in every world of the Kripke model*, *so it is true in the Kripke model*.

***Option 2*:** **□ ◊ p** — □ ◊ p *is not true in* x_2 *since* ◊ p *is not true in all worlds accessible from* x_2. *In particular*, ◊ p *is not true in* x_3 *since there are no worlds accessible from* x_3.

Option 3: □ (p ∨ q) — p ∨ q *is true in every world of the Kripke model*, *so it is true in any world accessible from every world, so it is true in the Kripke model*.

Option 4: p ∨ ◊ q — p *is true in worlds* x_2, x_3 *and* x_4, *and* ◊ q *is true in* x_1, *so* p ∨ ◊ q *is true in all worlds of the Kripke model*.

Option 5: □ p ∨ ◊ q — □ p *is true in worlds* x_3 *and* x_4 (*since* p *is true in all worlds accessible from* x_3 *and* x_4) *and* ◊ q *is true in worlds* x_1 *and* x_2 (*since* x_1 *and* x_2 *each have a world accessible from them in which* q *is true*).

QUESTION 6

If we interpret □ φ as "It ought to be that φ ", which of the following formulas correctly expresses the English sentence

It ought to be that if I am happy, I'm allowed to be unhappy.

where p stands for the declarative sentence "I am happy"?

Option 1:	$\Box p \to \Diamond \neg p$	*This formula states "If I ought to be happy, then I am allowed to be unhappy", which is not the same as the declarative sentence.*
Option 2:	$\Diamond \neg p \vee \neg \Box p$	*This formula is equivalent to* $\neg \Diamond \neg p \to \neg \Box p$, *which is equivalent to* $\Box p \to \neg \Box p$, *which states that "If I ought to be happy, then I ought not to be happy", which is not the same as the declarative sentence.*
***Option 3*:**	$\Box (p \to \neg \Box p)$	$\neg \Box p$ *is the same as* $\neg \neg \Diamond \neg p$ *which is the same as* $\Diamond \neg p$. *So the formula correctly expresses the sentence*.
Option 4:	$\Box (p \to \neg \Diamond p)$	*This formula states "It ought to be the case that if I am happy, then I am not allowed to be happy", which is not the same as the declarative sentence.*
Option 5:	It is impossible to translate this sentence into a formula of modal logic with the required interpretation.	*See comment on Option 3*.

QUESTION 7

If we interpret $\Box \phi$ as "It is necessarily true that ϕ", why should the formula scheme $\Box \phi \to \phi$ hold in this modality?

Option 1:	Because for all formulas ϕ, it is necessarily true that if ϕ then ϕ.	*This explanation does not correctly express the formula scheme. It represents the formula* $\Box (\phi \to \phi)$.
***Option 2*:**	**Because for all formulas ϕ, if ϕ is necessarily true, then it is true.**	*This explanation makes common sense.*
Option 3:	Because for all formulas ϕ, if ϕ is not possibly true, then it is true.	*This explanation represents the formula* $\neg \Diamond \phi \to \phi$, *which is equivalent to* $\neg \neg \Box \neg \phi \to \phi$, *which is equivalent to* $\Box \neg \phi \to \phi$. *This is not the same as the given formula scheme*.
Option 4:	Because for all formulas ϕ, ϕ is necessarily true if it is true.	This explanation represents the formula which is equivalent to $\phi \to \Box \phi$, *which is not the same as the given formula scheme*.
Option 5:	$\Box \phi \to \phi$ should not hold in this modality.	*Yes it should. See Table 5.7 in the prescribed book*.

QUESTION 8

If we interpret $\Box\phi$ as "After any execution of program P, ϕ holds", why should the formula scheme $\Box\phi \rightarrow \phi$ not hold in this modality?

***Option 1*:**	**Just because ϕ holds after every execution of P doesn't necessarily mean that ϕ holds before execution of P.**	*The formula scheme states "If ϕ holds after any execution of program* P, *then ϕ holds". The explanation follows from the common sense meaning of this sentence, which is incorrect.*
Option 2:	Because it is not the case that after any execution of P, if ϕ holds then ϕ holds.	*This explanation expresses the formula scheme* $\Box(\phi \rightarrow \phi)$ *which is not the same as the given formula scheme.*
Option 3:	Because if ϕ does not hold before execution of P, it doesn't necessarily mean that ϕ holds after any execution of P.	*This explanation expresses the formula scheme* $\phi \rightarrow \Box\phi$ *which is not the same as the given formula scheme.*
Option 4:	Because if ϕ does not hold after every execution of P, it doesn't necessarily mean that ϕ holds before any execution of P.	*This explanation expresses the formula scheme* $\neg\Box\phi \rightarrow \phi$. *Although it may be correct that this should not hold in this modality, this formula scheme is not the same as the given formula scheme.*
Option 5:	$\Box\phi \rightarrow \phi$ should hold in this modality, because if ϕ holds after any execution of P, then ϕ holds.	*This is not correct, because say ϕ represents the assertion that "`Hello world` is displayed on the screen." Then just because ϕ holds after program P is executed, doesn't mean that `Hello world` is always displayed on the screen (particularly before P is executed).*

