Tutorial Letter 101/3/2018

Formal Logic 2

COS2661

Semesters 1 and 2

School of Computing

This tutorial letter contains important information about your module.

BARCODE

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- COS2661 is a semester module. You need at least eight hours per week for this module.
- You have to purchase the prescribed book immediately.
- If you do not receive your study material immediately after registration, you have to download it so that you are able to start with your studies. See Section 1.1 for detail about the downloading of study material.
- To gain admission to the examination you have to submit Assignment 01 in time. Look at the due dates in this tutorial letter.

Dear Student

Welcome to COS2661. We hope that you will find it interesting and stimulating. You are most welcome to contact your lecturers for any academic queries regarding COS2661. See Sections 3.1 and 3.2 below. This is for academic issues only. (See Section 3.3 for information regarding other kinds of queries.) This tutorial letter contains general information about COS2661 - we discuss the tutorial matter, student support, the syllabus, a recommended study programme, and requirements for examination admission, semester mark and how to set about submitting assignments. In addition the letter contains Assignments 01, 02 and 03, first for the first semester and then for the second semester, as well as the solution to Assignment 03.

Some facts about the three assignments:

- The submission of Assignment 01 determines examination admission.
- Assignment 02 counts 70% of your semester mark.
- Assignment 03 is a self-assessment assignment and should not be submitted. ***Note, however, that it covers important study material.*** The model solution is given at the end of this tutorial letter, under addendum.

1 INTRODUCTION

Tutorial matter

The prescribed book (see Section 4.1) is essential study material. You will also need the tutorial letters that you will receive during the course of the semester. Tutorial letter 102 should be available when you register. This letter is your "study guide" and is meant to help you to master the material in the textbook. Some concepts are explained in more detail, examples are given, et cetera. You should use it together with the textbook. Please let us know if you find any errors in Tutorial letter 102.

Other tutorial letters will provide additional information (Tutorial Letters 103, 104, etc.) whilst others will discuss the assignments (Tutorial letters 201 and 202).

Some of the tutorial matter will not be available when you register. Tutorial matter that is not available when you register will be posted to you as soon as possible. It is not necessary to wait for the printed copies of these letters. The tutorial letters will be available on the Internet. Keep an eye upon myUnisa. One of the requirements for study at the School of Computing is to have regular access to myUnisa. It is therefore expected from you to download any study material from the Internet that, for whatever reason, is not available on paper in time. When you want to use this facility for the first time, you have to register. Go to my.unisa.ac.za and click on "Join myUnisa". Then follow the instructions on the screen. You will get a password for future use. The tutorial letters will be sent out in English only.

COS2661 is a semester module, therefore, time is of the utmost importance. You should start studying the module immediately after registration. This tutorial letter and Tutorial letter 102 (and the textbook) are the most important.

2 PURPOSE AND OUTCOMES

2.1 Purpose

The purpose of studying logic is to refine one's natural ability to reason and argue. Logic is concerned with training the mind to think clearly. The aim of logic is to secure clearness in the definition and arrangement of our ideas and other mental images, consistency in our judgments, and validity in our processes of inference. By studying logic a student is prepared to:

- confront the world with a firm grasp of logical thinking skills,
- view logic as a defensive tool, a tool that allows one to defend oneself against the onslaught of powerful persuasive appeals that bombard us daily,
- apply logic extensively in the fields of Artificial Intelligence, Computer Science and Philosophy.

2.2 Outcomes

At the end of this study you should:

- be imparted with strategies for thinking well,
- know about the common errors in reasoning which should be avoided and
- have developed effective techniques for evaluating arguments.

Abbreviated syllabus:

Propositional logic: logic of atomic sentences and the Boolean connectives; informal and formal proofs.
Quantifiers: logic of quantifiers; informal and formal proofs.

3 LECTURER(S) AND CONTACT DETAILS

3.1 Lecturer(s)

As mentioned in Tutorial letter COSALLF/301/4/2018, students may contact lecturers by mail, e-mail or telephone. We recommend the use of e-mail. COSALLF tutorial letter contains the names and contact details of your COS2661 lecturers. Students may also make appointments to see a lecturer, but this has to be done well in advance. Students should mention their student number in all communications with the lecturers.

3.2 Department

In the meantime, if you would like to speak to a lecturer, you may contact the secretary of the School of Computing at 011 670 9188. Remember to mention your student number. This is for academic queries only. Please do not contact the School about missing tutorial matter, cancellation of a module, payments, enquiries about the registration of assignments, and so on, but rather the relevant department as indicated in the brochure Study @ Unisa.

3.3 University

The brochure *Study @ Unisa* that you received with your tutorial matter contains information about computer laboratories, the library, myUnisa, assistance with study skills, et cetera. It also contains contact details of several Unisa departments, for example Examinations, Assignments, Despatch, Finances and Student Administration. Remember to mention your student number when contacting the University.

4 RESOURCES

4.1 Prescribed books

The prescribed book for this course is: Barwise, J. & J. Etchemendy, 2011. Language, proof and logic. Second Edition, Stanford: Center for the Study of Language and Information.

You have to purchase this book yourself at any of the official booksellers mentioned in the brochure *Study @ Unisa.* If you have difficulties with obtaining a copy of the book from these bookshops, please contact the Unisa Prescribed Book Section as soon as possible.

Together with this book you will receive a CD which contains the computer applications Tarski's world, Fitch, Boole, and Submit, and a manual that explains how to use them. All four of the applications run on Macintosh computers or PC's equipped with Microsoft Windows 95, 98, or NT 4.0. Three of these applications (Submit, Fitch, and Boole) are written using Java, and so require that your computer have the appropriate Java software installed on it. All the required software, including the necessary Java software, is included on the CD-ROM, though it may require installation.

4.2 Recommended books

There are no recommended books for this module.

4.3 Electronic reserves (e-reserves)

There are no e-Reserves for this module.

4.4 Library services and resources information

For brief information, go to www.unisa.ac.za/brochures/studies

For detailed information, go to http://www.unisa.ac.za/library. For research support and services of personal librarians, click on "Research support".

The library has compiled a number of library guides:

- finding recommended reading in the print collection and e-reserves – http://libguides.unisa.ac.za/request/undergrad
- requesting material – http://libguides.unisa.ac.za/request/request
- postgraduate information services – http://libguides.unisa.ac.za/request/postgrad
- finding, obtaining and using library resources and tools to assist in doing research – http://libguides.unisa.ac.za/Research_Skills
- how to contact the library/finding us on social media/frequently asked questions – http://libguides.unisa.ac.za/ask

5 STUDENT SUPPORT SERVICES

Important information appears in your *Study @ Unisa* brochure. The Student Services Bureau of Unisa provides support for students' in general academic matters, such as selecting appropriate modules, developing study skills, adapting to distance education or general difficulties with studies.

It is not necessary to make use of a computer in order to pass this module, but it is nevertheless highly recommended that you do so by applying the software supplied with the textbook. (See Section 4.1.) It will be very useful for mastering the material. In particular we strongly recommend that you use the software to do the exercises on Tarski worlds and formal proofs as given in the textbook.

You are encouraged to join fellow students and take an active part in the discussion forums on myUnisa.

You may also organise your own study group. However, we expect every member of a study group to write and submit the assignments on his or her own. Thus, discuss the problem, find solutions, etc., in the group, but then *do the assignment yourself and submit your own effort*. It is dishonest to submit the work of somebody else as your own. **Plagiarism** is the act of taking words, ideas and thoughts of others and passing them off as your own. It is a form of theft which involves a number of dishonest academic activities. The *Disciplinary Code for Students* (2004) is given to all students at registration. You are advised to study the Code, especially Sections 2.1.13 and 2.1.14 (2004:3-4). Kindly read the University's *Policy on Copyright Infringement and Plagiarism* as well.

6 STUDY PLAN

Use your *Study @ Unisa* brochure for general time management and planning skills. For this module we recommend that you use the study programme given below as a starting point. You will probably need to adapt this schedule, taking into account your other modules and your personal circumstances. You are expected to spend at least 8 hours per week on COS2661. Keep in mind that the skills you will learn in this course are progressive, i.e. you will not be able to understand or do the exercises in the later chapters if you skipped the earlier ones.

SEMESTER 1				
Week	**Date Monday**	**Activity: Textbook**	**Activity: Tutorial letter 102**	**Remarks**
1	23 January	Introduction and Chapter 1: 1.1 – 1.4	Chapter 1	
2	30 January	Chapter 2: 2.1 – 2.5	Chapter 2	
3	6 February	Chapter 3: 3.1 – 3.7	Chapter 3	
4	13 February	Chapter 4: 4.1 – 4.6	Chapter 4	Do Assignment 01 and submit it. The due date is **20 February 2018**.
5	20 February	Chapter 5: 5.1 – 5.4	Chapter 5	
6	27 February	Chapter 6: 6.1 – 6.6	Chapter 6	
7	5 March	Chapter 7: 7.1 – 7.4	Chapter 7	
8	12 March	Chapter 8: 8.1, 8.2, 8.4	Chapter 8	
9	19 March	Chapter 9: 9.1 – 9.6	Chapter 9	Do Assignment 02 and submit it. The due date is **26 March 2018**.
10	26 March	Chapter 10: 10.1 – 10.4	Chapter 10	
11	2 April	Chapter 11: 11.1 – 11.5, 11.7 – 11.8	Chapter 11	
12	9 April	Chapter 12: 12.1 – 12.4	Chapter 12	
13	16 April	Chapter 13: 13.1 – 13.3, 13.5	Chapter 13	
14	23 April	Chapter 14: 14.1	Chapter 14	Do Assignment 03 but do *not* submit it. The due date is **30 April 2018.**
15	30 April	Revision		
7 May until examination		Revision. Study ***all*** study material, including the solutions to the assignments. Read carefully through the examination tutorial letter.		
SEMESTER 2				
Week	**Date Monday**	**Activity: Textbook**	**Activity: Tutorial letter 102**	**Remarks**
1	9 July	Introduction and Chapter 1: 1.1 – 1.4	Chapter 1	
2	16 July	Chapter 2: 2.1 – 2.5	Chapter 2	

3	23 July	Chapter 3: 3.1 – 3.7	Chapter 3	Do Assignment 01 and submit it. The due date is **6 August 2018**.
4	30 July	Chapter 4: 4.1 – 4.6	Chapter 4	
5	6 August	Chapter 5: 5.1 – 5.4	Chapter 5	Do Assignment 02 and submit it. The due date is **10 September 2018**.
6	13 August	Chapter 6: 6.1 – 6.6	Chapter 6	
7	20 August	Chapter 7: 7.1 – 7.4	Chapter 7	
8	27 August	Chapter 8: 8.1, 8.2, 8.4	Chapter 8	
9	3 September	Chapter 9: 9.1 – 9.6	Chapter 9	
10	10 September	Chapter 10: 10.1 – 10.4	Chapter 10	Do Assignment 03 but do *not* submit it. The due date is **15 October 2018.**
11	17 September	Chapter 11: 11.1 – 11.5, 11.7 – 11.8	Chapter 11	
12	24 September	Chapter 12: 12.1 – 12.4	Chapter 12	
13	1 October	Chapter 13: 13.1 – 13.3, 13.5	Chapter 13	
14	8 October	Chapter 14: 14.1	Chapter 14	
15	15 October	Revision		
22 October until examination		Revision. Study ***all*** study material, including the solutions to the assignments. Read carefully through the examination tutorial letter.		

