UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

COS2661

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FORMAL LOGIC II

Duration 2 Hours

100 Marks

EXAMINERS

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SECOND MRS S VALLABHAPURAPU

Closed book examination

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This paper consists of 12 pages

INSTRUCTIONS:

1 This paper consists of two sections, Section A, 24 marks and Section B, 76 marks The total mark is 100

2 Answer all 12 questions of Section A and ALL questions of Section B in your answer book

3 Do all rough work in the answer book

4 Number your answers and label your rough work clearly

5 The mark for every question appears in brackets next to the question

6 The blocks first-order language (blocks FOL) is used in many questions

ALL THE BEST!

[TURN OVER]

All references to Tarski's World are as described in the prescribed book: Language, Proof and Logic

SECTION A

Question 1 **24 marks**

This section, consisting of twelve multiple choice questions, should be answered in your examination book (NOT on a multiple choice sheet). In each case, simply write down the number of the question followed by the number of the chosen option, for example, if you choose option 4 for question 1(i) you should write 1(i) 4.

TARSKI WORLD FOR QUESTIONS i AND ii

The Tarski world given below is used in questions i and ii of Section A The six blocks in the world are indicated as explained below the table An example of an entry is (**a** T, S) which indicates that the block in that location is called **a**, it is a tetrahedron, and it is small

			(**c** T,M)				
		(**b** D, S)			(**d** C, S)		
	(**a** C, L)						
						(**e** T, M)	(**f** T, S)

Shape indicated as follows
C: cube
T tetrahedron
D dodecahedron

Size indicated as follows
L large
M medium
S small

QUESTION 1(i) [2]

Consider the following two sentences in the blocks language The two sentences are followed by five options **Write down the number only of the correct option.**
Sentences

1 1 $\forall x$ (Cube(x) $\rightarrow$ $\exists y$ Smaller(x, y))
1 2 $\exists y \exists x$ (SameShape(x, y) $\wedge$ SameSize(x, y) $\wedge$ ($x \neq y$))

Options

1 Sentence 1 1 and sentence 1 2 are true in the Tarski world given above
2 Sentence 1 1 is true and sentence 1 2 is false in the Tarski world given above
3 Sentence 1 1 is false and sentence 1 2 is true in the Tarski world given above
4 Sentence 1 1 and sentence 1 2 are false in the Tarski world given above

QUESTION 1(ii) [2]

Consider the following two sentences in the blocks language The two sentences are followed by four options **Write down the number only of the correct option**

Sentences

2 1 $\exists x$ Cube(x) $\vee$ $\exists x$ Dodec(x)
2 2 $\forall x$ (Smaller(x, b) $\rightarrow$ Cube(x))

Options

1 Sentence 2 1 and sentence 2 2 are true in the Tarski world given above
2 Sentence 2 1 is true and sentence 2 2 is false in the Tarski world given above
3 Sentence 2 1 is false and sentence 2 2 is true in the Tarski world given above
4 Sentence 2 1 and sentence 2 2 are false in the Tarski world given above

QUESTION 1(iii) [2]

Consider the FOL sentence below and then write down the number of the option indicating a correct English translation

FOL sentence

$$\text{Smaller}(a, b) \rightarrow (\text{Cube}(a) \vee \neg \text{Cube}(b))$$

Options

1 Block **a** will be smaller than **b** if **a** is a cube or **b** is not a cube
2 Block **a** will only be smaller than **b** if **a** is a cube or **b** is not a cube
3 If **a** is smaller than **b**, one of them will be a cube
4 If **a** is smaller than **b**, **a** will be a cube

QUESTION 1(iv) [2]

Consider the FOL sentence below and then write down the number of the option indicating a correct English translation

FOL sentence

$$\forall x\,(\text{Tet}(x) \rightarrow \forall y\,(\text{Smaller}(y, x) \rightarrow \text{LeftOf}(y, x)))$$

Options

1 All blocks are tetrahedrons and they are arranged from small to large, left to right
2 All small blocks lie to the left of all medium and large blocks
3 All tetrahedrons lie in the rightmost column of the relevant Tarski world
4 Every tetrahedron lies to the right of all smaller blocks

QUESTION 1(v) [2]

Consider the English sentence below and then write down the number of the option indicating a correct FOL translation

English sentence

Block **a** is in the same column as blocks **b** and **c** but not in the same row as blocks **d** and **e**