QUESTION 9

If we interpret $\Box\phi$ as "Always in the future (where the future does not include the present) it will be true that ϕ", which of the following formulas should not be valid?

Option 1:	$\Box p \to \Box \Box p$	*This formula should be valid, because it states that if* p *will always be true in the future, then in the future*, p *will always be true in the future.*
Option 2:	$\neg \Box p \lor \neg \Diamond \Diamond \neg p$	*This formula is equivalent to* $\Box p \to \Box \Box p$, *because* $\neg \Diamond \neg \phi$ *is the same as* $\Box \phi$, *and* $\neg \phi_1 \lor \phi_2$ *is equivalent to* $\phi_1 \to \phi_2$. *So it should be valid according to the argument given for Option 1*.
Option 3:	$\neg \Diamond \neg p \to \Box p$	*This formula should be valid because it is equivalent to* $\Box p \to \Box p$.
Option 4:	$\neg \Box \Box p \to \Diamond \neg p$	*This formula is equivalent to* $\neg \Box \Box p \to \neg \Box \neg \neg p$, *which is equivalent to* $\Box p \to \Box \Box p$. *So it should be valid according to the argument given for Option 1*.
Option 5:	$\Box \Box p \lor \neg \Box p$	*This formula is equivalent to* $\Box p \to \Box \Box p$*, so it should be valid according to the argument given for Option 1*.

Please note that Question 9 was not marked, because the formulas in all the options are valid, and there is no invalid formula.

QUESTION 10

If we interpret $\Box \phi$ as "Agent A believes ϕ", what is the English translation of the formula $\Box p \to \neg \Diamond q$?

Option 1:	If Agent A believes p, then Agent B does not believe q.	$\Diamond q$ *does not refer to Agent B. It can be expressed as* "q *is consistent with Agent A's beliefs*".
Option 2:	Agent A believes that if p, then q is not consistent with Agent A's beliefs.	*This explanation expresses the formula* $\Box (p \to \neg \Diamond q)$ *which is not the same as the given formula*.
Option 3:	Agent A believes that if p, then Agent B does not believe q.	*See comment on Option 1.*
Option 4:	**If Agent A believes p, then Agent A believes not q.**	*This explanation expresses the formula* $\Box p \to \Box \neg q$ *which is the same as the given formula due to the duality of* $\Box$ *and* $\Diamond$.
Option 5:	Agent A believes p if Agent A believes not q.	*This explanation expresses the formula* $\Box \neg q \to \Box p$ *which is the converse of the given formula*.

QUESTION 11

If we interpret $\Box \phi$ as "Agent A believes ϕ", what formula will be correctly translated to English as

Agent A does not believe p or q.

***Option 1*:** **$\Diamond \neg (p \vee q)$** *This formula is equivalent to* $\neg \Box (p \vee q)$, *which is correctly translated to English as the given sentence*.

Option 2: $\Box \neg (p \vee q)$ *The translation of this formula is* "*Agent A believes that* p *or* q *is not true*." *This is not the same as the given sentence.*

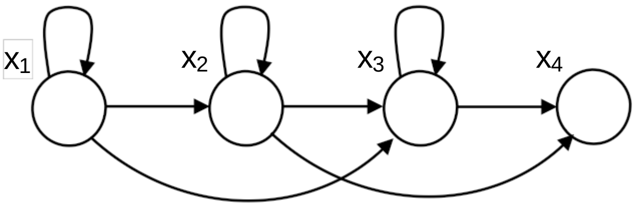
Option 3: $\neg \Box p \vee q$ *The translation of this formula is* "*Agent A does not believe* p, *or* q *is true*." *This is not the same as the given sentence.*

Option 4: $\Diamond \neg p \vee q$ *This formula is equivalent to* $\neg \Box p \vee q$ *due to the duality of* $\Box$ *and* $\Diamond$. *See comment on Option 3*.

