This paper consists of 13 pages.

INSTRUCTIONS:

1. This paper consists of two sections; Section A, 24 marks and Section B, 76 marks.
2. Answer all twelve questions of Section A and all six questions of Section B in your answer book.
3. Do all rough work in the answer book.
4. Number your answers and label your rough work clearly.
5. The mark for every question appears in brackets next to the question.

ALL THE BEST!

[TURN OVER]

All references to Tarski's World are as described in the prescribed book: Language, Proof and Logic

SECTION A

QUESTION 1 **24 marks**

This section, consisting of twelve multiple choice questions, should be answered in your examination book (NOT on a multiple choice sheet). In each case, simply write down the number of the question followed by the number of the chosen option, for example if you choose option 4 for question 1(i) you should write 1(i) 4.

TARSKI WORLD FOR QUESTIONS i, ii AND iii

The Tarski world given below is used in questions i, ii and iii of Section A. The symbols used to indicate the form of the blocks in the world are explained below the table. An example of an entry is (**a**: T, S) which indicates that the block in that location is called **a**, it is a tetrahedron, and it is small.

	c: D,S				d: C, L
a: T, S		b: C,S			

Shape indicated as follows:
C: cube
T: tetrahedron
D: dodecahedron

Size indicated as follows:
L: large
M: medium
S: small

Question 1(i) [2]

Consider the following two FOL sentences in the blocks language. The two sentences are followed by four options. **Write down the number only of the correct option.**

Sentences:

1.1 SameRow(c, d) $\wedge$ SameRow(a, b) $\wedge$ $\neg$SameSize(a,b)

1.2 Dodec(c) $\wedge$ Large(a)

Options:

1. Sentence 1.1 and sentence 1.2 are true in the Tarski world given above.
2. Sentence 1.1 is true and sentence 1.2 is false in the Tarski world given above.
3. Sentence 1.1 is false and sentence 1.2 is true in the Tarski world given above.
4. Sentence 1.1 and sentence 1.2 are false in the Tarski world given above.

Question 1(ii) [2]

Consider the following two FOL sentences in the blocks language. The two sentences are followed by four options. **Write down the number only of the correct option.**

Sentences

2.1 $\forall$y(Dodec(y) $\rightarrow$ SameCol(y,d))

2.2 $\forall$x(Tet(x) $\rightarrow$ (Small(x) $\wedge$ LeftOf(x, b)))

Options:

1. Sentence 2.1 and sentence 2.2 are true in the Tarski world given above.
2. Sentence 2.1 is true and sentence 2.2 is false in the Tarski world given above.
3. Sentence 2.1 is false and sentence 2.2 is true in the Tarski world given above.
4. Sentence 2.1 and sentence 2.2 are false in the Tarski world given above.

Question 1(iii) [2]

Consider the following two sentences in the blocks language. The two FOL sentences are followed by four options. **Write down the number only of the correct option.**

Sentences

3.1 $\forall$x(SameRow(x,c) $\rightarrow$ Cube(x))

3.2 BackOf(d,b)

Options:

1. Sentence 3.1 and sentence 3.2 are true in the Tarski world given above.
2. Sentence 3.1 is true and sentence 3.2 is false in the Tarski world given above.
3. Sentence 3.1 is false and sentence 3.2 is true in the Tarski world given above.
4. Sentence 3.1 and sentence 3.2 are false in the Tarski world given above.

Question 1(iv)

Consider the FOL sentence below and then write down the number of the option indicating a correct English translation:

FOL sentence:

$\forall y \forall x((Tet(x) \land Dodec(y)) \rightarrow SameSize(x, y))$

Options:

1. Every tetrahedron is the same size of every dodecahedron
2. Every dodecahedron is the same size of every tetrahedron.
3. Some of the tetrahedra are same size of dodecahedron.
4. Some of the dodecahedra are same size of tetrahedron.

Question 1(v) [2]

Consider the FOL sentence below and then write down the number of the option indicating a correct English translation:

FOL sentence:

Most x [(Cube(x), Most y (Tet(y), FrontOf(x, y)]

Options:

1. Most cubes are in front of most Tetrahedra.
2. Most cubes and Tetrahedra are in front of each other.
3. Most Tetrahedra are in front of Cubes.
4. Most Tetrahedra are in front of Most Cubes.

