UNISA university of south africa

COS3701

MAY/JUNE 2018

Theoretical Computer Science III

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Question No	Marks					
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	1		2		3	
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Subject

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UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

COS3701

May/June 2018

Theoretical Computer Science III

Duration 2 Hours

100 Marks

EXAMINATION PANEL AS APPOINTED BY THE DEPARTMENT.

Closed book examination

Examiners :

First : Prof I Sanders
Second : Mr K Halland
External Prof D Kourie (University of Stellenbosch)

Instructions:

1. Read these instructions *carefully.*
2. Answer all questions on the question paper in the spaces provided
3. Additional pages are provided in case they are necessary.
4. Pages are available for rough work. These pages will **not** be marked!
5 Write neatly and legibly
6. The mark for each question is given in brackets next to the question.
7. This paper consists of 24 pages

Good Luck

Question 1	Context Free Grammars	[16]

(a) Determine a regular expression for the language L over the alphabet $\{a, b\}$ that contains at least two ab substrings and ends on a b Notethat if the word ends in the substring ab, then at least two other occurrences of substring ab must precede this substring

For example, $aaaaabaaaabbbbbb$ and $ababab$ are words in L but $aaaaabaaaaab$ and $abab$ are not

(2)

(b) Design a deterministic finite automaton (DFA) that will recognise all of the words in L as defined above. (4)

(c) Use Theorem 21 to develop a context-free grammar (CFG) for the language L. (4)

(d) Convert the following CFG to Chomsky Normal Form (CNF).

$$S \rightarrow aYbZ \mid bXaY$$
$$X \rightarrow bY \mid \Lambda$$
$$Y \rightarrow aX \mid \Lambda$$
$$Z \rightarrow XaY$$

(6)

Step 1 Killing Λ Productions

Step 2 Killing Unit Productions

Step 3 Chomsky Normal Form

Question 2	Pushdown Automata	[10]

Build a deterministic pushdown automata (DPDA) that accepts the language $L = \{(aa)^{n+1}(aba)(b)^n \mid n \geq 0\}$ over the alphabet $\Sigma = \{a, b\}$ (10)

Question 3	Pumping Lemma	[12]

The pumping lemma with length for context-free languages (CFLs) can be stated as follows

Let L be a CFL generated by a CFG in CNF with p live productions

Then any word w in L with length $> 2^p$ can be broken into five parts:
$w = uvxyz$
such that
length$(vxy) \leq 2^p$
length$(x) > 0$
length(v) + length$(y) > 0$

and such that all the words uv^nxy^nz with $n \in \{2, 3, 4, \quad\}$ are also in the language L.

Use the pumping lemma with length to prove that the language

$$L = \{(a)^n(b)^{n+1}(aa)^n \mid n \geq 1\}$$

over the alphabet $\Sigma = \{a, b\}$ is non-context-free. (12)

The first step of the proof requires that we should assume that the language L is context-free

Next we need to choose a suitable word, say w from L which is long enough

Choose an appropriate word to pump.

The next step of the proof requires us to show the possible ways that the word w can be split up and still meet the restrictions listed above.

List the cases which need to be considered.

Case 1

Case 2

Case 3

Case 4

Case 5

Once we know what the cases are, we need to argue that each case would lead to a contradiction of our original assumption

Argue how each case above would lead to a contradiction of our assumption.

Case 1

Case 2

Case 3

Case 4

Case 5

Now complete the proof

Question 4	Closure	[13]

(a) Consider the language L generated by CFG1 given below

$$S \rightarrow AaaB$$
$$A \rightarrow abA \mid \Lambda$$
$$B \rightarrow baB \mid \Lambda$$

The language is

1. regular and context free,
2. nonregular and context free?

(1)

Write the number for the correct option below

[ ]

(b) Explain why you made the selection in part (a) above. (2)

(c) Consider the language L generated by CFG2 given below

$$S \rightarrow BSAA \mid \Lambda$$
$$A \rightarrow a$$
$$B \rightarrow b$$

The language is

1. regular and context free,
2. nonregular and context free?

(1)

Write the number for the correct option below

[ ]

(d) Explain why you made the selection in part c above. (2)

(e) Use the CFGs from the questions above (parts (a) and (c)) to generate a CFG that generates the union language L_u. (3)

(f) Is L_u

1. regular and context free
2. nonregular and context free
3. noncontext free?

Write the number for the correct option below

[ ] (1)

(g) Explain why you made the selection in part f above. (3)

Question 5	Decideability	[6]

Use the reformulated version of Theorem 42 to decide whether the grammar given below generates any words. (6)

$S \rightarrow AC$
$A \rightarrow CB \mid b$
$B \rightarrow CD$
$C \rightarrow DA$
$D \rightarrow a$

Step -1: Is S nullable?

