Tutorial letter 203/1/2018

Formal Logic 3

COS3761

Semester 1

School of Computing

Solutions to Assignment 3

BAR CODE

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SOLUTIONS FOR ASSIGNMENT 3 FIRST SEMESTER

Provided below are the questions, the **correct answers in bold**, and explanations, for all questions of the third assignment.

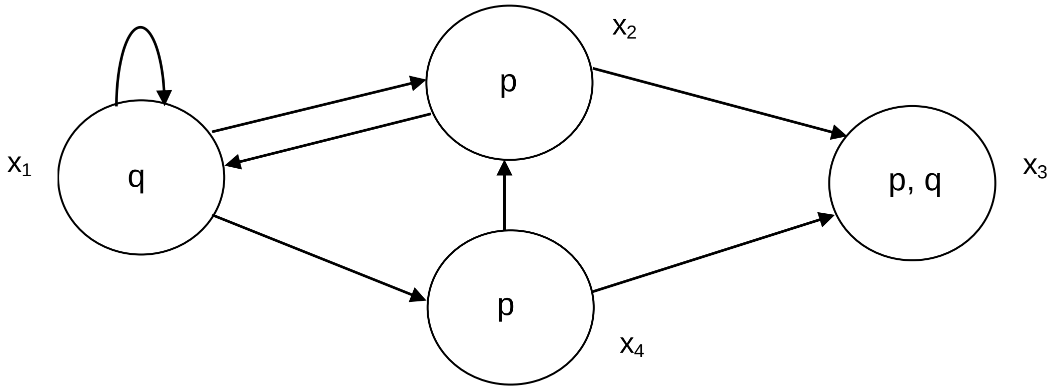


Figure 1: Kripke model used in Questions 1, 2, 3, 4 and 5

QUESTION 1

In which world of the Kripke model in Figure 1 is the formula $\Box p \wedge \Diamond \Box q$ true?

Option 1: x_1

Option 2: x_2

Option 3: x_3

***Option 4*: $\mathbf{x_4}$**

Option 5: The formula is not true in any world of the Kripke model.

QUESTION 2

Which of the following holds in the Kripke model given in Figure 1?

Option 1: $x_1 \Vdash \Box p$

Option 2: $x_2 \Vdash \Box (p \wedge q)$

Option 3: $x_3 \Vdash \Box p \wedge \Diamond q$

***Option 4*: $\mathbf{x_3 \Vdash \Box \Box \neg p}$**

Option 5: $x_4 \Vdash p \rightarrow q$

QUESTION 3

Which of the following does not hold in the Kripke model given in Figure 1?

Option 1: $x_1 \Vdash \Diamond q$

Option 2: $x_1 \Vdash \Box (p \vee q)$

Option 3: $x_2 \Vdash \Diamond p \wedge \Box q$

Option 4: $x_3 \Vdash \Box (p \wedge \neg q)$

***Option 5*: $x_4 \Vdash \Box (p \to q)$**

QUESTION 4

Which of the following formulas is true in the Kripke model given in Figure 1?

Option 1: $\Diamond p$

Option 2: $\Box q$

Option 3: $\Box \Diamond q$

***Option 4*: $\Box (p \vee q)$**

Option 5: $q \to p$

QUESTION 5

Which of the following formulas is false in the Kripke model given in Figure 1?

Option 1: $p \vee q$

***Option 2*: $\Box \Diamond p$**

Option 3: $\Box (p \vee q)$

Option 4: $p \vee \Diamond q$

Option 5: $\Box p \vee \Diamond q$

QUESTION 6

If we interpret $\Box \phi$ as "It ought to be that ϕ", which of the following formulas correctly expresses the English sentence

It ought to be that if I am happy, I'm allowed to be unhappy.

where p stands for the declarative sentence "I am happy"?

Option 1: $\Box p \to \Diamond \neg p$

Option 2: $\Diamond \neg p \vee \neg \Box p$

***Option 3*: $\Box (p \to \neg \Box p)$**

Option 4: $\Box (p \to \neg \Diamond p)$

Option 5: It is impossible to translate this sentence into a formula of modal logic with the required interpretation.

QUESTION 7

If we interpret $\Box \phi$ as "It is necessarily true that ϕ", why should the formula scheme $\Box \phi \to \phi$ hold in this modality?

Option 1: Because for all formulas ϕ, it is necessarily true that if ϕ then ϕ.

***Option 2*: Because for all formulas ϕ, if ϕ is necessarily true, then it is true.**

Option 3: Because for all formulas ϕ, if ϕ is not possibly true, then it is true.

Option 4: Because for all formulas ϕ, ϕ is necessarily true if it is true.

Option 5: $\Box \phi \to \phi$ should not hold in this modality.

QUESTION 8

If we interpret $\Box \phi$ as "After any execution of program P, ϕ holds", why should the formula scheme $\Box \phi \to \phi$ not hold in this modality?