Question 1(vi) [2]

Which of the options below correctly states how we can determine if a given atomic sentence of FOL is TT-Possible?

Options:

1. Build the truth table for the sentence and inspect the truth values below the main connective. If all these values are T, the sentence is a TT Possible
2. Build the truth table for the sentence and inspect the truth values below the main connective. If there is atleast one row of the truth table assigns to true, the sentence is TT- possible.
3. Build the truth table for the sentence and inspect the truth values below the main connective. If there is at least one F, the sentence is a TT Possible.
4. Build the truth table for the sentence and inspect the truth values below the main connective. If all these values are F, the sentence is a TT Possible.

Question 1(vii) [2]

Consider the FOL sentence below and write down the number of the option giving a correct Disjunctive Normal Form (DNF) of the sentence.

$$\neg[(A \lor \neg B) \land \neg(A \land D)] \lor [C \land (A \lor D)]$$

Options:

1. $(A \land B) \lor (\neg A \land \neg D) \lor (C \land A) \lor (C \land D)$
2. $(\neg A \land \neg B) \lor (\neg A \lor \neg D) \lor (C \land A) \lor (C \land D)$
3. $(\neg A \land B) \lor (A \land D) \lor (C \land A) \lor (C \land D)$
4. $(\neg A \land B) \land (\neg A \land \neg D) \lor (C \land A) \lor (C \land D)$.

Question 1(viii)

Consider the FOL sentence below and write down the number of the option giving a correct Negation Normal Form (NNF) of the sentence.

¬[Angry(max) ∨ ¬(¬Loves(carl, claire) ∨ ¬¬loves(claire, carl))]

Options:

1. ¬Angry(max) ∧ ¬Loves(carl, claire) ∨ Loves(claire, carl))]
2. ¬Angry(max) ∨ ¬(¬Loves (carl, claire) ∨ ¬Loves(claire, carl))
3. ¬Angry(max) ∨ (¬¬Loves (carl, claire) ∧ ¬¬Loves (claire, carl))
4. ¬Angry(max) ∨ Loves (carl, claire) ∧ Loves(claire, carl)).

Question 1(ix) [2]

Given the following three sentences in the blocks language and the joint truth table below, write down the number of the option giving the correct interpretation of the truth table:

1. ¬P ∨ Q
2. ¬P ∨ R
3. (Q ∧ R) → ¬ P

¬P	**Q**	**R**	**¬P ∨ Q**	**¬ P ∨ R**	**Q ∧ R**	(Q ∧ R) → ¬ P
T	T	T	T	T	T	T
T	T	F	F	T	F	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	F
F	T	F	F	F	F	T
F	F	T	F	T	F	T
F	F	F	F	F	F	T

Options:

1. The third sentence is neither a tautological consequence nor a logical consequence of the first and second sentences.
2. The third sentence is a tautological consequence but not a logical consequence of the first and second sentences.
3. The third sentence is a tautological consequence and a logical consequence of the first and second sentences.
4. The third sentence is a logical consequence but not a tautological consequence of the first and second sentences.

Question 1(x) [2]

Consider the partial truth table below for the following sentence and then write down the number of the option giving the correct interpretation of the truth table:

$(A \rightarrow B) \wedge (B \rightarrow C) \rightarrow (A \rightarrow C)$

A	B	C	$A \rightarrow B$	$B \rightarrow C$	$(A \rightarrow B) \wedge (B \rightarrow C)$	$A \rightarrow C$	$(A \rightarrow B) \wedge (B \rightarrow C) \rightarrow (A \rightarrow C)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Options:

1. The sentence is a tautology.
2. The sentence is logically equivalent to $(A \rightarrow (B \wedge B)) \rightarrow (C \rightarrow (A \rightarrow C))$
3. The sentence is not a tautology but TT-possible.
4. The sentence is not well-formed.

Question 1(xi) [2]

Consider the argument below and then write down the number of the option giving the correct evaluation of the argument:

All that glitters is made of gold.
Necklace is made of gold

Necklace is glittering.

Options:

1. The argument is both sound and valid.
2. The argument is sound but not valid.
3. The argument is valid but not sound.
4. The argument is neither valid nor sound.

