Tutorial Letter 202/2/2018

Formal Logic 2

COS2661

Semester 2

School of Computing

Solutions to Assignment 2

Bar code

Dear student,

This tutorial letter contains the solutions to Assignment 2.

TUTORIAL MATTER

You should already have received the material listed below. If any of it is missing, please contact the Department of Despatch. You may also download it from the Internet – see tutorial letter COSALLF/301/4/2018.

Tutorial letters:

COSALLF/301/4/2018

General information concerning the School of Computing and study at Unisa

Lecturers' names and contact information

COS2661/101/3/2018

Information about COS2661 and the assignments

COS2661/102/3/2018

Tutorial letter serving as study guide

COS2661/103/3/2018

Mathematics that may be needed

COS2661/201/2/2018

Solutions to assignment 1 of second semester of 2018

COS2661/202/2/2018

This letter, solutions to assignment 2 of second semester of 2018

Solutions to Assignment 2 Questions

QUESTION 1 **[12]**

Important: ***An argument is invalid when it is possible for all the premises to be true but the conclusion not***.

- In order to show (informally) that an argument is *valid*, one assumes that the premises are true. Then one uses proof by contradiction or proof by cases and show that the conclusion *has to be true* also.
- In order to show that an argument in *not valid*, one assumes that the premises are true and then sketches a situation where the conclusion is *not true*.

Question 1.1 **(4)**

The argument is valid. We give an informal proof, using proof by contradiction:

Assume Carl is not happy. Then, because the second premise has to be true, Max cannot be home. Also, because the third premise has to be true, Claire cannot be home. But this contradicts the first premise that requires at least one of Max or Claire to be home. This means that our assumption that Carl was not happy was wrong and we have to retract it. This means that we may conclude that Carl is happy. The argument is valid.

Question 1.2 **(4)**

The argument is valid. We give two informal proofs, first using proof by contradiction and then proof by cases.

Proof by contradiction:
Assume John is not at work. Then, because the third premise has to be true, Carol is not on holiday. So, according to the first premise, Sue is on holiday (to make the first part of the premise true). Furthermore, also according to the first premise, Mark is at work (to make the second part of the premise true). But if Sue is on holiday and Mark is at work, the second premise is not true. We arrive at a contradiction (remember that all the premises have to be true). This means that our assumption that John is not at work was wrong and we have to retract is. This means that we may conclude that John is at work. The argument is valid.
Proof by cases:

Let us split the third premise into two cases.

- Assume Carol is not on holiday. Then, according to the first premise, Sue is on holiday. If Sue is on holiday, the second premise requires Mark to be not at work. But if Mark is not at work, John has to be at work according to the first premise. So we arrive at the required conclusion.
- Assume John is at work. This is immediately our required conclusion.

In both cases we get the conclusion that John is at work. The argument is valid.

Question 1.3 **(4)**

The argument is not valid. To show this, we sketch a situation where all three premises are true but the conclusion is not true:

Let both **a** and **b** be cubes.

Then

Cube(a) ∨ Tet(b) and

¬ Dodec(a) ∨ ¬ Cube(a) and

¬ Tet(b) ∨ ¬ Dodec(b)

are all true but

Tet(b)

is not true.

QUESTION 2 **[25]**

Question 2.1 **(7)**

1	$C \rightarrow (A \vee B)$	
2	$C \wedge \neg A \wedge \neg B$	
3	C	$\wedge$ Elim: 2
4	$\neg A$	$\wedge$ Elim: 2
5	$\neg B$	$\wedge$ Elim: 2
6	$A \vee B$	$\rightarrow$ Elim: 1, 3
7	A	
8	$\perp$	$\perp$ Intro: 4, 7
9	B	
10	$\perp$	$\perp$ Intro: 5, 9
11	$\perp$	$\vee$ Elim: 6, 7-8, 9-10

Question 2.2 **(9)**

1	$A \rightarrow B$	
2	$(B \vee D) \rightarrow E$	
3	$F \rightarrow \neg E$	
4	A	
5	B	$\rightarrow$ Elim: 1, 4
6	$B \vee D$	$\vee$ Intro: 5
7	E	$\rightarrow$ Elim: 2, 6
8	F	
9	$\neg E$	$\rightarrow$ Elim: 3, 8
10	$\perp$	$\perp$ Intro: 5, 9
11	$\neg F$	$\neg$ Intro: 8-10
12	$A \rightarrow \neg F$	$\rightarrow$ Intro: 4-11

Question 2.3 **(9)**

1	$A \wedge (B \vee C)$	
2	$A \rightarrow D$	
3	$B \rightarrow \neg E$	
4	E	
5	A	$\wedge$ Elim: 1
6	D	$\rightarrow$ Elim: 2, 5
7	$B \vee C$	$\wedge$ Elim: 1
8	B	
9	$\neg E$	$\rightarrow$ Elim: 3, 8
10	$\perp$	$\perp$ Intro: 5, 9
11	C	$\perp$ Elim:
12	C	
13	C	Reit: 12
14	C	$\vee$ Elim: 7, 8-11, 12-13
15	$C \wedge D$	$\wedge$ Intro: 14, 6
16	$E \rightarrow (C \wedge D)$	$\rightarrow$ Intro: 4-15

QUESTION 3 **[45 3 marks each]**

If any of the following answers are not clear, please work through the relevant pages in your textbook again. (An *argument of a predicate* is explained in section 1.2 of your textbook. This is not the same as the *arguments* of question 1 above.)

- Always remember that an argument of a predicate *cannot* be a predicate. It is therefore *not* permissible to have something like Looses(team(x), blue).
- Also keep in mind the arity of every predicate, i.e. the number of arguments that it should have. The predicate Play should, for example, always have two arguments while the predicate Injured should always have one argument.