7 PRACTICAL WORK AND WORK-INTEGRATED LEARNING

There are no practicals for this module.

8 ASSESSMENT

8.1 Assessment criteria

The marks that you obtain for Assignments 01 and 02 form the semester mark for COS2661.

8.2 Assessment plan

The semester mark forms 20% and the examination 80% of the final mark for the module. The weights of the COS2661 assignments are indicated in the table below.

Assignment	Weight
01	30%
02	70%
03	0%

An example follows: Suppose a student gets 60% for Assignment 01 and 45% for Assignment 02. In order to calculate the semester mark, the mark obtained for the specific assignment is multiplied by the weight. This then forms part of the 20% that the semester mark contributes to the final mark. Therefore:

Assignment	**Marks obtained**	**Weight**	**Contribution to semester mark**	
01	60%	30%	60/100 x 30/100 x 20	3.60
02	45%	70%	45/100 x 70/100 x 20	6.30
Total				9.90

In this example the student has a semester mark of 9.90. The semester mark will not form part of the final mark of a supplementary examination.

8.3 Assignment numbers

8.3.1 General assignment numbers

Assignments are numbered consecutively per module, starting from 01.

8.3.2 Unique assignment numbers

Semester	Assignment No:	Unique Number
Semester 1	Assignment 01	867221
	Assignment 02	853449
Semester 2	Assignment 01	865521
	Assignment 02	845524

8.4 Assignment due dates

Assignment number	**Due dates of COS2661 assignments**		**Weight for semester mark**
	Semester 1	**Semester 2**	
01 (multiple choice) Compulsory for admission to the examination	20 February 2018	6 August 2018	30%
02 (written assignment)	26 March 2018	10 September 2018	70%
03 (self-assessing)	30 April 2018	15 October 2018	0%

8.5 Submission of assignments

To do assignments is extremely important for mastering the study material. We strongly advise you to do all three assignments.

The three assignments of both the first and second semesters are given in Section 8.4 of this tutorial letter. The tutorial matter to study for each assignment appears in the study timetables in Section 6 and at the start of each assignment. Give yourself enough time to do the assignments properly, realising that an hour or two will not be sufficient. Follow the procedures in this tutorial letter, in Tutorial letter COSALLF/301/4/2018 and in the brochure *Study @ Unisa* when submitting your assignments.

- ***Assignment 01*** is a multiple choice assignment and has to be submitted either on a mark reading sheet (by post) or electronically through myUnisa. There is no extension of the due date for this assignment.

- ***Assignment 02*** may be submitted (i) electronically as a .pdf file (by using myUnisa) or (ii) through normal post inside an assignment cover with all the particulars filled in on the cover. The semester system does not allow for late submission of the assignment. The work that you have to study during the last weeks of the semester is very important and there will be examination questions set on it. However, because of the constraints of the semester system.

- ***Assignment 03*** is a self-assessment assignment. Thus you should not submit it. The model solution is given at the end of this tutorial letter. Compare your effort to it.

All students will receive a discussion of Assignments 01 and 02 (Tutorial letters 201 and 202). Carefully compare these discussions with your own answers and make sure that you understand any differences.

You will receive a percentage mark for Assignments 01 and 02 if you submit them by the respective deadlines. The marks obtained for these two assignments form the semester mark for this module – see Section 8.1. Also note that examination admission is dependent on the submission of Assignment 01 – see Section 9.

It is possible that not all questions of Assignment 02 will be marked. Please note that you will obtain 0% if you do not submit the question(s) that is/are marked. The unmarked questions will be considered to be self-assessment questions. In those cases, compare your effort with the model solution. As mentioned above, we discuss all the questions of every assignment in the relevant tutorial letter that you will receive after every due date and that will be available on myUnisa.

For detailed information on assignments, please refer to the *Study @ Unisa* brochure, which you received with your study package.

To submit an assignment via *my*Unisa:

- Go to *my*Unisa.
- Log in with your student number and password.
- Select the module.
- Click on assignments in the menu on the left-hand side of the screen.
- Click on the assignment number you wish to submit.
- Follow the instructions.

8.6 The assignments

THE ASSIGNMENTS OF THE FIRST SEMESTER

ASSIGNMENT 01 FIRST SEMESTER

(MULTIPLE CHOICE)

SUBMISSION: On multiple choice form or electronically through myUnisa

Please note that Assignment 01 has to be submitted in order to gain examination admission. It will be to your own advantage to check whether the assignment has been registered on the system after a few days.
If you want to submit the assignment electronically and myUnisa is off-line during that time, you need not contact us, because we will be aware of it. Simply submit it as soon as myUnisa is available again.

Due date	20 February 2018
Extension	No extension
Tutorial matter	Textbook chapters Chapter 1: 1.1 – 1.4 Chapter 2: 2.1 – 2.5 Chapter 3: 3.1 – 3.7 Chapter 4: 4.1 – 4.6 Tutorial letter 102 chapters 1, 2, 3 and 4
Weight of contribution to semester mark	30%
Unique number	**867221**
Questions	20 questions, with 4 options each. Choose one option in every question.

You will need predicates and names in many of the questions.

- You should use the interpretation of the blocks language predicates as given on page 22 of your textbook.
- You should use the interpretation of other names and predicates as given on page 30 of your textbook, except when explicitly instructed differently in the question.
  If you have access to a computer, we recommend that you use the Tarski world software (supplied with the textbook) to help you to choose the correct option in questions 1, 2 and 3.

back

	d: C, M		**e**: D, S						
left						**b**: C, L			right
		c: D, S							
	a: T, M						**f:** T, S		

front

Figure 1 – used in questions 1 and 2

See page 10 of tutorial letter 102 for an explanation of the entries in the figure.

QUESTION 1

Given the Tarski world in Figure 1 and the seven sentences below, which of the options is correct?

Sentence 1: Large(a) $\vee$ Large(b)

Sentence 2: SameShape(a,b) $\vee$ SameSize(a,d)

Sentence 3: Between(e,d,b)

Sentence 4: Small(f) $\wedge$ ($\neg$ Cube(f) $\vee$ $\neg$ Tet(a))

Sentence 5: LeftOf(f,b) $\vee$ RightOf(a,d)

Sentence 6: $\neg$ (Cube(a) $\wedge$ Dodec(c))

Sentence 7: $\neg$ Medium(b) $\wedge$ Medium(d).

Option 1: Sentences 1, 2, 3, 4, 5 and 7 are true and sentence 6 is false.

Option 2: Sentences 1, 2, 3 and 7 are true and sentences 4, 5 and 6 are false

Option 3: Sentences 1, 2, 4, 6 and 7 are true and sentences 3 and 5 are false.

Option 4: All seven sentences are true.

QUESTION 2

You have to change the sentence

$\neg$ (Dodec(a) $\wedge$ $\neg$ Tet(c)) $\wedge$ (Large(c) $\vee$ Medium(f))

which is false in the Tarski world given in Figure 1 so that it becomes true in that world. Which of the options below will be a correct change?

Option 1: $\neg$ (Dodec(a) $\wedge$ $\neg$ Tet(c)) $\wedge$ (Large(c) $\wedge$ Medium(f))

Option 2: $\neg$ (Dodec(a) $\wedge$ $\neg$ Tet(c)) $\vee$ (Large(c) $\vee$ Medium(f))

Option 3: (Dodec(a) $\wedge$ $\neg$ Tet(c)) $\wedge$ (Large(c) $\vee$ Medium(f))

Option 4: $\neg$ (Dodec(a) $\vee$ $\neg$ Tet(c)) $\wedge$ (Large(c) $\wedge$ Medium(f)).

QUESTION 3

Given the following seven sentences, you have to build a Tarski world in which all the sentences will be true. Which of the options below is a world where this is the case?

Sentence 1: Between(a,b,c)

Sentence 2: Tet(f) $\wedge$ (b = e)

Sentence 3: Cube(e) $\vee$ $\neg$ Cube(a)

Sentence 4: Medium(c) $\wedge$ Larger(e,c)

Sentence 5: $\neg$ SameShape(b,f) $\wedge$ SameSize(b,f)

Sentence 6: RightOf(f,c)

Sentence 7: SameRow(e,f) $\wedge$ $\neg$ SameRow(a,b).

Option 1:

back

	c:								
	a: D, S								
left									right
	b, **e**: D, L				**f**: T, M				

front

Tarski world question 3 option 1

Option 2:

back

	c: T, M								
left									right
		a: D, S							
	b, **e**: D, L				**f**: T, L				

front

Tarski world question 3 option 2

Option 3:

back

	c:								
left	**a**: D, S								right
	b, **e**: D, S				**f**: T, S				

front

Tarski world question 3 option 3

Option 4:

back

	c: T, M								
left									right
	a: D, S								
	b, **e**: D, L				**f**: T, L				

front

Tarski world question 3 option 4

QUESTION 4

Given the following sentence in FOL, which of the options below is a correct English translation?

Person(pris) $\vee$ Pet(pris)

Option 1: Pris is a person or a pet.

Option 2: Pris is the name of a person and also the name of her pet.

Option 3: In this world all persons are called Pris or all pets are called Pris.

Option 4: Person versus Pet.

QUESTION 5

Given the following sentence in FOL, which of the options below is a correct English translation?

Hungry(max, 2:00) $\wedge \neg$ Hungry(carl, 2:00) $\wedge$ Pet(carl) $\wedge$ Fed(max, carl, 2:00)

Option 1: Max and the pet Carl were both hungry at 2 pm, Jan 2, 2001, so Max fed Carl.

Option 2: Max and Carl were both hungry and t is less than t′.