Question 1(xii) [2]

Consider the argument below and then write down the number of the option giving the correct evaluation of the argument:

If Professor Mark comes to class on time, then he is a good teacher.
Professor Mark is a good teacher.

Therefore, he comes to class on time.

Options:

1. The argument is both valid and sound.
2. The argument is not valid but sound.
3. The argument is valid but not sound.
4. The argument is not valid and not sound.

SECTION B
76 marks

Table 2 below lists the names and predicates that should be used in questions 2 of Section B.

English	FOL
Names	
David	david
Temba	temba
Patty	patty
Cathy	cathy
Cyprian College	cc
Predicates	
x is a student in y	Student(x, y)
x is a teacher	Teacher(x)
x is a principal	Principal(x)
x teaches y	Teach(x, y)
x heads at y	Head(x, y)

Table 2

QUESTION 2 **[12]**

Question 2(a) **(6)**

Translate the following two English sentences into a first-order logic (FOL) sentence, using the names and predicates given in Table 2:

(i) If David is a student at Cyprian College, Patty is the teacher who teaches him.

Student(david, cc) → (Teacher(patty) ∧ Teach(patty, david))

(ii) Unless Patty is a student there, Temba is the principal heading Cyprian College.

¬Student(patty, cc) → Principal(themba) ∧ Head(themba, cc)

Question 2(b) **(6)**

Translate the following first-order logic (FOL) sentence into a English sentence, using the names and predicates given in Table 2:

(i) Student(cathy, cc) $\rightarrow$ (Principal(david) $\wedge$ Teach(david, cathy) $\wedge$ Teacher(patty) $\wedge$ Teach(patty, cathy))

If Cathy is a student at Cyprian College, David and Patty are, respectively, the principal and teacher teaching her.

(ii) $\forall$x(Student(x, cc) $\rightarrow$ (Teach(patty, x) $\vee$ Teach(cathy, x)))

All students at Cyprian College are being taught by Patty or Cathy.

QUESTION 3 **[6]**

Given the following sentence of FOL, which of the options below is an equivalent sentence in disjunctive normal form (DNF)?

$\neg((A \vee B) \wedge C) \vee ((A \vee C) \wedge D)$

$\neg((A \vee B) \wedge C) \vee ((A \vee C) \wedge D) \equiv (\neg(A \vee B) \vee \neg C) \vee ((A \vee C) \wedge D)$ DeMorgan

$\equiv (\neg(A \vee B) \vee \neg C) \vee ((A \wedge D) \vee (C \wedge D))$ Distributive

$\equiv (\neg A \wedge \neg B) \vee \neg C \vee (A \wedge D) \vee (C \wedge D)$ DeMorgan

QUESTION 4 **[6]**

Construct a truth table for the following FOL sentence and show that it is a contingent:

$(A \vee \neg B) \wedge \neg(A \vee C)$

A	**B**	**C**	**$\neg$B**	**A $\vee$ $\neg$B**	**A $\vee$ C**	**$\neg$(A $\vee$ C)**	**(A $\vee$ $\neg$B) $\wedge$ $\neg$(A $\vee$ C)**
T	T	T	F	T	T	F	F
T	T	F	F	T	T	F	F
T	F	T	T	T	T	F	F
T	F	F	T	T	T	F	F
F	T	T	F	F	T	F	F
F	T	F	F	F	F	T	F
F	F	T	T	T	T	F	F
F	F	F	T	T	F	T	T

Since the final column has both true and false truth values showing that the statement is a contingent.

Question 5 **[12 Marks]**

Question 5(i) Give an informal proof of the following argument: **(6)**

b is small unless it's a cube.
If **c** is small, then either **d** or **e** is too.
If **d** is small, then **c** is not.
If **b** is a cube, then **e** is not small.

If **c** is small, then so is **b**.

Proof:

Assume that c is small. We want to prove that b is small. Assume, by way of contradiction, that b is not small. Then by the first premise, b is a cube. Then by the final premise, e is not small. But then by the second premise, d is small. But the third premise assures us that if d is small, then c isn't small, contradicting our initial assumption. Hence, using proof by contradiction and conditional proof, we see that if c is small, then so is b.