Step 0: Convert the CFG to CNF.

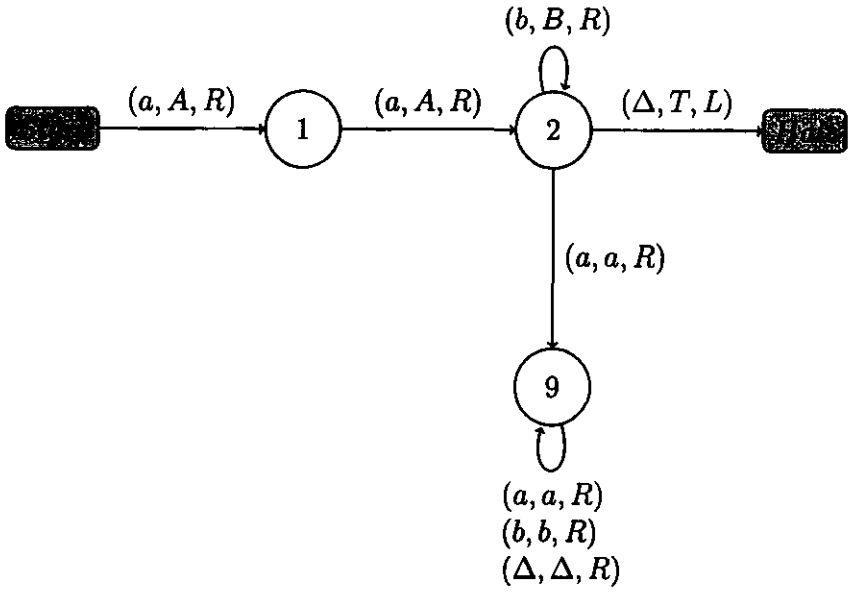
Is there a production of the form $S \rightarrow t$ where t is a terminal?

Step 1:

Step 2:

Question 6	Tracing a Turing Machine	[7]

Consider the Turing Machine (TM) T (over the input alphabet $\Sigma = \{a, b\}$) given below.



Hint: Trace the execution of the TM on a few strings of as and bs so that you can see how it works.

(a) What is the shortest word that would be accepted by T? (1)

(b) What is accept(T)? (2)

(c) What is reject(T)? (2)

(d) What is loop(T)? (1)

(e) What is left on the tape if T halts? (1)

Question 7	Building a Turing Machine	[12]

Build a Turing Machine (TM) that

- accepts all words in $\{(b)^{n+2}a^n \mid n \geq 1\}$,
- loops forever on all words starting with a, and
- rejects all other words.

Assume that the alphabet is $\Sigma = \{a, b\}$

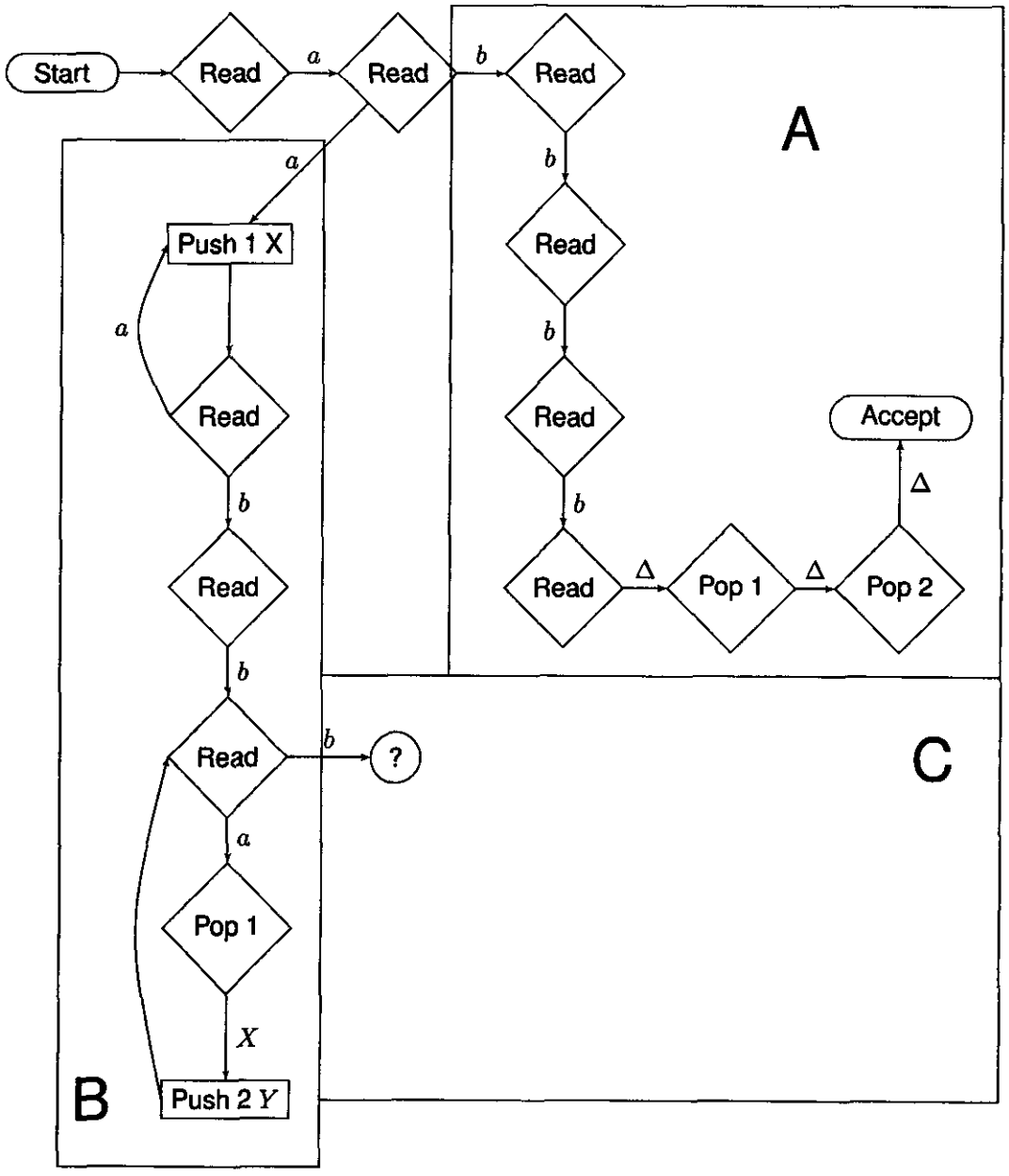
Hint:
Write out your solution as high level pseudocode before you start drawing your TM. This will help you to formulate an approach to solving the problem. (12)