Option 3: At 2 pm, Jan 2, 2001 Max was hungry but the pet Carl was not.

Option 4: At 2 pm, Jan 2, 2001 Max was hungry and fed the pet Carl who was not hungry.

QUESTION 6

Given the following sentence in FOL, which of the options below is a correct English translation?

Pet(pris) $\wedge$ (Owned(max, pris, 2:00) $\vee$ Owned(claire, pris, 2:00))

Option 1: Pris owned either Max or Claire at 2 pm, Jan 2, 2001.

Option 2: Pris is a pet that was owned by Max or Claire at 2 pm, Jan 2, 2001.

Option 3: Pris is a pet that was owned by Max and Claire at 2 pm, Jan 2, 2001

Option 4: Pris is either a pet or was owned by Max or Claire at 2 pm, Jan 2, 2001.

QUESTION 7

Given the following sentence in English, which of the options below is a correct translation in FOL?

Claire is not a student and not a pet.

Option 1: $\neg$ Student(claire) $\vee$ $\neg$ Pet(claire)

Option 2: $\neg$ (Student(claire) $\wedge$ Pet(claire))

Option 3: $\neg$ Student(claire) $\wedge$ $\neg$ Pet(claire)

Option 4: $\neg$ (Student $\vee$ Pet)(claire).

QUESTION 8

Given the following sentence in English, which of the options below is a correct translation in FOL?

Although Scruffy was not hungry, he was fed by Max at 2 pm, Jan 2, 2001.

Option 1: $\neg$ Hungry(scruffy, 2:00) $\vee$ Fed(scruffy, max, 2:00)

Option 2: $\neg$ Hungry(scruffy, 2:00) $\wedge$ Fed(scruffy, max, 2:00)

Option 3: $\neg$ Hungry(scruffy, 2:00) $\wedge$ Fed(max, scruffy, 2:00)

Option 4: $\neg$ Hungry(scruffy, 2:00) $\vee$ Fed(max, scruffy, 2:00).

QUESTION 9

Given the following sentence in English, which of the options below is a correct translation in FOL?

Claire was hungry at 2 pm, Jan 2, 2001 or 2 pm is not earlier than 2:05.

Option 1: Hungry(claire, 2:00) $\vee$ (2:00 > 2:05)

Option 2: Hungry(claire, 2:00) $\vee$ $\neg$ (2:00 < 2:05)

Option 3: Hungry(claire, 2:00) $\wedge$ $\neg$ (2:00 < 2:05)

Option 4: Hungry(claire, 2:00) $\wedge$ (2:00 > 2:05).

QUESTION 10

Complete the following truth table on some scrap paper and then indicate which option below reflects the last column correctly:

P	**Q**	**R**	**$\neg$ Q**	**P $\vee$ $\neg$Q**	**$\neg$ (P $\vee$ $\neg$Q)**	**(Q $\wedge$ R)**	**$\neg$ (P $\vee$ $\neg$Q) $\vee$ (Q $\wedge$ R)**

Option 1:

$\neg$ (P $\vee$ $\neg$Q) $\vee$ (Q $\wedge$ R)
T
F
F
F
T
T
F
F

Option 2:

$\neg$ (P $\vee$ $\neg$Q) $\vee$ (Q $\wedge$ R)
T
F
T
F
T
T
F
T

Option 3:

$\neg$ (P $\vee$ $\neg$Q) $\vee$ (Q $\wedge$ R)
F
F
T
F
T
T
F
F

Option 4:

¬ (P ∨ ¬Q) ∨ (Q ∧ R)
T
T
T
T
T
T
F
T

QUESTION 11

Which of the options below correctly states how we can determine if a given sentence of FOL built up from atomic sentences by means of truth functional connectives is a tautology?

Option 1: Build the truth table for the sentence and inspect the truth values below the main connective. If there is at least one T, the sentence is a tautology.

Option 2: Build the truth table for the sentence and inspect the truth values below the main connective. If there are more occurrences of T than F, the sentence is a tautology.

Option 3: Build the truth table for the sentence and inspect the truth values below the main connective. If all these values are T, the sentence is a tautology.

Option 4: Build the truth table for the sentence and inspect the truth values below the main connective. If all these values are F, the sentence is a tautology.

QUESTION 12

Which of the options below correctly states how we can determine if a given sentence of FOL built up from atomic sentences by means of truth functional connectives is TT- possible?

Option 1: Build the truth table for the sentence and inspect the truth values below the main connective. If there is at least one T, the sentence is TT-possible.

Option 2: Build the truth table for the sentence and inspect the truth values below the main connective. If there are more occurrences of T than F, the sentence is TT-possible.

Option 3: Build the truth table for the sentence and inspect the truth values below the main connective. If all these values are T, the sentence is TT-possible.

Option 4: Build the truth table for the sentence and inspect the truth values below the main connective. If all these values are F, the sentence is TT-possible.

The following partially constructed truth table is used in questions 13, 14 and 15. The blocks language is used for all the sentences.

	Tet(a)	**a = b**	**Cube(b)**	¬ **(a = b)**	....	some sentence
	T	T	F	T		
	T	T	F	F		
	T	F	T	T		
	T	F	T	F		
	F	T	F	T		
	F	T	F	F		
	F	F	T	T		
	F	F	T	F		

QUESTION 13

Suppose we want to determine if a certain sentence in the blocks language built from the three atomic sentences **Tet(a), a = b** and **Cube(b)** is a tautological consequence of **Cube(b)** and ¬ **(a = b)**. We build the complete truth table and then inspect the last column. The sentence is a tautological consequence of the two sentences **Cube(b)** and ¬ **(a = b)** if

Option 1: only the entry in line 8 is F

Option 2: the entry in line 7 is T

Option 3: the entry in line 1 is T and the entry in all other lines is F

Option 4: the entry in lines 3 and 7 is T.

QUESTION 14

Suppose we want to determine if a certain sentence in the blocks language built from the three atomic sentences **Tet(a), a = b** and **Cube(b)** is a logical consequence of **Cube(b)** and ¬ **(a = b)**. We build the complete truth table and then inspect the last column. The sentence is a logical consequence of the two sentences **Cube(b)** and ¬ **(a = b)** if

Option 1: only the entry in line 8 is F

Option 2: the entry in line 7 is T

Option 3: T the entry in line 1 is T and the entry in all other lines is F

Option 4: the entry in lines 3 and 7 is T.

QUESTION 15

Suppose we want to determine if two sentences in the blocks language, both built from the three atomic sentences **Tet(a), a = b** and **Cube(b)**, are tautological equivalent. We build the complete truth table where the last two columns indicate the respective truth values of the two sentences. We then inspect the last two columns. The two sentences are tautological equivalent if

Option 1: at least one of the two columns consist of T only
Option 2: the last two columns are identical
Option 3: the entries in the last two columns are F in lines 1, 3 and 7
Option 4: the entries in the last two columns are T in lines 2, 4, 6 and 7.

QUESTION 16

Consider the following argument:

| Some soccer fans pay R1000 for a ticket to the match between SA and Spain.
| Rob is a soccer fan.
|---
| Rob pays R1000 for a ticket to the match between SA and Spain.

Which of the following options is a proof of the validity or not of the argument?

Option 1: The argument is valid because the chances are very good that Rob has enough money for this.
Option 2: The argument is not valid and we prove nonconsequence as follows: The first statement only states that some soccer fans pay R1000, thus there are fans that do not pay as much. Rob may be one of those who do not pay R1000. Hence the first two statements are true but the conclusion is not.
Option 3: The argument is not valid because no soccer fan pays R1000 for any ticket.
Option 4: The argument is not valid because Rob is not a soccer fan.

QUESTION 17

Consider the following argument where the blocks language is used:

| Larger(a,b)
| Larger(c,b)
|---
| Larger(a,c)

Which of the following options is a proof of the validity or not of the argument?

Option 1: The argument is valid because, if both **a** and **c** are larger than **b**, then **a** has to be larger than **c**.

Option 2: The argument is not valid and we prove nonconsequence as follows: If both **a** and **c** are larger than **b**, then **a** and **c** should be two names for the same thing and thus one cannot be larger than the other. Hence the first two statements are true but the conclusion is not.

Option 3: The argument is not valid and we prove nonconsequence as follows: Suppose **a** is of medium size, **b** is small and **c** is large. Then the first two statements are true but the conclusion is not.

Option 4: The argument is not valid because it is impossible to allocate sizes like.

QUESTION 18

One or more statement has been omitted from the formal proof (in the blocks language) below. Which option indicates the complete proof?

```
| 1  LeftOf(a, b)
| 2  f = a
|----
|  ...
|  ...
|  LeftOf(f, b)
```

Option 1:

```
| 1    LeftOf(a, b)
| 2    f = a
|----
| 3    f = f            = Intro
| 4    a = f            = Elim  2, 3
| 5    LeftOf(f, b)     = Elim  1, 4
```

Option 2:

```
| 1    LeftOf(a, b)
| 2    f = a
|----
| 3    a = a            = Intro
| 4    a = f            = Elim  2, 3
| 5    LeftOf(f, b)     = Elim  1, 4
```

Option 3:

1	LeftOf(a, b)	
2	f = a	
3	a = f	= Elim 2
4	LeftOf(f, b)	= Elim 1, 3

Option 4:

1	LeftOf(a, b)	
2	f = a	
3	LeftOf(a, b)	= Elim 1, 2

QUESTION 19

Given the following sentence of FOL, which of the options below is an equivalent sentence in disjunctive normal form (DNF)?

$\neg (A \lor (B \lor \neg C)) \lor ((A \lor C) \lor \neg D)$

Option 1: $(\neg A \land \neg (B \lor \neg C)) \lor (A \land \neg D) \lor (C \land \neg D)$

Option 2: $(\neg A \land \neg (B \lor \neg C)) \lor (A) \land (C \land \neg D)$

Option 3: $(\neg A \land \neg B \land C) \lor A \lor C \lor \neg D$

Option 4: $(\neg A \lor \neg B) \land \neg C \land (\neg A \lor \neg C) \land \neg D$.

QUESTION 20

Given the following sentence of FOL, which of the options below is an equivalent sentence in conjunctive normal form (CNF)?

$\neg ((A \land B) \lor C) \land \neg (A \land C) \land \neg D$

Option 1: $(A \lor \neg B) \land \neg C \land (\neg A \lor \neg C) \land \neg D$

Option 2: $(\neg A \lor \neg B) \land \neg C \land (\neg A \lor \neg C) \land \neg D$

Option 3: $(\neg (A \land B) \land \neg C) \land (\neg A \lor \neg C) \land \neg D$

Option 4: $(\neg (A \land B) \land \neg C) \land \neg (A \lor \neg C) \land \neg D$.