***Option 1*: Just because ϕ holds after every execution of P doesn't necessarily mean that ϕ holds before execution of P.**

Option 2: Because it is not that case that after any execution of P, if ϕ holds then ϕ holds.

Option 3: Because if ϕ does not hold before execution of P, it doesn't necessarily mean that ϕ holds after any execution of P.

Option 4: Because if ϕ does not hold after every execution of P, it doesn't necessarily mean that ϕ holds before any execution of P.

Option 5: $\Box \phi \to \phi$ should hold in this modality, because if ϕ holds after any execution of P, then ϕ holds.

QUESTION 9

If we interpret $\Box \phi$ as "Always in the future (where the future does not include the present) it will be true that ϕ", which of the following formulas should be valid?

Option 1: $\Box p \to \Box \Box p$

Option 2: $\neg \Box p \lor \neg \Diamond \Diamond \neg p$

Option 3: $\neg \Diamond \neg p \to \Box p$

Option 4: $\neg \Box \Box p \to \Diamond \neg p$

***Option 5*: All of the above are valid**

QUESTION 10

If we interpret $\Box \phi$ as "Agent A believes ϕ", what is the English translation of the formula $\Box p \to \neg \Diamond q$?

Option 1: If Agent A believes p, then Agent B does not believe q.

Option 2: Agent A believes that if p, then q is not consistent with Agent A's beliefs.

Option 3: Agent A believes that if p, then Agent B does not believe q.

***Option 4*: If Agent A believes p, then Agent A believes not q.**

Option 5: Agent A believes p if Agent A believes not q.

QUESTION 11

If we interpret $\Box \phi$ as "Agent A believes ϕ", what formula will be correctly translated to English as

Agent A does not believe p or q.

***Option 1*: $\Diamond \neg (p \lor q)$**

Option 2: $\Box \neg (p \lor q)$

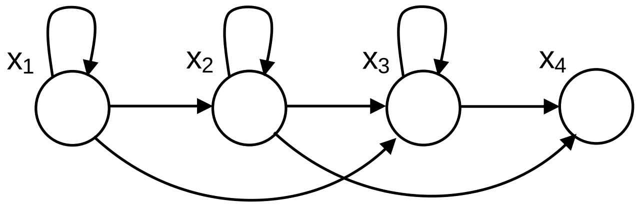
Option 3: $\neg \Box p \lor q$

Option 4: $\Diamond \neg p \lor q$

Option 5: $\Box \neg p \land \Box \neg q$

QUESTION 12

Consider the following Kripke frame:



Which of the following modal logics does this frame conform to?

Option 1: KT

Option 2: KB

Option 3: KD

***Option 4*: K4**

Option 5: KT45

The following natural deduction proof (without reasons) is referred to in Questions 13, 14 and 15:

1	$\Box (p \land q)$
2	$p \land q$
3	p
4	q
5	$\Box p$
6	$\Box q$
7	$\Box p \land \Box q$
8	$\Box (p \land q) \rightarrow (\Box p \land \Box q)$

QUESTION 13

How many times are □ elimination and introduction rules used in the above proof?

Option 1: None

Option 2: □ elimination and □ introduction are both only used once.

***Option 3*: □ elimination is used only once but □ introduction twice.**

Option 4: □ elimination is used twice but □ introduction only once.

Option 5: □ elimination and □ introduction are both used twice.

QUESTION 14

What is the correct reason for steps 1, 2 and 3 of the above proof?

Option 1:	1	assumption
	2	axiom T in line 1
	3	$\land$e 2
Option 2:	1	premise
	2	$\Box$i 1
	3	$\Box$e 2
Option 3:	1	premise
	2	assumption
	3	$\land$i 2
Option 4:	1	$\Box$i 1
	2	axiom T in line 2
	3	$\land$i 3,4
***Option 5*:**	**1**	**assumption**
	2	**$\Box$e 1**
	3	**$\land$e 2**

QUESTION 15

What sequent is proved by the above proof?

Option 1: $\Box (p \land q) \vdash \Box (p \land q) \rightarrow (\Box p \land \Box q)$

Option 2: $\Box (p \land q) \vdash \Box p \land \Box q$

***Option 3*:** **$\vdash \Box (p \land q) \rightarrow (\Box p \land \Box q)$**

Option 4: $\vdash \Box p \land \Box q$

Option 5: It's impossible to say without the reasons.

The following incomplete natural deduction proof is referred to in Questions 16 and 17:

1	$\Box \neg \Box \neg \Box p$	assumption
2		
3	$\neg \Box p$	assumption
4		
5	$\bot$	$\neg$e 2,4

6	□ p	PBC 3-5
7		
8	□ ¬ □ ¬ □ p → □ □ p	→i 1-7

QUESTION 16

Rules T, 4 and 5 are used in the missing lines of the above proof. Which rule is used in which line?