Options

1 $\text{SameCol(a, b)} \vee \text{SameCol(a, c)} \wedge \neg (\text{SameRow(a, d)} \vee \text{SameRow(a, e)})$
2 $\text{SameCol(a, b)} \vee \text{SameCol(a, c)} \vee \neg \text{SameRow(a, d)} \vee \neg \text{SameRow(a, e)}$
3 $\text{SameCol(a, b)} \wedge \text{SameCol(a, c)} \wedge \neg \text{SameRow(a, d)} \wedge \neg \text{SameRow(a, e)}$
4 $\text{SameCol(a, b, c)} \wedge \neg \text{SameRow(a, d, e)}$

QUESTION 1(vi) [2]

Consider the English sentence below and then write down the number of the option indicating a correct FOL translation

English sentence

There is a cube or a dodecahedron that is larger than all other blocks

Options

1 $\forall x ((\text{Cube}(x) \vee \text{Dodex}(x)) \rightarrow \forall y (y \neq x \rightarrow \text{Larger}(x, y)))$
2 $\exists x ((\text{Cube}(x) \vee \text{Dodex}(x)) \wedge \forall y \text{ Larger}(x, y))$
3 $\exists x (\text{Cube}(x) \wedge \forall y \text{ Larger}(x, y)) \vee \exists x (\text{Dodec}(x) \wedge \forall y (\text{Larger}(x, y))$
4 $\exists x (\text{Cube}(x) \wedge \forall y (y \neq x \rightarrow \text{Larger}(x, y))) \vee \exists x (\text{Dodec}(x) \wedge \forall y (y \neq x \rightarrow \text{Larger}(x, y)))$

QUESTION 1(vii) [2]

Consider the FOL sentence below and write down the number of the option giving a correct Conjunctive Normal Form (CNF) of the sentence (Do not guess but derive the CNF by rough work)

$$\neg A \wedge \neg (A \vee \neg B) \wedge (\neg C \vee (D \wedge A))$$

Options

1 $\neg A \wedge \neg (A \vee \neg B) \wedge (\neg C \vee D) \wedge (\neg C \vee A)$
2 $\neg A \wedge B \wedge (\neg C \vee D) \wedge (\neg C \vee A)$
3 $\neg A \wedge B \wedge (\neg C \vee D \vee A)$
4 $(\neg A \vee \neg B) \wedge (\neg C \vee D) \wedge (\neg C \vee A)$

QUESTION 1(viii) [2]

Given the following three sentences in the blocks language and the joint truth table below, write down the number of the option giving the correct interpretation of the truth table

P1 $\neg$ Dodec(a) $\vee$ Small(a)
P2 $\neg$ Dodec(a) $\vee$ Large(a)
P3 (Small(a) $\wedge$ Large(a)) $\rightarrow$ $\neg$ Dodec(a)

$\neg$ Dodec(a)	Small(a)	Large(a)	Small(a) $\wedge$ Large(a)	$\neg$ Dodec(a) $\vee$ Small(a)	$\neg$ Dodec(a) $\vee$ Large(a)	(Small(a) $\wedge$ Large(a)) $\rightarrow$ $\neg$ Dodec(a)
T	T	T	T	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	F
F	T	F	F	T	T	T
F	F	T	F	F	T	T
F	F	F	F	F	F	T

Options

1 The third sentence is neither a tautological consequence nor a logical consequence of the first and second sentences
2 The third sentence is a tautological consequence and a logical consequence of the first and second sentences
3 The third sentence is a tautological consequence but not a logical consequence of the first and second sentences
4 The third sentence is a logical consequence but not tautological consequence of the first and second sentences

QUESTION 1(ix) [2]

Given the following three sentences in the blocks language and the joint truth table below, write down the number of the option giving the correct interpretation of the truth table

P1 $\neg$ Large(a) $\wedge$ Small(a)
P2 Tet(b)
P3 $\neg$ Tet(b) $\vee$ $\neg$ Large(a)

¬ Large(a)	Small(a)	Tet(b)	¬ Tet(b)	¬ Large(a) ∧ Small(a)	¬ Tet(b) ∨ ¬ Large(a)
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	F	T
F	F	T	F	F	F
F	F	F	T	F	T

Options

1 The third sentence is neither a tautological consequence nor a logical consequence of the first and second sentences
2 The third sentence is a tautological consequence and a logical consequence of the first and second sentences
3 The third sentence is a tautological consequence but not a logical consequence of the first and second sentences
4 The third sentence is a logical consequence but not tautological consequence of the first and second sentences

QUESTION 1(x) [2]

Construct a truth table for the following FOL sentence (as rough work) and then write down the number of the option giving the correct interpretation of the truth table

$$(A \rightarrow (B \wedge C)) \leftrightarrow ((\neg B \vee \neg C) \rightarrow \neg A)$$