Option 5: $\Box \neg p \wedge \Box \neg q$ *This formula is equivalent to* $\Box (\neg p \wedge \neg q)$ *due to equivalences 5.3 on page 314 of the prescribed book, which is equivalent to* $\Box \neg (p \vee q)$ *due to De Morgan's rule. See comment on Option 2.*

QUESTION 12

Consider the following Kripke frame:



Which of the following modal logics does this frame conform to?

Option 1: KT *The relation is not reflexive because not all worlds are accessible from themselves, in particular* x_4.

Option 2: KB *The relation is not symmetric, because not all pairs of worlds are mutually accessible. For example*, x_2 *is accessible from* x_1, *but* x_1 *is not accessible from* x_2.

Option 3: KD *The relation is not serial, because not every world has a world which is accessible from it, in particular* x_4.

***Option 4*:** **K4** *The relation is transitive, because for all pairs of worlds* x_i *and* x_k *such that there is an* x_j *such that* x_j *is accessible from* x_i, *and* x_k *is accessible from* x_j, *then* x_k *is accessible from* x_i.

Option 5: KT45 *The relation is not functional because every world is not accessible by a maximum of one other world. For example*, x_3 *is accessible from three worlds, namely* x_1, x_2 *and* x_3.

The following natural deduction proof (without reasons) is referred to in Questions 13, 14 and 15:

1	$\Box (p \wedge q)$
2	$p \wedge q$
3	p
4	q
5	$\Box p$
6	$\Box q$
7	$\Box p \wedge \Box q$
8	$\Box (p \wedge q) \rightarrow (\Box p \wedge \Box q)$

QUESTION 13

How many times are □ elimination and introduction rules used in the above proof?

Option 1:	None	*See comment on Option 3.*
Option 2:	□ elimination and □ introduction are both only used once.	*See comment on Option 3.*
***Option 3*:**	**□ elimination is used only once but □ introduction twice.**	*□ elimination is used in line 2, and □ introduction is used in lines 5 and 6. □ elimination always occur somewhere inside a dashed box, □ and introduction always occurs somewhere after a dashed box.*
Option 4:	□ elimination is used twice but □ introduction only once.	*See comment on Option 3.*
Option 5:	□ elimination and □ introduction are both used twice.	*See comment on Option 3.*

QUESTION 14

What is the correct reason for steps 1, 2 and 3 of the above proof?

Option 1:	1 2 3	assumption axiom T in line 1 $\land$e 2	*Although line 1 is an assumption, line 2 is inside a dashed box, so it utilises* $\Box$ *elimination, not axiom T.*
Option 2:	1 2 3	premise $\Box$i 1 $\Box$e 2	*The first line in a solid box is always an assumption.*
Option 3:	1 2 3	premise assumption $\land$i 2	*The first line in a solid box is always an assumption.*
Option 4:	1 2 3	$\Box$i 1 axiom T in line 2 $\land$i 3,4	*The first line in a solid box is always an assumption.*
***Option 5*:**	**1** **2** **3**	**assumption** $\Box$**e 1** $\land$**e 2**	*The first formula inside a (solid) box is always an assumption, so the reason for line 1 is correct. Line 2 is inside a dashed box, so this is the application of* $\Box$ *elimination. Line 3 is indeed an application of* $\land$ *elimination applied to line 2.*

QUESTION 15

What sequent is proved by the above proof?

Option 1:	$\Box (p \land q) \quad \Box (p \land q) \rightarrow (\Box p \land \Box q)$	*There are no premises in the sequent because there are no assumptions outside of boxes in the proof.*
Option 2:	$\Box (p \land q) \quad \Box p \land \Box q$	*If the final step was omitted, as well as the box around lines 1 to 7, then this would be a valid sequent for the proof.*
***Option 3*:**	$\mathbf{\Box (p \land q) \rightarrow (\Box p \land \Box q)}$	*Since* $\Box (p \land q)$ *is an assumption, and* $\Box p \land \Box q$ *is proved in consequence inside the same box, we can use* $\rightarrow$ *introduction to prove* $\Box (p \land q) \rightarrow (\Box p \land \Box q)$ *outside the box. Since no other assumptions outside the box were used, there are no premises of the sequent.*
Option 4:	$\Box p \land \Box q$	$\Box p \land \Box q$ *is proved on the basis of the assumption* $\Box (p \land q)$, *so it is not a valid sequent on its own.*
Option 5:	It's impossible to say without the reasons.	*This is incorrect because you can fill in the reasons first.*

The following incomplete natural deduction proof is referred to in Questions 16 and 17:

1	$\Box \neg \Box \neg \Box p$	assumption
2		
3	$\neg \Box p$	assumption
4		
5	$\bot$	¬e 2,4
6	$\Box p$	PBC 3-5
7		
8	$\Box \neg \Box \neg \Box p \rightarrow \Box \Box p$	→i 1-7

QUESTION 16

Rules T, 4 and 5 are used in the missing lines of the above proof. Which rule is used in which line?