End of Assignment 1: Semester 1

ASSIGNMENT 02 FIRST SEMESTER

SUBMISSION: Printouts or electronically through myUnisa (as one .pdf file)

It will be to your own advantage to check whether the assignment has been registered on the system after a few days. If you want to submit the assignment electronically and myUnisa is off-line during that time, you need not contact us, because we will be aware of it. Simply submit it as soon as myUnisa is available again.

Due date	25 March 2018
Extension	No extension
Tutorial matter	Textbook: All previous work and Chapter 5: 5.1 – 5.4 Chapter 6: 6.1 – 6.6 Chapter 7: 7.1 – 7.4 Chapter 8: 8.1, 8.2, 8.4 Chapter 9: 9.1 – 9.6 Tutorial letter 102 chapters 1 to 9
Weight of contribution to semester mark	70%
Unique number	**853449**

QUESTION 1 **[12]**

Consider the arguments below and decide whether they are valid. If they are, write down an informal proof, phrased in complete, well- formed English sentences. When you use proof by case or proof by contradiction, say so. You do not have to be explicit about the use of simple proof steps like conjunction elimination. If the argument is invalid, construct a counter example.

Question 1.1

1	Dodec(a) ∨ Dodec(b)
2	¬ Dodec(a) ∨ Cube(c)
3	¬ Dodec(b) ∨ Cube(c)
	Cube(c)

Question 1.2

1	Anna or Ben is not shopping.
2	Ben is shopping or Ben and Anna are married.
3	Anna and Ben are not married or Anna is shopping.
	Ben and Anna are married.

Question 1.3

1	Student(mary) ∨ Hungry(mary, 2:00)
2	¬ Hungry(mary, 2:00) ∨ ¬ Pet(rex)
3	Pet(rex) ∨ ¬ Student(mary)
	Student(mary)

QUESTION 2 **[25]**

In this question you have to construct formal proofs using the natural deduction rules. The Fitch system makes use of these rules.

A summary of the rules of natural deduction is given on pages 573 to 578 of your textbook. Consult this when you do question 2. Remember that De Morgan's laws and other tautologies are not permissible natural deduction rules. You are also not allowed to use Taut Con, Ana Con or FO Con. It is important to number your statements, to indicate subproofs and at each step to give the rule that you are using.
Hint: If you have access to a computer, take advantage of the fact and use Fitch.

Question 2.1 **(7)**

Using the natural deduction rules, give a formal proof that the following sentences are inconsistent:

1 $P \wedge R$

2 $R \rightarrow \neg P$

Question 2.2 **(9)**

Using the natural deduction rules, give a formal proof of

$P \rightarrow S$

from the premises

1 $P \rightarrow (Q \vee R)$

2 $Q \rightarrow S$

3 $R \rightarrow S$

Question 2.3 **(9)**

Using the natural deduction rules, give a formal proof of

$(P \rightarrow Q) \rightarrow ((\neg P \rightarrow Q) \rightarrow Q)$

from the premise

1 $P \vee \neg P$

English	FOL
Names	
Betty	betty
Thabo	thabo
Kitty	kitty
Durban	durban
Bloemfontein	bloemfontein
Predicates	
x is a city	City(x)
x drives a lorry in y	Drives(x, y)
x is a driver	Driver(x)
x obeys the traffic rules	Obeys(x)
x is involved in an accident	Accident(x)

Table 1 used to answer questions 3 and 4

QUESTION 3 **[45]**

In this question you have to translate English sentences into sentences of First Order Logic using the predicates and names given in Table 1.

Question 3.1

Whenever Thabo drives a lorry in Durban, he is involved in an accident.

Question 3.2

Kitty is involved in an accident only if she does not obey the traffic rules.

Question 3.3

Unless Betty obeys the traffic rules when she drives a lorry in Bloemfontein, she is involved in an accident.

Question 3.4

Durban or Bloemfontein is a city.

Question 3.5

If we assume that Kitty drives a lorry in Durban, either Thabo or Betty drives a lorry in Bloemfontein.

Question 3.6

Neither Kitty nor Betty obeys the traffic rules.

Question 3.7

Thabo is involved in accident if and only if both he and Betty do not obey the traffic rules.

Question 3.8

None of Thabo, Kitty or Betty is involved in an accident.

Question 3.9

If Kitty obeys the traffic rules and Betty does not drive a lorry in Bloemfontein, Thabo is not involved in an accident.

Question 3.10

Thabo obeys the traffic rules only if he drives a lorry in Durban.

Question 3.11

All drivers are involved in accidents.

Question 3.12

Some people driving lorries in Durban obey the traffic rules.

Question 3.13

Somebody driving a lorry in Bloemfontein does not obey the traffic rules but is not involved in an accident.

Question 3.14

If anyone drives a lorry in Bloemfontein and does not obey the traffic rules, he or she is involved in an accident.

Question 3.15

Not all drivers do not obey the traffic rules.

QUESTION 4 **[18]**

In this question you have to translate sentences of First Order Logic into English sentences using the predicates and names given in Table 1.

Question 4.1

$\neg \text{Drives}(\text{kitty}, \text{durban})$

Question 4.2

$\text{Obey}(\text{betty}) \rightarrow \text{Accident}(\text{kitty})$

Question 4.3

$(\text{Accident}(\text{betty}) \wedge \text{Accident}(\text{kitty})) \leftrightarrow \neg (\text{Obey}(\text{betty}) \vee \text{Obey}(\text{kitty}))$

Question 4.4

$\exists x\, (\text{Driver}(x) \wedge \text{Obey}(x))$

Question 4.5

$\text{Driver}(\text{thabo}) \rightarrow \exists x\, (\text{City}(x) \wedge \neg \text{Drives}(\text{thabo}, x))$

Question 4.6

$\forall x\, ((\text{Driver}(x) \wedge \text{Obey}(x)) \rightarrow \neg \text{Accident}(x))$

End of Assignment 2: Semester 1

ASSIGNMENT 03 FIRST SEMESTER

Submission: The assignment should NOT be submitted.

You should assess it yourself when you have completed it. The model solution is given at the end of this tutorial letter. ***Note that there will be examination questions set on this part of the study material also.***

Due date	30 April 2018
Extension	Not applicable
Tutorial matter	Textbook: All previous material and Chapter 10: 10.1 – 10.4 Chapter 11: 11.1 – 11.5, 11.7, 11.8 Chapter 12: 12.1 – 12.4 Chapter 13: 13.1, 13.2, 13.3, 13.5 Chapter 14: 14.1 Tutorial letter 102 chapters 1 to 14
Weight of contribution to semester mark	0%
No unique number	

QUESTION 1

Use De Morgan's laws and show that

$\neg$ ($\exists$x Brillig(x) $\land$ $\forall$y Gimble(y))

is logically equivalent to

$\forall$x $\exists$y ($\neg$ Brillig(x) $\lor$ $\neg$ Gimble(y))

QUESTION 2

Below a Tarski world is given followed by eleven sentences. Eight of the sentences are true in the given world. Which three sentences are false in the given world? If you have access to a computer, use the Tarski software to check your answers.

back

left / right

a: C, S							
d: T, S							
e: C, S			**b**: D, L			**c**: T, M	

front

Tarski World: Question 2

Sentences:

1 $\exists x\, \text{Between}(x, e, a) \land \exists x\, \text{Between}(x, e, c)$

2 $\forall x\, (\text{Cube}(x) \rightarrow \exists y\, \exists z\, (\text{Dodec}(y) \land \text{Tet}(z) \land \text{RightOf}(y, x) \land \text{LeftOf}(x, z)))$

3 $\neg \forall x\, (\text{Cube}(x) \rightarrow \text{Small}(x))$

4 $\forall x\, (\text{Small}(x) \lor \text{Medium}(x) \lor \text{Dodec}(x))$

5 $\forall x\, (\text{Dodec}(x) \land \text{Large}(x))$

6 $\exists y\, \forall x\, (\text{Medium}(y) \rightarrow (\text{Tet}(y) \land (\text{Cube}(x) \rightarrow \text{Larger}(y, x))))$

7 $\exists y\, ((\text{Dodec}(y) \land \text{Small}(y)) \lor (\text{Tet}(y) \land \text{Medium}(y)))$

8 $\exists x\, \exists y\, (\text{Cube}(x) \land \text{Tet}(y) \land \neg \text{SameSize}(x, y))$

9 $\forall x\, \exists y\, \text{SameCol}(x, y)$

10 $\exists y\, \forall x\, \text{SameCol}(x, y)$

11 $\text{Cube}(d) \rightarrow \forall x\, \forall y\, \text{SameShape}(x, y)$

English	FOL
Names	
Abe	abe
Beauty	beauty
Carol	carol
Predicates	
x is a typist	Typist(x)
x is a farmer	Farmer(x)
x is the sister of y	Sister(x, y)
x and y are cousins	Cousin(x, y)
x is in the library	Library(x)

Table 2 used in questions 3 and 4

QUESTION 3

Translate the following FOL sentences into English sentences using the predicates and names given in the table above.

Question 3.1

$\forall x\ (Farmer(x) \rightarrow \exists y\ (Sister(y, x) \vee Cousin(y, x)))$

Question 3.2

$\exists x\ (Typist(x) \wedge Cousin(x, abe) \wedge Sister(x, carol))$

Question 3.3

$Typist(abe) \rightarrow \forall x\ (Farmer(x) \rightarrow Library(x))$

Question 3.4

$\forall x\ ((Typist(x) \vee Farmer(x)) \wedge (Cousin(x, beauty) \vee Cousin(x, carol)))$

Question 3.5

$\exists x \forall y\ (\neg\ Cousin(x, y))$

QUESTION 4

Translate the following English sentences into FOL sentences using the predicates and names given in the table above.

Question 4.1

Abe has a cousin who is a farmer.

Question 4.2

Everybody has a sister who is a typist in the library.

Question 4.3

All farmers have sisters.

Question 4.4

If a farmer is in the library, he has a cousin who is a typist.

Question 4.5

Both Carol and Beauty have cousins but Abe does not have a cousin.

QUESTION 5

One of the arguments below is valid and the other not. If the argument is valid, give an *informal* proof. If the argument is not valid, construct a Tarski world which is a counterexample.