Question 5(ii) **(6)**

In the following argument below, identify the premises and conclusion by putting the argument into Fitch format. Then say whether the argument is valid or not.

Anyone who wins an academy award is famous. Meryl Streep won the academy award. Hence, Meryl Streep is famous.

1. Anyone who wins an academy award is famous.
2. Meryl Streep won the academy award.

3. Hence, Meryl Streep is famous.

The argument is valid, but not sound. (Various people have won awards, but are not famous)

Question 6 **[30 Marks]**

Question 6(i) **(8)**

Using the natural deduction rules, give a formal proof that the following three sentences are inconsistent:

1. $P \wedge Q$
2. $\neg P \vee \neg R$
3. $Q \rightarrow R$

1.	$P \wedge Q$	
2.	$\neg P \vee \neg R$	
3.	$Q \rightarrow R$	
4.	$\neg P$	
5.	P	$\wedge$ Elim: 1
6.	$\perp$	$\perp$ Intro: 4, 5
7.	$\neg Q$	
8.	R	$\wedge$ Elim: 1
9.	Q	$\rightarrow$ Elim: 3, 8
10.	$\perp$	$\perp$ Intro: 7, 9
11.	$\perp$	$\vee$ Elim: 2, 4-6, 7-10

Question 6(ii) **(12)**

Give formal proof of the argument below using the rules of natural deduction. You may not use Taut Con or FO Con or Ana Con. It is important to number to your statement to indicate sub proofs and at each step to give the rule that you are using.

$\neg M \rightarrow N$
$R \rightarrow (P \vee Q)$
$P \rightarrow \neg R$
$M \rightarrow \neg Q$

$R \rightarrow N$

Solution

1. $\neg M \rightarrow N$
2. $R \rightarrow (P \vee Q)$
3. $P \rightarrow \neg R$
4. $M \rightarrow \neg Q$

5.	R	
6.	$P \vee Q$	$\rightarrow$ Elim: 2, 5
7.	P	
8.	$\neg R$	$\rightarrow$ Elim: 3, 7
9.	$\perp$	$\perp$ Intro: 5, 8
10	N .	$\perp$ Elim: 9
11.	Q	
12.	M	
13.	$\neg Q$	$\rightarrow$ Elim: 4, 12
14.	$\perp$	$\perp$ Intro: 11, 13
15.	$\neg M$	$\neg$ Intro: 12-14
16.	N	$\rightarrow$ Elim: 1, 15
17.	N	$\vee$ Elim: 6, 7-10, 11-16
18.	$R \rightarrow N$	$\rightarrow$ Intro: 5-17

Question 6(iii) **(10)**

Give formal proofs of the arguments below using the rules of natural deduction. You may not use Taut Con or FO Con or Ana Con. It is important to number your statements, to indicate sub proofs and at each step to give the rule that you are using.

1. ∃x¬Large(x)
2. ∀x(Large(x) ∨ Small(x))

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3. ∃x Small(x)

1. ∃x¬Large(x)	
2. ∀x(Large(x) ∨ Small(x))	
3. $\boxed{a}$ ¬Large(a)	
4. Large(a) ∨ Small(a)	∀x Elim: 2
5. Large(a)	
6. ⊥	⊥ Intro: 3, 5
7. Small(a)	⊥ Elim: 6
8. Small(a)	
9. Small(a)	Reit: 8
10. Small(a)	∨ Elim: 4, 5 – 7, 8 - 9
11. ∃xSmall(x)	∃x Intro: 10
12. ∃x Small(x)	∃x Elim: 1, 3 - 11

QUESTION 7 **[10 Marks]**

Draw a Tarski world serving as counter example to show that no proof is possible for the argument below.
Note: smaller (y, x) means y is smaller than x.

$\forall x(Tet(x) \lor \neg\exists y Smaller(y, x))$

$\forall x(Tet(x) \rightarrow \neg\exists y Smaller(y, x))$

Solution

$\forall x(Tet(x) \lor \neg \exists y Smaller(y, x))$

$\forall x(Tet(x) \rightarrow \exists y Smaller(y, x))$

In the counter-example below the premise is true but the conclusion not.

	a: T, M	**b**: C, S					