Option 1:	Rule T is used in line 2, rule 4 is used in line 4 and rule 5 is used in line 7.	*If we use rule T in line 2, we will get a negated formula. We can only use rule 4 on a positive formula, so the only valid application would be on line 1 to produce* $\Box \Box \neg \Box \neg \Box p$. *Similarly, we can only use rule 5 on a negated formula, so the only valid applications would be on lines 2 or 3 to produce* $\Box \neg\Box \neg \Box p$ *or* $\Box \neg \Box p$. *Neither of these options would make step 8 correct*.
Option 2:	Rule 4 is used in line 2, rule 5 is used in line 4 and rule T is used in line 7.	*If rule 4 is applied in line 2, it will produce the formula* $\Box \Box \neg \Box \neg \Box p$. *To be able to use ¬e 2,4 in line 5, the formula in line 4 would have to be* $\neg \Box \Box \neg \Box \neg \Box p$. *Using rule 5 in line 4 cannot produce this*.
***Option 3*:**	**Rule T is used in line 2, rule 5 is used in line 4 and rule 4 is used in line 7.**	*Since ¬e 2,4 is used in line 5, the formulas in lines 2 and 4 must be the complements of one another. We therefore need to use rule T in line 2 to strip a box from the formula in line 1, and rule 5 in line 4 to add a box to the formula in line 3. Finally, we need to use rule 4 in line 7 to add a box to the formula in line 6.*
Option 4:	Rule 5 is used in line 2, rule T is used in line 4 and rule 4 is used in line 7.	*Rule 5 cannot be used in line 2 because the only line it can be applied to is line 1, and the formula in line 1 is not a negated formula.*
Option 5:	Rule 4 is used in line 2, rule T is used in line 4 and rule 5 is used in line 7.	*If rule 4 is applied in line 2, it will produce the formula* $\Box \Box \neg \Box \neg \Box p$. *To be able to use ¬e 2,4 in line 5, the formula in line 4 would have to be* $\neg \Box \Box \neg \Box \neg \Box p$. *Using rule T in line 4 cannot produce this*.

QUESTION 17

What formulas belong in the missing lines of the above proof?

***Option 1*: ¬ □ ¬ □ p in line 2, □ ¬ □ p in line 4 and □ □ p in line 7**

Option 2: □ □ ¬ □ ¬ □ p in line 2, □ ¬ □ ¬ □ p in line 4 and □ □ p in line 7

Option 3: ¬ □ ¬ □ p in line 2, ¬ □ ¬ □ p in line 4 and p in line 7

Option 4: □ □ ¬ □ ¬ □ p in line 2, ¬ □ ¬ □ p in line 4 and p in line 7

Option 5: □ ¬ □ p in line 2, ¬ □ ¬ □ p in line 4 and □ ¬ □ ¬ □ p → □ □ p in line 7

The fact that ¬e 2,4 is used in line 5 means that the formulas in lines 2 and 4 must be the complements of one another. This immediately eliminates options 2, 3 and 4. It would be meaningless to use the formula □ ¬ □ ¬ □ p → □ □ p *in line 7, as in option 5, since step 8 would be redundant (and incorrect). The formulas given in option 1, together with the reasons of option 1 of Question 16, will give a correct proof.*

QUESTION 18

What proof strategy would you use to prove the following sequent:

$\vdash_{KT4}$ □ □ (p ∧ q) → (□ p ∧ □ q)

***Option 1*:**

- **Open a solid box and start with □ □ (p ∧ q) as an assumption.**
- **Use axiom T to remove one □.**
- **Open a dashed box and use □ elimination to get p ∧ q.**
- **Use ∧ elimination twice to obtain the separate atomic formulas.**
- **Close the dashed box and use □ introduction twice, i.e. once on each of the atomic formulas.**
- **Combine □ p and □ q using ∧ introduction.**
- **Close the solid box and use → introduction on the first and last formulas to get the result.**

You can confirm that this is a correct proof strategy by writing out the proof, following the instructions step by step.