Question 5.1

$\forall y\ (Cube(y) \lor Dodec(y))$

$\forall y\ (Cube(y) \rightarrow Large(y))$

$\exists y\ \neg\ Large(y)$

$\exists y\ Dodec(y)$

Question 5.2

$\forall$y (Cube(y) $\vee$ Dodec(y))
$\forall$y ($\neg$ Small(y) $\rightarrow$ Tet(y))

$\neg$ $\exists$y Small(y)

QUESTION 6

In this question you have to construct ***formal proofs*** using the natural deduction rules. The Fitch system makes use of these rules.

A summary of the rules of natural deduction is given on pages 573 to 578 of your textbook. Consult this when you do this question. Remember that De Morgan's laws and other tautologies are not permissible natural deduction rules. You are also not allowed to use Taut Con, Ana Con or FO Con. It is important to number your statements, to indicate subproofs and at each step to give the rule that you are using.

Hint: If you have access to a computer, take advantage of the fact and use Fitch.

Question 6.1

Using the natural deduction rules, give a formal proof of

$\exists$x (Large(x) $\wedge$ LeftOf(x, b))

from the premises

$\forall$x (Cube(x) $\rightarrow$ Large(x))
$\forall$y (Large(y) $\rightarrow$ LeftOf(y, b))
$\exists$x Cube(x)

Question 6.2

Using the natural deduction rules, give a formal proof of the following argument:

$\forall$x [(AtHome(x) $\vee$ InLibrary(x)) $\rightarrow$ (Happy(x) $\wedge$ Reading(x))]
$\forall$y [(Smiling(y) $\vee$ Happy(y)) $\rightarrow$ InLibrary(y)]
$\exists$x Smiling(x)

$\exists$x (Smiling(x) $\wedge$ Happy(x))

Question 6.3

Using the natural deduction rules, give a formal proof of

$$\forall x\, [P(x) \rightarrow Q(x)] \rightarrow [\forall x P(x) \rightarrow \forall x Q(x)]$$

from no premises.

Question 6.4

Using the natural deduction rules, give a formal proof of

$$\exists x\, \text{Dodec}(x)$$

from the premises

$$\forall y\, [\text{Cube}(y) \vee \text{Dodec}(y)]$$

$$\forall x\, [\text{Cube}(x) \rightarrow \text{Large}(x)]$$

$$\exists x\, \neg \text{Large}(x)$$

QUESTION 7

Translate the following English sentence into an FOL sentence:

There are at most three cubes.

End of Assignment 3: Semester 1

THE ASSIGNMENTS OF THE SECOND SEMESTER

ASSIGNMENT 01 SECOND SEMESTER

(MULTIPLE CHOICE)

SUBMISSION: On multiple choice form or electronically through myUnisa

Please note that Assignment 01 has to be submitted in order to gain examination admission. It will be to your own advantage to check whether the assignment has been registered on the system after a few days. If you want to submit the assignment electronically and myUnisa is off-line during that time, you need not contact us, because we will be aware of it. Simply submit it as soon as myUnisa is available again.	
Due date	6 August 2018
Extension	No extension
Tutorial matter	Textbook chapters Chapter 1: 1.1 – 1.4 Chapter 2: 2.1 – 2.5 Chapter 3: 3.1 – 3.7 Chapter 4: 4.1 – 4.6 Tutorial letter 102 chapters 1, 2, 3 and 4
Weight of contribution to semester mark	30%
Unique number	**865521**
Questions	20 questions, with 4 options each. Choose one option in every question.

You will need predicates and names in many of the questions.

- You should use the interpretation of the blocks language predicates as given on page 22 of your textbook.
- You should use the interpretation of other names and predicates as given on page 30 of your textbook, except when explicitly instructed differently in the question.

If you have access to a computer, we recommend that you use the

- Tarski world software (supplied with the textbook) to help you to choose the correct option in questions 1, 2 and 3 and
- Fitch software (supplied with the textbook) to help you to choose the correct option in question 18.

back

left	**f**: D, L				**c**: D, L				right
			b: T, M						
					e: T, S				
	a: C, S						**d**: C, M		

front

Figure 1 – used in questions 1 and 2

See page 10 of tutorial letter 102 for an explanation of the entries in the figure.

QUESTION 1

Given the Tarski world in Figure 1 and the seven sentences below, which of the options is correct?

Sentence 1: Cube(a) $\wedge$ (Cube(c) $\vee$ Large(c))

Sentence 2: Cube(a) $\wedge$ (Cube(c) $\wedge$ Large(c))

Sentence 3: Between(b,a,d)

Sentence 4: $\neg$ (Dodec(b) $\wedge$ $\neg$ Tet(f))

Sentence 5: Cube(a) $\vee$ $\neg$ Cube(e)

Sentence 6: $\neg$ SameSize(b,e)

Sentence 7: Medium(a) $\vee$ $\neg$ Large(c).

Option 1: Sentences 1, 2, 3 and 4 are true and sentences 5, 6 and 7 are false.

Option 2: Sentences 1, 4, 5, 6 and 7 are true and sentences 2 and 3 are false.

Option 3: Sentences 1, 4, 5 and 6 are true and sentences 2, 3 and 7 are false.

Option 4: All seven sentences are true.

QUESTION 2

You have to change the sentence

$(\neg \text{Cube}(a) \land \neg \text{Dodec}(a)) \lor (\neg \text{Tet}(c) \land \neg \text{Dodec}(c))$

which is false in the Tarski world given in Figure 1 so that it becomes true in that world. Which of the options below will be a correct change?

Option 1: $(\neg \text{Cube}(a) \land \neg \text{Dodec}(a)) \land (\neg \text{Tet}(c) \land \neg \text{Dodec}(c))$

Option 2: $(\text{Cube}(a) \lor \neg \text{Dodec}(a)) \land (\neg \text{Tet}(c) \lor \neg \text{Dodec}(c))$

Option 3: $(\text{Cube}(a) \land \text{Dodec}(a)) \lor (\text{Tet}(c) \land \text{Dodec}(c))$

Option 4: $(\neg \text{Cube}(a) \lor \neg \text{Dodec}(a)) \land \neg(\neg \text{Tet}(c) \lor \neg \text{Dodec}(c))$.

QUESTION 3

Given the following seven sentences, you have to build a Tarski world in which all the sentences will be true. Which of the options below is a world where this is the case?

Sentence 1: $\text{Tet}(a) \lor \neg \text{Tet}(b)$

Sentence 2: $\text{Small}(d) \land \text{Small}(e)$

Sentence 3: $\text{Cube}(e) \land \neg \text{Dodec}(d)$

Sentence 4: $a = f \land \neg \text{Tet}(a)$

Sentence 5: $\text{SameSize}(a,e) \lor \text{SameSize}(b,e)$

Sentence 6: $\text{RightOf}(b,a) \land (\neg \text{Small}(b) \land \neg \text{Medium}(b))$

Sentence 7: $\text{Between}(d,e,f)$.

Option 1:

back

left

a,**f**: C, L							
d: T, S							
e: C, S			**b**: D, L				

right

front

Tarski world question 3 option 1

Option 2:

back

left

a: C, S							
f: C, S							
d: T, S							
e: C, S			**b**: D, L				

right

front

Tarski world question 3 option 2

Option 3:

back

left / right

e: C, S	**d**: T, S		**b**: D, L			**a**,**f** C, S	

front

Tarski world question 3 option 3

Option 4:

back

left / right

a,**f**: C, S							
d: T, S							
e: C, S			**b**: D, L				

front

Tarski world question 3 option 4

QUESTION 4

Given the following sentence in FOL, which of the options below is a correct English translation?

Angry(max, 2:00) $\wedge$ Angry(claire, 2:00)

Option 1: Max and Claire were both angry at 2 pm, Jan 2, 2001.

Option 2: Either Max or Claire was angry at 2 pm, Jan 2, 2001.

Option 3: Max was angry at Claire at 2 pm, Jan 2, 2001.

Option 4: Claire was angry at Max at 2 pm, Jan 2, 2001.

QUESTION 5

Given the following sentence in FOL, which of the options below is a correct English translation?

Student(claire) $\wedge$ Pet(folly) $\wedge$ Fed(claire, folly, 2:05) $\wedge$ Hungry(folly, 2:00)

Option 1: Claire is a student who owns and feeds the pet called Folly at 2 pm, Jan 2, 2001.

Option 2: Claire is a student and she fed the pet called Folly before he became hungry.

Option 3: Claire is a student who owns and feeds a pet called Folly at 2:05 pm, Jan 2, 2001.

Option 4: Claire is a student who feeds a pet called Folly at 2:05 pm, Jan 2, 2001, 5 minutes after Folly was hungry.

QUESTION 6

Given the following sentence in FOL, which of the options below is a correct English translation?

$\neg$ Student(folly) $\wedge$ Hungry(folly, 2:00)

Option 1: Folly is not a student and he was not hungry at 2 pm, Jan 2, 2001.

Option 2: Folly is not a student and was hungry at 2 pm, Jan 2, 2001.

Option 3: Either Folly is not a student or he was hungry at 2 pm, Jan 2, 2001.

Option 4: Either Folly is not a student or he was not hungry at 2 pm, Jan 2, 2001.

QUESTION 7

Given the following sentence in English, which of the options below is a correct translation in FOL?

Claire is not a student.

Option 1: $\neg$ Person(claire) $\wedge$ $\neg$ Student(claire)

Option 2: $\neg$ Student(Person(claire))

Option 3: $\neg$ Student(claire)

Option 4: $\neg$ ($\neg$ Student(claire) $\vee$ Student(claire)).

QUESTION 8

Given the following sentence in English, which of the options below is a correct translation in FOL?

Claire gave the pet, Folly, to Max at 2 pm, Jan 2, 2001.

Option 1: Person(claire) $\wedge$ Gave(folly, max, 2:00)

Option 2: Pet(folly) $\wedge$ Gave(claire, folly, max, 2:00)

Option 3: Pet(folly) $\wedge$ Person(claire) $\wedge$ Gave(folly, max, 2:00)

Option 4: Gave(claire, Pet(folly), max, 2:00).

QUESTION 9

Given the following sentence in English, which of the options below is a correct translation in FOL?

2 pm, Jan 2, 2001 is not earlier than 5 minutes later.

Option 1: $\neg$ (2:00 $<$ 2:05)

Option 2: 2:00 $<$ 2:05

Option 3: 2:00 $\neg$ $<$ 2:05

Option 4: 2:00 $>$ 2:05.