Additional space for answering Question 7

Question 8	2PDA	[11]

Consider the partially complete 2PDA given below. This 2PDA, when complete, should recognise the language L as defined below:

$L = \{(a)^{n+1}bb(a)^nbb(a)^n \mid n \geq 0\}$ (Assume that the alphabet is $\Sigma = \{a, b\}$.)



(a) What is the purpose of the section of the 2PDA illustrated in box A? Explain in detail how this purpose is achieved. (3)

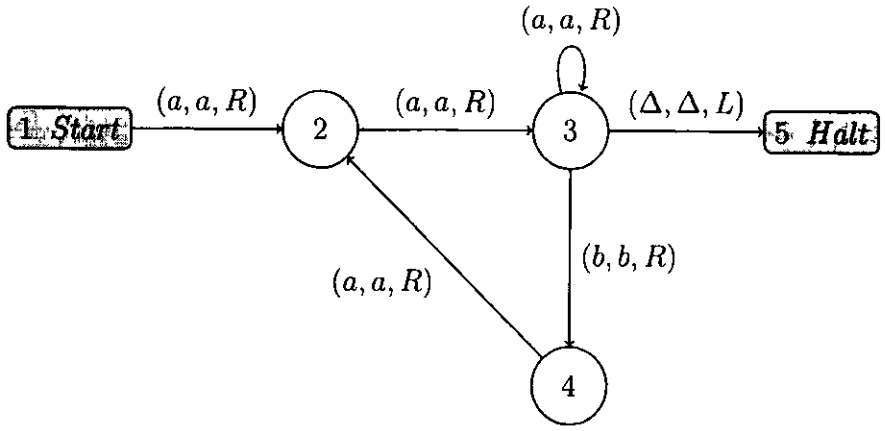
(b) What is the purpose of the section of the 2PDA illustrated in box B? Explain clearly what is being done here and how it affects or is affected by the overall design of the 2PDA. (4)

(c) Complete the 2PDA by showing what should be included in Box C of the diagram shown above.

Note that the edges shown in the diagram are the only edges that are needed between what is outside of Box C and what is needed inside Box C (4)

Question 9 Turing Machine Encodings **[9]**

Consider the Turing Machine T given below.



(a) Convert T into a summary table. (3)

1				
2				
3				
3				
3				
4				

(b) Convert T into CWL (code word language) (3)

(c) Which language is accepted by T? (1)

(d) Does the code word of T belong to ALAN? (Motivate your answer) (2)

Note that the definition provided for ALAN in Cohen holds for question 9(d).

Question 10	Chomsky Hierarchy and Computability	[4]

(a) What type of language is a Type-2 language in the Chomsky Hierarchy and what is the *acceptor* for such a language? (2)

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(b) What is *Church's Thesis*? (2)

Total: 100

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