UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

COS3761

May/June 2016

FORMAL LOGIC III

Duration 2 Hours

100 Marks

EXAMINATION PANEL AS APPOINTED BY THE DEPARTMENT.

Closed book examination.

This paper consists of 6 pages

INSTRUCTIONS:

1 Answer all three questions
2 Do all rough work in the answer book.
3 Number your answers and label your rough work clearly
4 The mark for every question appears in brackets next to the question

ALL THE BEST!

[TURN OVER]

QUESTION 1 **[25]**

Question 1.1

Consider the following propositional symbols and their intended meanings

p	It is windy
q	It is cloudy
r	There is a dust storm
s	There is a hail storm
t	Temperature is above 20°C

(i) Express the following declarative sentence in propositional logic using the propositional symbols as given above

There is a hail storm only if it is cloudy and the temperature is not above 20°C (2)

(ii) Express the following propositional logic formula in English where the propositional symbols q, r and s have the meanings given above

$p \land (t \to r)$ (2)

Question 1.2

Use the basic natural deduction rules for propositional logic to prove the validity of the following sequents

(i) $q \to r \vdash (p \to q) \to (p \to r)$ (6)

(ii) $q \to (p \to r), \neg r, q \vdash \neg p$ (6)

Question 1.3

Show that the following sequent is not valid by giving an appropriate valuation

$p \to (\neg q \lor r), \neg r \vdash \neg q \to \neg p$ (4)

Explain why your valuation proves that the sequent is not valid.

Question 1.4

Use the HORN algorithm to prove that the following Horn formula is satisfiable Show each step

$(\top \to p) \land (p \to r) \land (p \land \top \to q) \land (r \land q \to \bot)$ (5)

QUESTION 2 [37]

Question 2.1

Consider the following predicate, constant and function symbols and their intended meanings.

S(x)	x is a soccer player
R(x)	x is a rugby player
C(x)	x is a sport club
W(x)	x wins
P(x, y)	x plays for y
M(x, y)	x plays against y (think of M as indicating a match)
a	Angelina
b	Ben
l	Lavender team
b(x)	brother of x

(i) Express the following predicate logic formula in English, where the symbols have the meanings as given above

$\forall x\, (C(x) \rightarrow \exists y\, (C(y) \wedge M(x, y) \wedge \neg W(x)))$ (3)

(ii) Express the following declarative sentence in predicate logic using the symbols as given above

If Angelina's brother plays for some club, all rugby players play for the Lavender team (3)

Question 2.2

Let

- A, B and P be predicate symbols, with 1, 2 and 3 arguments respectively,
- a, b are constant symbols, and
- x, y are variables

State which of the following are well formed formulas

(i) $\forall x \exists a \exists b\, P(x, a, b)$
(ii) $(\forall x\, (A(x) \rightarrow B(x, y)))$
(iii) $\exists x\, (A(x) \wedge \vee B(x, y))$
(iv) $P \rightarrow A$
(v) $\neg(\neg P(a, a, a))$ (5)

Question 2.3

Consider the following formula ϕ where P is a predicate symbol with two arguments and Q is a predicate symbol with three arguments

$$(\exists x\, P(x, y) \lor \exists y\, P(x, y)) \rightarrow \forall z\, Q(x, y, z)$$

(i) Draw the parse tree of ϕ (3)

(ii) Mark the free and bound variables on the tree (2)

(iii) Let f be a function with one argument Is f(x) free for y in ϕ? Explain your answer (2)

Question 2.4

Using the basic natural deduction rules for predicate logic, prove the validity of the following sequent

$$\forall x\, (A(x) \lor B(x)),\ \forall x\, (A(x) \rightarrow C(x)),\ \exists x \neg C(x) \vdash \exists x\, B(x)$$ (9)

Question 2.5

Let ϕ be the following formula

$$\forall x\, \forall y\, [R(x, y) \rightarrow (\neg R(x, x) \land \neg R(y, y))]$$

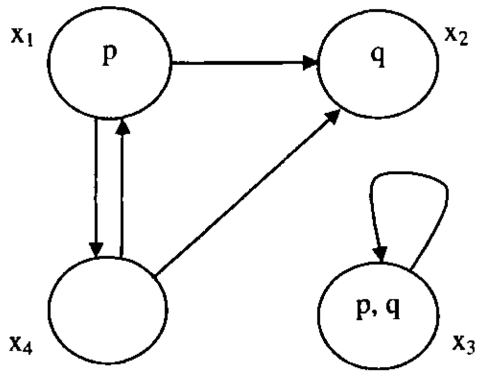
(i) Show that ϕ is satisfiable by constructing a mathematical model (where the universe *A* of concrete values is the set of positive integers) and a non-mathematical model (where the universe *A* of concrete values is {a, b, c}) Explain why your models satisfy ϕ (5)

(ii) Show that ϕ is not valid by constructing a mathematical and a non-mathematical model (where the universe *A* of concrete values is the same as for (i)) Explain why your models falsify ϕ (5)

QUESTION 3 **[38]**

Question 3.1

Consider the following Kripke model with worlds x_1, x_2, x_3 and x_4



(i) For each of the following relations, determine whether it holds in the above Kripke model and give reasons for your answer

(a) $x_1 \Vdash \Box \Diamond q$
(b) $x_2 \Vdash \Box p \rightarrow \neg p$
(c) $x_3 \Vdash \Diamond \Diamond p$
(d) $x_4 \Vdash \Box (p \vee q)$ (8)

(ii) Write down any modal logic formula containing at least one modal connective which is not valid but is true in the above Kripke model Explain your answer (5)

Question 3.2

Suppose we want to re-engineer basic modal logic to fit the following reading of $\Box \phi$

$\Box \phi$ The agent believes that ϕ

For each of the following two formulas, state whether it should be valid or not and explain your answer in both cases

(i) $\phi \rightarrow \Box \phi$ (3)

(ii) $\Box \phi \rightarrow \Box \Box \phi$ (3)

Question 3.3

Say we interpret the modal operators □ and ◊ to represent the temporal notions "always" and "sometime" (in the future) respectively, and say we interpret the propositional letters p and q to mean "I will be rich" and "I will be happy" respectively

(i) Express the following modal logic formula in English

$\Box p \rightarrow \Diamond \neg q$ (2)

(ii) Translate the following declarative sentence into modal logic

Even though I will never be rich, sometime in the future I will be happy (3)

Question 3.4

(i) Express the following modal logic formula in English where K_i is read as "Agent i knows that"

$K_1 p \wedge K_1 \neg K_2 q$ (2)

(ii) Express the following sentence in modal logic where K_i is read as "Agent i knows that"

Not every agent knows not q (Assume there are 5 agents) (3)

Question 3.5

Using the basic natural deduction rules, the □ introduction and elimination rules of the basic modal logic K as well as the following three additional rules for KT45

$$\frac{\Box \phi}{\phi}\ T \qquad \frac{\Box \phi}{\Box \Box \phi}\ 4 \qquad \frac{\neg \Box \phi}{\Box \neg \Box \phi}\ 5$$

prove the validity of the following sequent

$\Box (p \rightarrow q) \vdash_{KT45} \neg \Box \neg p \rightarrow \neg \Box \neg q$ (9)

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