QUESTION 10

Complete the following truth table on some scrap paper and then indicate which option below reflects the last column correctly:

P	**Q**	**R**	**¬ P**	**¬P ∧ Q**	**P ∨ R**	**(¬P ∧ Q) ∧ (P ∨ R)**
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Option 1:

(¬P ∧ Q) ∧ (P ∨ R)
F
T
T
T
T
F
F
F

Option 2:

(¬P ∧ Q) ∧ (P ∨ R)
F
F
T
T
F
F
T
T

Option 3:

(¬P ∧ Q) ∧ (P ∨ R)
T
F
F
T
T
F
F
T

Option 4:

$(\neg P \land Q) \land (P \lor R)$
F
F
F
F
T
F
F
F

QUESTION 11

Which of the options below correctly states how we can determine if a given sentence of FOL built up from atomic sentences by means of truth functional connectives is a tautology?

Option 1: Build the truth table for the sentence and inspect the truth values below the main connective. If there is at least one T, the sentence is a tautology.

Option 2: Build the truth table for the sentence and inspect the truth values below the main connective. If there are more occurrences of T than F, the sentence is a tautology.

Option 3: Build the truth table for the sentence and inspect the truth values below the main connective. If all these values are F, the sentence is a tautology.

Option 4: Build the truth table for the sentence and inspect the truth values below the main connective. If all these values are T, the sentence is a tautology.

QUESTION 12

Which of the options below correctly states how we can determine if a given sentence of FOL built up from atomic sentences by means of truth functional connectives is TT- possible?

Option 1: Build the truth table for the sentence and inspect the truth values below the main connective. If there is at least one T, the sentence is TT-possible.

Option 2: Build the truth table for the sentence and inspect the truth values below the main connective. If there are more occurrences of T than F, the sentence is TT-possible.

Option 3: Build the truth table for the sentence and inspect the truth values below the main connective. If all these values are F, the sentence is TT-possible.

Option 4: Build the truth table for the sentence and inspect the truth values below the main connective. If all these values are T, the sentence is TT-possible.

The following partially constructed truth table is used in questions 13, 14 and 15. The blocks language is used for all the sentences.

	Small(b)	**Cube(a)**	**¬ Cube(a)**	**Large(b)**	…… …… …….	some sentence
	T	T	F	T		
	T	T	F	F		
	T	F	T	T		
	T	F	T	F		
	F	T	F	T		
	F	T	F	F		
	F	F	T	T		
	F	F	T	F		

QUESTION 13

Suppose we want to determine if a certain sentence in the blocks language built from the three atomic sentences **Small(b), Cube(a)** and **Large(b)** is a tautological consequence of ¬ **Cube(a)** and **Large(b).** We build the complete truth table and then inspect the last column. The sentence is a tautological consequence of the two sentences ¬ **Cube(a)** and **Large(b)** if

Option 1: only the entry in line 8 is F

Option 2: the entry in line 7 is T

Option 3: the entry in lines 3 and 7 is T

Option 4: the entry in line 1 is T and the entry in all other lines is F

QUESTION 14

Suppose we want to determine if a certain sentence in the blocks language built from the three atomic sentences **Small(b), Cube(a)** and **Large(b)** is a logical consequence of ¬ **Cube(a)** and **Large(b)**. We build the complete truth table and then inspect the last column. The sentence is a tautological consequence of the two sentences ¬ **Cube(a)** and **Large(b)** if

Option 1: only the entry in line 8 is F

Option 2: the entry in line 7 is T

Option 3: the entry in lines 3 and 7 is T

Option 4: the entry in line 1 is T and the entry in all other lines is F

QUESTION 15

Suppose we want to determine if two sentences in the blocks language, both built from the three atomic sentences **Small(b), Cube(a)** and **Large(b)**, are tautological equivalent. We build the complete truth table where the last two columns indicate the respective truth values of the two sentences. We then inspect the last two columns. The two sentences are tautological equivalent if

Option 1: at least one of the two columns consists of T only

Option 2: the entry in rows 1, 2, 3 and 4 is T in at least one of the columns

Option 3: the last two columns are identical

Option 4: either all the entries are T or all the entries are F in at least one of the columns

QUESTION 16

Consider the following argument:

Nearly all soccer players have a sense of humour.
Pete plays soccer.

Pete has a sense of humour.

Which of the following options is a proof of the validity or not of the argument?

Option 1: The argument is valid because the chances are very good that Pete has a sense of humour since most soccer players do.

Option 2: The argument is not valid and we prove nonconsequence as follows: According to the first statement not all soccer players have a sense of humour. Pete may be one of the exceptions who do not have a sense of humour. Hence the first two statements are true but the conclusion is not.

Option 3: The argument is not valid because no soccer player has a sense of humour.

Option 4: The argument is not valid because Pete does not play soccer.

QUESTION 17

Consider the following argument where the blocks language is used:

RightOf(a,b)
RightOf(c,b)

RightOf(a,c)

Which of the following options is a proof of the validity or not of the argument?

Option 1: The argument is valid because, if both **a** and **c** are to the right of **b**, then **a** has to be to the right of **c**

Option 2: The argument is not valid and we prove nonconsequence as follows: If both **a** and **c** are to the right of **b**, then **a** and **c** should be two names for the same thing and thus one cannot be to the right of the other. Hence the first two statements are true but the conclusion is not.

Option 3: The argument is not valid because it is impossible to arrange three things like this.

Option 4: The argument is not valid and we prove nonconsequence as follows: Suppose **a** is in the position directly right of **b** (in the same row) and **c** is in the position directly right of **a** (also in the same row). Then the first two statements are true but the conclusion is not.

QUESTION 18

One or more statement has been omitted from the formal proof (in the blocks language) below. Which option indicates the complete proof?

1	SameRow(a, b)	
2	c = b	
	SameRow(a, c)	

Option 1:

1	SameRow(a, b)	
2	c = b	
3	c = c	= Intro
4	b = c	= Elim 2, 3
5	SameRow(a, c)	= Elim 1, 4

Option 2:

1	SameRow(a, b)	
2	c = b	
3	b = b	= Intro
4	b = c	= Elim 2, 3
5	SameRow(a, c)	= Elim 1, 4

Option 3:

1	SameRow(a, b)	
2	c = b	
3	b = c	= Elim 2
4	SameRow(a, c)	= Elim 1, 3

Option 4:

1	SameRow(a, b)	
2	c = b	
3	SameRow(a, c)	= Elim 1, 2

QUESTION 19

Given the following sentence of FOL, which of the options below is an equivalent sentence in disjunctive normal form (DNF)?

$\neg(\neg A \lor B) \lor \neg(B \land \neg C) \lor (D \lor \neg A)$

Option 1: $(A \land \neg B) \lor (\neg B \lor C) \lor D \lor A$

Option 2: $(A \land \neg B) \lor (\neg B \lor C) \lor D \land \neg A$

Option 3: $(A \land \neg B) \lor \neg(B \land \neg C) \lor (D \lor \neg A)$

Option 4: $(A \land \neg B) \lor \neg B \lor C \lor D \lor \neg A.$

QUESTION 20

Given the following sentence of FOL, which of the options below is an equivalent sentence in conjunctive normal form (CNF)?

$(A \land \neg B) \lor (A \lor C) \land \neg (B \land C)$

Option 1: $(A \lor C) \land (\neg B \lor A \lor C) \land (\neg B \lor \neg C)$

Option 2: $(A \lor C) \land (B \lor A \lor C) \land (\neg\neg B \lor \neg C)$

Option 3: $\neg (A \lor C) \land (\neg B \lor A \lor C) \land (\neg B \lor \neg C)$

Option 4: $(\neg A \lor C) \land (\neg B \lor A \lor C) \land (\neg B \lor \neg C)$.

End of Assignment 1: Semester 2

ASSIGNMENT 02 SECOND SEMESTER

SUBMISSION: Printouts or electronically through myUnisa (as one .pdf file)

It will be to your own advantage to check whether the assignment has been registered on the system after a few days. If you want to submit the assignment electronically and myUnisa is off-line during that time, you need not contact us, because we will be aware of it. Simply submit it as soon as myUnisa is available again.	
Due date	10 September 2018
Extension	No extension
Tutorial matter	Textbook: All previous work and Chapter 5: 5.1 – 5.4 Chapter 6: 6.1 – 6.6 Chapter 7: 7.1 – 7.4 Chapter 8: 8.1, 8.2, 8.4 Chapter 9: 9.1 – 9.6 Tutorial letter 102 chapters 1 to 9
Weight of contribution to semester mark	70%
Unique number	845524

QUESTION 1 **[12]**

Consider the arguments below and decide whether they are valid. If they are, write down an informal proof, phrased in complete, well- formed English sentences. When you use proof by case or proof by contradiction, say so. You do not have to be explicit about the use of simple proof steps like conjunction elimination. If the argument is invalid, construct a counter example. In questions 1.1 and 1.3 we assume we deal with the blocks language as given on page 22 of your textbook.

Question 1.1

1 Home(max) $\vee$ Home(claire)

2 ¬ Home(max) $\vee$ Happy(carl)

3 ¬ Home(claire) $\vee$ Happy(carl)

Happy(carl)

Question 1.2

1	Sue or Carol is on holiday but Mark or John is at work.
2	Either Sue is not on holiday or Mark is not at work.
3	Either Carol is not on holiday or John is at work.
	John is at work.

Question 1.3

1	Cube(a) $\vee$ Tet(b)
2	$\neg$ Dodec(a) $\vee$ $\neg$ Cube(a)
3	$\neg$ Tet(b) $\vee$ $\neg$ Dodec(b)
	Tet(a)

QUESTION 2 **[25]**

In this question you have to construct formal proofs using the natural deduction rules. The Fitch system makes use of these rules.

A smmary of the rules of natural deduction is given on pages 573 to 578 of your textbook. Consult this when you do question 2. Remember that De Morgan's laws and other tautologies are not permissible natural deduction rules. You are also not allowed to use Taut Con, Ana Con or FO Con. It is important to number your statements, to indicate subproofs and at each step to give the rule that you are using.
Hint: If you have access to a computer, take advantage of the fact and use Fitch.