Question 3.1

If Richard plays for Red Arrows, he gets injured:

Play(richard, red) $\rightarrow$ Injured(richard)

The use of the material conditional symbol $\rightarrow$ is explained in section 7.1 of your textbook. Here we have the *if...then* situation.

Question 3.2

Vince gets injured only if he plays for Red Arrows:

Injured(vince) $\rightarrow$ Play(vince, red)

The use of the material conditional symbol $\rightarrow$ is explained in section 7.1 of your textbook. Here we have the *only if* situation.

Question 3.3

Unless Felicity is a teacher, Vince and Richard will both play for Red Arrows:

$\neg$ Teacher(felicity) $\rightarrow$ (Play(vince, red) $\wedge$ Play(richard, red))

- The use of the material conditional symbol $\rightarrow$ is explained in section 7.1 of your textbook. Here we have the *unless* situation.
- Note that we cannot have Play(vince $\wedge$ richard, red). You have to use the $\wedge$ symbol between two predicates to translate the given English sentence correctly.

Question 3.4

Blue Bucks do not loose against Red Arrows:

$\neg$ Loose(blue, red)

Note that neither ¬ Loose(red, blue) nor Loose(red, blue) would be a correct translation. Can you see why?

Question 3.5

If we assume that Vince plays for Blue Bucks, Felicity also plays for Blue Bucks but Richard plays for Red Arrows:

Play(vince, blue) $\rightarrow$ (Play(felicity, blue) $\wedge$ Play(richard, red))

- The use of the material conditional symbol $\rightarrow$ is explained in section 7.1 of your textbook. Here we have the *if...then* situation.
- Note that both *but* and *and* are translated into FOL using $\wedge$. You may read about this on page 85.

Question 3.6

Red Arrows loose against Blue Bucks, or Blue Bucks loose against Red Arrows:

Loose(red, blue) $\vee$ Loose(blue, red)

The use of the $\vee$ symbol is explained in section 3.3 of your textbook.

Question 3.7

Richard plays for Blue Bucks if and only if Felicity and Vince are both teachers or neither of them is a teacher:

Play(richard, blue) $\leftrightarrow$ (Teacher(felicity) $\wedge$ Teacher(vince)) $\vee$ (¬Teacher(felicity) $\wedge$ ¬Teacher(vince))

- The use of the $\leftrightarrow$ symbol for *if and only if* is explained in section 7.2 of your textbook.
- Note the use of brackets in the translated sentence.

Question 3.8

Richard is either a teacher or a player but not both:

$$(Player(richard) \lor Teacher(richard)) \land \neg (Player(richard) \land Teacher(richard))$$

An equivalent translation will be:

$$(Player(richard) \lor Teacher(richard)) \land (\neg Player(richard) \lor \neg Teacher(richard))$$

The correct placing of brackets is essential in this sentence.

Question 3.9

If Felicity and Vince get injured while they play for Blue Bucks, Richard will play for Red Arrows:

$$(Injured(felicity) \land Injured(vince) \land Play(felicity, blue) \land Play(vince, blue)) \rightarrow Play(richard, red)$$

Note how the phrase "Felicity and Vince get injured while they play for Blue Bucks" is translated.

Question 3.10

Felicity is neither a teacher nor a soccer player:

$$\neg Teacher(felicity) \land \neg Player(felicity)$$

An equivalent translation will be:

$$\neg (Teacher(felicity) \lor Player(felicity))$$

Question 3.11

No teachers are soccer players:

$$\forall x\ (Teacher(x) \rightarrow \neg Player(x))$$

See section 9.5 for the Aristotelian form *No P's are Q's*.

Question 3.12

Some teachers do not play soccer:

$$\exists x\ (Teacher(x) \land \neg Player(x))$$

See section 9.5 for the Aristotelian form *Some P's are not Q's*.

Question 3.13

Someone is a soccer player and a teacher:

$$\exists x\ (Teacher(x) \land Player(x))$$

The use of the ∃ symbol is explained in section 9.2 of your textbook.

Question 3.14

Everyone gets injured when Vince plays for Blue Bucks:

$$Play(vince, blue) \rightarrow \forall x\ Injured(x)$$

The use of the ∀ symbol is explained in section 9.2 of your textbook.

Question 3.15

Someone gets injured if Red Arrows loose against Blue Bucks:

$$Loose(red, blue) \rightarrow \exists x\ Injured(x)$$

The use of the ∃ symbol is explained in section 9.2 of your textbook.

QUESTION 4 **[18 3 marks each]**

Question 4.1

$\neg$ Player(felicity):

Felicity does not play soccer.

Question 4.2

Play(vince, red) $\rightarrow$ Injured(richard):

If Vincent plays for the Red Arrows, Richard gets injured.

Question 4.3

(Injured(vince) $\wedge$ Loose(red, blue)) $\leftrightarrow$ (Play(richard, blue) $\vee$ Play(felicity, blue)):

Vincent gets injured when the Red Arrows loose against the Blue Bucks if and only if Richard or Felicity plays for the Blue Bucks.

Question 4.4

$\exists$x (Team(x) $\wedge$ Loose(blue, x)):

The Blue Bucks loose against some team.

Question 4.5

Play(vince, red) $\rightarrow$ $\exists$x (Player(x) $\wedge$ Injured(x)):

If Vincent plays for the Red Arrows, some player gets injured.

Question 4.6

$\forall$x ((Team(x) $\wedge$ Play(felicity, x)) $\rightarrow$ Loose(x, red)):

If Felicity plays for any team, that team loose against the Red Arrows.