Option 2:	• Start with □ □ (p ∧ q) as a premise. • Use axiom T twice to remove both □ to get p ∧ q. • Use ∧ elimination once to obtain the separate atomic formulas. • Use axiom 4 twice, i.e. once on each atomic formula to add a □. • Combine □ p and □ q using ∧ introduction. • Use → introduction on the first and last formulas to get the result.	*The final step of this proof strategy is incorrect, as you can only use* → i *outside a solid box that starts with the antecedent and ends with the consequent. Furthermore, when you start with a premise, then the sequent which you are proving should have that formula to the left of the turnstile.*
Option 3:	• Start with □ □ (p ∧ q) as a premise. • Open a dashed box and use □ elimination to get □ (p ∧ q). • Open another dashed box and use □ elimination to get p ∧ q. • Use ∧ elimination twice to obtain the separate atomic formulas. • Close the first dashed box and use □ introduction on the first atomic formula. • Close the second dashed box and use □ introduction on the second atomic formula. • Combine □ p and □ q using ∧ introduction. • Use → introduction on the first and last formulas to get the result.	*See comment on Option 2.*
Option 4:	• Open a solid box and start with □ □ (p ∧ q) as an assumption. • Open a dashed box and use □ elimination to get □ (p ∧ q). • Use axiom T to remove one □ to get p ∧ q. • Use ∧ elimination twice to obtain the separate atomic formulas. • Use axiom 4 twice, i.e. once on each atomic formula to add a □. • Close the dashed box and combine □ p and □ q using ∧ introduction. • Close the solid box and use → introduction on the first and last formulas to get the result.	*The problem with this proof strategy is that it uses rule 4 to add boxes to* p *and* q*. This is not allowed. Rule 4 can only be used to add a box to a formula which already starts with a box.*
Option 5:	This is not a valid sequent in KT4.	*It is, since there is a correct natural deduction proof for it. See comment on Option 1.*

QUESTION 19

If we interpret $K_i \phi$ as "Agent i knows ϕ", what is the English translation of the formula $K_1 K_2 \neg p \rightarrow q$?

Option 1:	Agent 1 knows that agent 2 doesn't know p implies q.	*This explanation expresses the formula* $K_1 \neg K_2 p \rightarrow q$, *which is not the same as the given formula.*
Option 2:	Agent 1 knows that agent 2 knows that not p implies q.	*This explanation expresses the formula* $K_1 K_2 (\neg p \rightarrow q)$, *which is not the same as the given formula.*
Option 3:	Agent 1 knows that if agent 2 doesn't know p, then q.	*This explanation expresses the formula* $K_1 (\neg K_2 p \rightarrow q)$, *which is not the same as the given formula.*
Option 4:	Agent 1 knows that if agent 2 knows not p, then q.	*This explanation expresses the formula* $K_1 ((K_2 \neg p) \rightarrow q)$, *which is not the same as the given formula.*
***Option 5*:**	**If agent 1 knows that agent 2 knows not p, then q.**	*This explanation expresses the formula* $(K_1 K_2 \neg p) \rightarrow q$, *which is the same as the given formula due to the precedence of the modal operators over implication.*

QUESTION 20

If we interpret $K_i \phi$ as "Agent i knows ϕ", what formula of modal logic is correctly translated to English as

Agent 1 knows p but he doesn't know that agent 2 knows q.

Option 1:	$K_1 (p \wedge \neg K_2 q)$	*This formula can be translated to English as follows: "Agent 1 knows* p *and that agent 2 doesn't know* q*." This is not the same as the required translation.*
Option 2:	$K_1 (p \wedge K_2 \neg q)$	*This formula can be translated to English as follows: "Agent 1 knows* p *and that agent 2 knows not* q*." This is not the same as the required translation.*
***Option 3*:**	$\mathbf{K_1 p \wedge \neg K_1 K_2 q}$	*This formula is correctly translated to the given sentence.*
Option 4:	$K_1 p \wedge K_1 \neg K_2 q$	*This formula can be translated to English as follows: "Agent 1 knows* p *and he knows that agent 2 doesn't know* q*." This is not the same as the required translation.*
Option 5:	$K_1 p \wedge K_1 K_2 \neg q$	*This formula can be translated to English as follows: "Agent 1 knows* p *and he knows that agent 2 knows not* q*." This is not the same as the required translation.*