Question 2.1 **(7)**

Using the natural deduction rules, give a formal proof that the following two sentences are inconsistent:

1 $C \rightarrow (A \vee B)$

2 $C \wedge \neg A \wedge \neg B$

Question 2.2 **(9)**

Using the natural deduction rules, give a formal proof of

$A \rightarrow \neg F$

from the three premises

1 $A \rightarrow B$

2 $(B \vee D) \rightarrow E$

3 $F \rightarrow \neg E$

Question 2.3 **(9)**

Using the natural deduction rules, give a formal proof of

$E \rightarrow (C \wedge D)$

from the three premises

1 $A \wedge (B \vee C)$

2 $A \rightarrow D$

3 $B \rightarrow \neg E$

English	FOL
Names	
Richard	richard
Vince	vince
Felicity	felicity
Red Arrows	red
Blue Bucks	blue
Predicates	
x is a soccer player	Player(x)
x is a teacher	Teacher(x)
x is a team	Team(x)
x plays for y	Play(x, y)
x looses against y	Loose(x, y)
x gets injured	Injured(x)

Table 2 used in questions 3 and 4

QUESTION 3 [45]

In this question you have to translate English sentences into sentences of First Order Logic, using the predicates and names given in Table 1.

Question 3.1

If Richard plays for Red Arrows, he gets injured.

Question 3.2

Vince gets injured only if he plays for Red Arrows.

Question 3.3

Unless Felicity is a teacher, Vince and Richard will both play for Red Arrows.

Question 3.4

Blue Bucks do not loose against Red Arrows.

Question 3.5

If we assume that Vince plays for Blue Bucks, Felicity also plays for Blue Bucks but Richard plays for Red Arrows.

Question 3.6

Red Arrows loose against Blue Bucks, or Blue Bucks loose against Red Arrows.

Question 3.7

Richard plays for Blue Bucks if and only if Felicity and Vince are both teachers or neither of them is a teacher.

Question 3.8

Richard is either a teacher or a player but not both.

Question 3.9

If Felicity and Vince get injured while they play for Blue Bucks, Richard will play for Red Arrows.

Question 3.10

Felicity is neither a teacher nor a soccer player.

Question 3.11

No teachers are soccer players.

Question 3.12

Some teachers do not play soccer.

Question 3.13

Someone is a soccer player and a teacher.

Question 3.14

Everyone gets injured when Vince plays for Blue Bucks.

Question 3.15

Someone gets injured if Red Arrows loose against Blue Bucks.

QUESTION 4 **[18]**

In this question you have to translate sentences of First Order Logic into English sentences, using the predicates and names given in Table 1.

Question 4.1

$\neg$ Player(felicity)

Question 4.2

Play(vince, red) $\rightarrow$ Injured(richard)

Question 4.3

(Injured(vince) $\wedge$ Loose(red, blue)) $\leftrightarrow$ (Play(richard, blue) $\vee$ Play(felicity, blue))

Question 4.4

$\exists$x (Team(x) $\wedge$ Loose(blue, x))

Question 4.5

Play(vince, red) $\rightarrow$ $\exists$x (Player(x) $\wedge$ Injured(x))

Question 4.6

$\forall$x ((Team(x) $\wedge$ Play(felicity, x)) $\rightarrow$ Loose(x, red))

End of Assignment 2: Semester 2

ASSIGNMENT 03 SECOND SEMESTER

Submission: The assignment should NOT be submitted.

You should assess it yourself when you have completed it. The model solution is given at the end of this tutorial letter. ***Note that there will be examination questions set on this part of the study material also.***

Due date	15 October 2018
Extension	Not applicable
Tutorial matter	Textbook: All previous material and Chapter 10: 10.1 – 10.4 Chapter 11: 11.1 – 11.5, 11.7, 11.8 Chapter 12: 12.1 – 12.4 Chapter 13: 13.1, 13.2, 13.3, 13.5 Chapter 14: 14.1 Tutorial letter 102 Chapters 1 to 14
Weight of contribution to semester mark	0%
No unique number	

QUESTION 1

Use De Morgan's laws and show that

$$\neg (\exists x \text{ Brillig}(x) \land \forall y \text{ Gimble}(y))$$

is logically equivalent to

$$\forall x \exists y (\neg \text{ Brillig}(x) \lor \neg \text{ Gimble}(y))$$

QUESTION 2

Below a Tarski world is given followed by eleven sentences. Eight of the sentences are true in the given world. Which three sentences are false in the given world? If you have access to a computer, use the Tarski software to check your answers.

back

left / right

a: C, S							
d: T, S							
e: C, S			**b**: D, L			**c**: T, M	

front

Tarski world: Question 2

Sentences:

1 $\exists x\ \text{Between}(x, e, a) \land \exists x\ \text{Between}(x, e, c)$

2 $\forall x\ (\text{Cube}(x) \rightarrow \exists y\ \exists z\ (\text{Dodec}(y) \land \text{Tet}(z) \land \text{RightOf}(y, x) \land \text{LeftOf}(x, z)))$

3 $\neg \forall x\ (\text{Cube}(x) \rightarrow \text{Small}(x))$

4 $\forall x\ (\text{Small}(x) \lor \text{Medium}(x) \lor \text{Dodec}(x))$

5 $\forall x\ (\text{Dodec}(x) \land \text{Large}(x))$

6 $\exists y\ \forall x\ (\text{Medium}(y) \rightarrow (\text{Tet}(y) \land (\text{Cube}(x) \rightarrow \text{Larger}(y, x))))$

7 $\exists y\ ((\text{Dodec}(y) \land \text{Small}(y)) \lor (\text{Tet}(y) \land \text{Medium}(y)))$

8 $\exists x\ \exists y\ (\text{Cube}(x) \land \text{Tet}(y) \land \neg \text{SameSize}(x, y))$

9 $\forall x\ \exists y\ \text{SameCol}(x, y)$

10 $\exists y\ \forall x\ \text{SameCol}(x, y)$

11 $\text{Cube}(d) \rightarrow \forall x\ \forall y\ \text{SameShape}(x, y)$

English	FOL
Names	
Abe	abe
Beauty	beauty
Carol	Carol
Predicates	
x is a typist	Typist(x)
x is a farmer	Farmer(x)
x is the sister of y	Sister(x, y)
x and y are cousins	Cousin(x, y)
x is in the library	Library(x)

Table 2 used in questions 3 and 4

QUESTION 3

Translate the following FOL sentences into English sentences using the predicates and names given in Table 2 above.

Question 3.1

$\forall x$ (Farmer(x) $\rightarrow$ $\exists y$ (Sister(y, x) $\vee$ Cousin(y, x)))

Question 3.2

$\exists x$ (Typist(x) $\wedge$ Cousin(x, abe) $\wedge$ Sister(x, carol))

Question 3.3

Typist(abe) $\rightarrow$ $\forall x$ (Farmer(x) $\rightarrow$ Library(x))

Question 3.4

$\forall x$ ((Typist(x) $\vee$ Farmer(x)) $\wedge$ (Cousin(x, beauty) $\vee$ Cousin(x, carol)))

Question 3.5

$\exists x \, \forall y \, (\neg \text{Cousin}(x, y))$

QUESTION 4

Translate the following English sentences into FOL sentences using the predicates and names given in the table above.

Question 4.1

Abe has a cousin who is a farmer.

Question 4.2

Everybody has a sister who is a typist in the library.

Question 4.3

All farmers have sisters.

Question 4.4

If a farmer is in the library, he has a cousin who is a typist.

Question 4.5

Both Carol and Beauty have cousins but Abe does not have a cousin.

QUESTION 5

One of the arguments below is valid and the other not. If the argument is valid, give an *informal* proof. If the argument is not valid, construct a Tarski world which is a counterexample.

Question 5.1

$\forall y \, (\text{Cube}(y) \lor \text{Dodec}(y))$

$\forall y \, (\text{Cube}(y) \rightarrow \text{Large}(y))$

$\exists y \, \neg \text{Large}(y)$

$\exists y \, \text{Dodec}(y)$

Question 5.2

| $\forall y\ (Cube(y) \lor Dodec(y))$
| $\forall y\ (\neg Small(y) \to Tet(y))$
|—
| $\neg \exists y\ Small(y)$

QUESTION 6

In this question you have to construct ***formal proofs*** using the natural deduction rules. The Fitch system makes use of these rules.

A summary of the rules of natural deduction is given on pages 557 to 560 of your textbook. Consult this when you do this question. Remember that De Morgan's laws and other tautologies are not permissible natural deduction rules. You are also not allowed to use Taut Con, Ana Con or FO Con. It is important to number your statements, to indicate subproofs and at each step to give the rule that you are using.

Hint: If you have access to a computer, take advantage of the fact and use Fitch.

Question 6.1

Using the natural deduction rules, give a formal proof of

$\exists x\ (Large(x) \land LeftOf(x, b))$

from the premises

$\forall x\ (Cube(x) \to Large(x))$

$\forall y\ (Large(y) \to LeftOf(y, b))$

$\exists x\ Cube(x)$

Question 6.2

Using the natural deduction rules, give a formal proof of the following argument:

| $\forall x\ [\ (AtHome(x) \lor InLibrary(x)) \to (Happy(x) \land Reading(x))\ ]$
| $\forall y\ [\ (Smiling(y) \lor Happy(y)) \to InLibrary(y)\ ]$
| $\exists x\ Smiling(x)$
|—
| $\exists x\ (\ Smiling(x) \land Happy(x)\ )$

Question 6.3

Using the natural deduction rules, give a formal proof of

$\forall x[P(x) \rightarrow Q(x)] \rightarrow [\forall x P(x) \rightarrow \forall x Q(x)]$

from no premises.

Question 6.4

Using the natural deduction rules, give a formal proof of

$\exists x \text{Dodec}(x)$

from the premises

$\forall y[\text{Cube}(y) \vee \text{Dodec}(y)]$

$\forall x[\text{Cube}(x) \rightarrow \text{Large}(x)]$

$\exists x \neg \text{Large}(x)$

QUESTION 7

Translate the following English sentence into an FOL sentence:

There are at most three cubes.

END OF QUESTIONS FOR ASSIGNMENTS 1, 2 and 3 OF BOTH SEMESTERS

8.7 Other assessment methods

There are no other assessment methods for this module.

8.8 The examination

Use your *Study @ Unisa* brochure for general examination guidelines and examination preparation guidelines.

Make a note of your examination dates and arrange with your employer for leave in good time. The COS2661 examination will be in May or June if you are registered for the first semester and in October or November if you are registered for the second semester. Check for clashes on the examination timetable and should there be any between your modules, discuss them with the Student Administration department.

To gain admission to the examination, you have to submit (the multiple choice) Assignment 01:

- ***before or on 12 March*** if you are registered for the first semester, and
- ***before or on 3 September*** if you are registered for the second semester.

A detailed exam tutorial letter will follow six weeks before you write your exam.