Option 1: Rule T is used in line 2, rule 4 is used in line 4 and rule 5 is used in line 7.

Option 2: Rule 4 is used in line 2, rule 5 is used in line 4 and rule T is used in line 7.

***Option 3*: Rule T is used in line 2, rule 5 is used in line 4 and rule 4 is used in line 7.**

Option 4: Rule 5 is used in line 2, rule T is used in line 4 and rule 4 is used in line 7.

Option 5: Rule 4 is used in line 2, rule T is used in line 4 and rule 5 is used in line 7.

QUESTION 17

What formulas belong in the missing lines of the above proof?

***Option 1*: ¬ □ ¬ □ p in line 2, □ ¬ □ p in line 4 and □ □ p in line 7**

Option 2: □ □ ¬ □ ¬ □ p in line 2, □ ¬ □ ¬ □ p in line 4 and □ □ p in line 7

Option 3: ¬ □ ¬ □ p in line 2, ¬ □ ¬ □ p in line 4 and p in line 7

Option 4: □ □ ¬ □ ¬ □ p in line 2, ¬ □ ¬ □ p in line 4 and p in line 7

Option 5: □ ¬ □ p in line 2, ¬ □ ¬ □ p in line 4 and □ ¬ □ ¬ □ p → □ □ p in line 7

QUESTION 18

What proof strategy would you use to prove the following sequent:

$\vdash_{KT4}$ □ □ (p ∧ q) → (□ p ∧ □ q)

***Option 1*: Open a solid box and start with □ □ (p ∧ q) as an assumption.**
Use axiom T to remove one □.

Open a dashed box and use □ elimination to get p ∧ q.

Use ∧ elimination twice to obtain the separate atomic formulas.

Close the dashed box and use □ introduction twice, i.e. once on each of the atomic formulas.

Combine □ p and □ q using ∧ introduction.

Close the solid box and use → introduction on the first and last formulas to get the result.

Option 2: Start with $\Box \Box (p \wedge q)$ as a premise.
Use axiom T twice to remove both $\Box$ to get $p \wedge q$.
Use $\wedge$ elimination once to obtain the separate atomic formulas.
Use axiom 4 twice, i.e. once on each atomic formula to add a $\Box$.
Combine $\Box p$ and $\Box q$ using $\wedge$ introduction.
Use $\rightarrow$ introduction on the first and last formulas to get the result.

Option 3: Start with $\Box \Box (p \wedge q)$ as a premise.
Open a dashed box and use $\Box$ elimination to get $\Box (p \wedge q)$.
Open another dashed box and use $\Box$ elimination to get $p \wedge q$.
Use $\wedge$ elimination twice to obtain the separate atomic formulas.
Close the first dashed box and use $\Box$ introduction on the first atomic formula.
Close the second dashed box and use $\Box$ introduction on the second atomic formula.
Combine $\Box p$ and $\Box q$ using $\wedge$ introduction.
Use $\rightarrow$ introduction on the first and last formulas to get the result.

Option 4: Open a solid box and start with $\Box \Box (p \wedge q)$ as an assumption.
Open a dashed box and use $\Box$ elimination to get $\Box (p \wedge q)$.
Use axiom T to remove one $\Box$ to get $p \wedge q$.
Use $\wedge$ elimination twice to obtain the separate atomic formulas.
Use axiom 4 twice, i.e. once on each atomic formula to add a $\Box$.
Close the dashed box and combine $\Box p$ and $\Box q$ using $\wedge$ introduction.
Close the solid box and use $\rightarrow$ introduction on the first and last formulas to get the result.

Option 5: This is not a valid sequent in KT4.

QUESTION 19

If we interpret $K_i \phi$ as "Agent i knows ϕ", what is the English translation of the formula $K_1 K_2 \neg p \rightarrow q$?

Option 1: Agent 1 knows that agent 2 doesn't know p implies q.

Option 2: Agent 1 knows that agent 2 knows that not p implies q.

Option 3: Agent 1 knows that if agent 2 doesn't know p, then q.

Option 4: Agent 1 knows that if agent 2 knows not p, then q.

***Option 5*: If agent 1 knows that agent 2 knows not p, then q.**

QUESTION 20

If we interpret $K_i \phi$ as "Agent i knows ϕ", what formula of modal logic is correctly translated to English as

Agent 1 knows p but he doesn't know that agent 2 knows q.

Option 1: $K_1 (p \wedge \neg K_2 q)$

Option 2: $K_1 (p \wedge K_2 \neg q)$

***Option 3*: $K_1 p \wedge \neg K_1 K_2 q$**

Option 4: $K_1 p \wedge K_1 \neg K_2 q$

Option 5: $K_1 p \wedge K_1 K_2 \neg q$