9 FREQUENTLY ASKED QUESTIONS

The *Study @ Unisa* brochure contains an A-Z guide of the most relevant study information.

10 SOURCES CONSULTED

- Prescribed book
- COS2661 Study guide
- Mathematical background tutorial letter.

11 IN CLOSING

In your studies this year you will acquire skills that you will apply in further studies. If you study hard and purposefully, you are on your way to success. We hope that you will enjoy your studies at Unisa. Everything of the best with your studies this year!

12 ADDENDUM

12.1 Solution to Assignment 03

Below is the solution to Assignment 03.

QUESTION 1

$\neg(\exists x\ Brillig(x) \wedge \forall y\ Gimble(y)) \equiv (\neg \exists x\ Brillig(x)) \vee (\neg \forall y\ Gimble(y))$ page 83

$\equiv (\forall x \neg Brillig(x)) \vee (\exists y \neg Gimble(y))$ page 279

$\equiv \forall x (\neg Brillig(x)) \vee \exists y (\neg Gimble(y))$

$\equiv \forall x \exists y (\neg Brillig(x) \vee \neg Gimble(y))$

QUESTION 2

Sentences 3, 5 and 10 are false and all the other sentences are true in the given Tarski world.

Sentence 3: This sentence is false, because in all cases, if a block is a cube then it is small.

Sentence 5: This sentence is false, because although the only dodecahedron is large, *not* all blocks are dodecahedrons.

Note: *Sentence 9* is true, because it is true that for all blocks there is a block in the same column (even if it is the same block).

Sentence 10: This sentence is false, because it is not true that there exists a block that is in the same column as all the other blocks.

Sentence 11: This sentence is true because, if the left-hand side of a sentence A → B is false, the sentence is true. In this case Cube(d) is false, therefore the sentence is true regardless whether the right hand side is true or not.

QUESTION 3

Question 3.1

Every farmer has either a sister or a cousin.

Question 3.2

There is a typist who is Abe's cousin and Carol's sister.

Question 3.3

If Abe is a typist, all farmers are in the library.

Question 3.4

Everybody is either a typist or a farmer and is a cousin of either Beauty or Carol.

In order to translate the given FOL sentence correctly, you have to take note of the brackets.

Question 3.5

Somebody is no one's cousin.

QUESTION 4

You should note that the material implication symbol ($\rightarrow$) is normally used when dealing with "for all", $\forall$x, and that the conjunction symbol ($\wedge$) is normally used when dealing with "there exists", $\exists$. This also means that you should think very carefully when both $\forall$ and $\exists$ are used.

Question 4.1

$\exists$x (Farmer(x) $\wedge$ Cousin(x, abe))

Question 4.2

$\forall$x $\exists$y (Sister(y, x) $\wedge$ Typist(y) $\wedge$ Library(y))

Question 4.3

$\forall$x $\exists$y (Farmer(x) $\rightarrow$ Sister(y, x))

Question 4.4

$\forall$x ((Farmer(x) $\wedge$ Library(x)) $\rightarrow$ $\exists$y (Cousin(x, y) $\wedge$ Typist(y)))

Question 4.5

$\exists$x Cousin(x, carol) $\wedge$ $\exists$x Cousin(x, beauty) $\wedge$ $\neg$ $\exists$x Cousin(x, abe)

QUESTION 5

Question 5.1

This argument is valid. We show it as follows:

Suppose there is not a dodecahedron. Then, according to the first premise, everything is a cube. Then, according to the second premise, everything is large. But the third premise states that there is something that is not large. Thus we have a contradiction. This means that our assumption is not correct. Therefore we retract our assumption (that there is no dodecahedron) and conclude that there is actually a dodecahedron.

Question 5.2

This argument is not valid. In the Tarski world below both premises are true but the conclusion (that there is no small thing) is not true. There actually are small things in the constructed Tarski world. This is what a counterexample is, namely a *specific* situation (or world) where the premises are true but the conclusion is false. There are many counterexamples – yours could be quite different from ours.

back

	c: D, S								
left									right
	a: D, S								
								d: C, S	
	b: D, S				**e**: C, S				

front

Tarski world: Question 5.2

QUESTION 6

Question 6.1

Many students have difficulty with the application of the ∃ elimination rule. In many cases when the premise starts with ∃x, i.e. when the premise is ∃x ϕ, we need to use the ∃ elimination rule. That means that we need to introduce a new name and then assume ϕ with x substituted by the new name. Below this is done in line 4. The ∃ elimination rule is cited in line 11, outside the subproof containing the new name a. The ∃ Intro rule, on the other hand, is cited inside the subproof – see line 10.

1	∀x (Cube(x) → Large(x))	
2	∀y (Large(y) → LeftOf(y, b))	
3	∃x Cube(x)	
4	[a] Cube(a)	
5	Cube(a) → Large(a)	∀ Elim: 1
6	Large(a)	→ Elim: 4, 5
7	Large(a) → LeftOf(a, b)	∀ Elim: 2
8	LeftOf(a, b)	→ Elim: 6, 7
9	Large(a) ∧ LeftOf(a, b)	∧ Intro: 6, 8
10	∃x (Large(x) ∧ LeftOf(x, b))	∃ Intro: 9
11	∃x (Large(x) ∧ LeftOf(x, b))	∃ Elim: 3, 4 - 10

Question 6.2

This question again involves both quantifiers. Pay special attention to the positions where the rules are cited: both applications of the∀ Elim rule (lines 5 and 6) as well as the application of the ∃ Intro rule (line 13) are *inside* the subproof starting with the choice of a, while the ∃ Elim rule is cited *outside* the subproof (line 14).

1	$\forall x\ [\ (AtHome(x)\ \lor\ InLibrary(x))\ \to\ (Happy(x)\ \land\ Reading(x))]$		
2	$\forall y\ [\ (Smiling(y)\ \lor\ Happy(y))\ \to\ InLibrary(y)]$		
3	$\exists x\ Smiling(x)$		
4	a	Smiling(a)	
5	$(AtHome(a)\ \lor\ InLibrary(a))\ \to\ (Happy(a)\ \land\ Reading(a))$		∀ Elim: 1
6	$(Smiling(a)\ \lor\ Happy(a))\ \to\ InLibrary(a)$		∀ Elim: 2
7	$Smiling(a)\ \lor\ Happy(a)$		∨ Intro: 4
8	InLibrary(a)		→ Elim: 6, 7
9	$AtHome(a)\ \lor\ InLibrary(a)$		∨ Intro: 8
10	$Happy(a)\ \land\ Reading(a)$		→ Elim: 5, 9
11	Happy(a)		∧ Elim: 10
12	$Smiling(a)\ \land\ Happy(a)$		∧ Intro: 4, 11
13	$\exists x\ (\ Smiling(x)\ \land\ Happy(x)\ )$		∃ Intro: 12
14	$\exists x\ (\ Smiling(x)\ \land\ Happy(x)\ )$		∃ Elim: 3, 4 - 13

Question 6.3

The goal involves the material conditional symbol →. In such cases we need a subproof starting with the assumption of the formula on the left hand side of the conditional symbol → and ending with the formula on the right hand side of the symbol →. The question is made more complicated by the fact that there are actually three → symbols in the goal. The formula on the left hand side of the main connective of the goal is $\forall x\ (P(x)\ \to\ Q(x))$ and that is the reason for its assumption in line 2. This subproof ends in line 9 with the formula on the right hand side of the main connective of the goal, namely $\forall x\ P(x)\ \to\ \forall x\ Q(x)$. Inside this subproof we need a sub-subproof starting with the assumption of $\forall x\ P(x)$ in line 3 and ending on $\forall x\ Q(x)$ in line 8.

We actually need a sub-sub-subproof (lines 4 to 7) because we need to introduce a new name. You should further note the application of the ∀rules: Both applications of the ∀ Elim rule (lines 5 and 6) are *inside* the subproof containing the new name b, while the application of the ∀ Intro rule (line 8) is *outside* this subproof.

1		
2	$\forall x\,(P(x) \rightarrow Q(x))$	
3	$\forall x\, P(x)$	
4	b	
5	$P(b)$	$\forall$ Elim 3
6	$P(b) \rightarrow Q(b)$	$\forall$ Elim 2
7	$Q(b)$	$\rightarrow$ Elim 5, 6
8	$\forall x\, Q(x)$	$\forall$ Intro 4 - 7
9	$\forall x\, P(x) \rightarrow \forall x\, Q(x)$	$\rightarrow$ Intro 3 - 8
10	$\forall x\,(P(x) \rightarrow Q(x)) \rightarrow (\forall x\, P(x) \rightarrow \forall x\, Q(x))$	$\rightarrow$ Intro 2 - 9

Question 6.4

When you work through this proof, you should keep in mind our comments at questions 6.1, 6.2 and 6.3 above regarding the application of the elimination and introduction rules of ∀ and ∃.

Also, note how the ∨ elimination rule is applied:

- there are two subproofs (lines 7 to 10 and lines 11 to 12),
- each starting with one of the disjuncts of the sentence in line 5 (namely Cube(a) ∨ Dodec(a)) and
- ending with the same sentence (Dodec(a) in this case) and then
- this same sentence (namely Dodec(a)) is deduced by using the ∨ Elim rule outside these subproofs (in line 13).

1	$\forall y\, (Cube(y) \lor Dodec(y))$	
2	$\forall x\, (Cube(x) \rightarrow Large(x))$	
3	$\exists x\, \neg Large(x)$	
4	[a] $\neg Large(a)$	
5	$Cube(a) \lor Dodec(a)$	$\forall$ Elim 1
6	$Cube(a) \rightarrow Large(a)$	$\forall$ Elim 2
7	$Cube(a)$	
8	$Large(a)$	$\rightarrow$ Elim 7, 6
9	$\bot$	$\bot$ Intro 4, 8
10	$Dodec(a)$	$\bot$ Elim 9
11	$Dodec(a)$	
12	$Dodec(a)$	Reit 11
13	$Dodec(a)$	$\lor$ Elim 5, 7 - 10, 11 - 12
14	$\exists x\, Dodec(x)$	$\exists$ Intro 13
15	$\exists x\, Dodec(x)$	$\exists$ Elim 3, 4 - 14

QUESTION 7

Refer to the last sections of Tutorial letter 102 regarding this question.

$$\forall x\, \forall y\, \forall z\, \forall u\, ((Cube(x) \land Cube(y) \land Cube(z) \land Cube(u)) \rightarrow ((x = y) \lor (x = z) \lor (x = u) \lor (y = z) \lor (y = u) \lor (z = u)))$$