Options

1 The sentence is a contradiction
2 The sentence is a tautology
3 The sentence is logically equivalent to $(A \vee (B \wedge \neg C))$
4 The sentence is not well-formed

QUESTION 1(xi) [2]

Consider the argument below and then write down the number of the option giving the correct evaluation of the argument

It is against the law to drive a car if you are younger than 16 years
Richard drives his mother's car
Richard is 14 years old
∴
Richard breaks the law

Options

1 The argument is both sound and valid
2 The argument is sound but not valid
3 The argument is valid but not sound
4 The argument is neither sound nor valid

QUESTION 1(xii) [2]

Consider the argument below and then write down the number of the option giving the correct evaluation of the argument

All coffee shop owners make lots of money
Pete is a waiter in a coffee shop
Jack is a poor student
∴
Pete and Jack do not have much money

Options

1 The argument is both sound and valid
2 The argument is sound but not valid
3 The argument is valid but not sound
4 The argument is neither sound nor valid

SECTION B
76 marks

Table 1 below lists the names and predicates that should be used in questions 2 and 3 of Section B

English	FOL	Comment
Names		
Tom	tom	The name of a boy
Pamela	pam	The name of a girl
Cape Town	cape	The name of a city
Predicates		
x and y are friends	Friends(x, y)	
x is a city	City(x)	
x drives in y	Drives(x, y)	
x studies for y hours	Studies(x, y)	
x is more than y	MoreThan(x, y)	

Table 1

QUESTION 2 **[12]**

Question 2(a) **(6)**

Translate the following English sentence into a first-order logic (FOL) sentence, using the names and predicates given in Table 1

Whenever Pamela and Tom both study, she studies for more than 2 hours and he studies for not more than 1 hour

Question 2(b) **(6)**

Translate the following English sentence into a first-order logic (FOL) sentence, using the names and predicates given in Table 1

Some friend of Pamela, but not of Tom, does not study but drives around in Cape Town

QUESTION 3 **[12]**

Question 3(a) **(6)**

Translate the following first-order logic (FOL) sentence into an English sentence, using the names and predicates given in Table 1

$\forall x$ (Studies(tom, x) $\rightarrow$ $\forall y$ (Friends(tom, y) $\rightarrow$ $\forall z \neg$ Studies(y, z)))

Question 3(b) **(6)**

Translate the following first-order logic (FOL) sentence into an English sentence, using the names and predicates given in Table 1

$\exists x$ (City(x) $\wedge$ Drives(pam, x)) $\wedge \neg$ Drives(pam, cape)

QUESTION 4 **[16]**

Question 4(a) **(8)**

Consider the following argument and then answer the questions given below

Mushrooms may be poisonous
Fred went to the *Silver Lining Restaurant* for dinner
Fred had mushroom soup
Fred was extremely ill after dinner
$\vdash$
Fred ate poisonous mushrooms

(i) Is the argument valid? Explain your answer

(ii) Is the argument sound? Explain your answer

Question 4(b) **(8)**

Consider the following argument and then answer the questions given below

Anyone who walks more than one km per day will not get fat
Anyone who eats at least one apple per day has a lovely skin
Patricia walks 3 km and eats two apples per day
∴
Patricia will not get fat and has a lovely skin

(i) Is the argument valid? Explain your answer

(ii) Is the argument sound? Explain your answer

QUESTION 5 **[8]**

Show that the following argument is valid by giving an ***informal proof***, phrased in complete, well-formed English sentences Use proof by cases and/or proof by contradiction

Beauty and Tim are married and live in a flat
At 18 00 Beauty is always either at the gym or in the kitchen of the flat
Tim is always either at work or with Beauty
It is 18 00
Beauty is not at the gym
The building where Tim works is locked up for the night
∴
Tim is in the kitchen

QUESTION 6 **[22]**

Give ***formal proofs*** of the arguments below using the rules of natural deduction You may ***not*** use Taut Con or FO Con or Ana Con It is important to number your statements, to indicate subproofs and at each step to give the rule that you are using

Question 6(a) **(10)**

$$\begin{array}{|l} \neg D \\ A \rightarrow B \\ B \rightarrow (C \vee D) \\ A \vee C \\ \hline\hline C \end{array}$$

Question 6(b) **(8)**

$$\begin{array}{|l} \forall x\ \text{Student}(x) \vee \forall x\ \text{Hungry}(x) \\ \hline\hline \forall x\ (\text{Student}(x) \vee \text{Hungry}(x)) \end{array}$$

Question 6(c) **(4)**

Prove that the following two premises are contradictory

$\forall x$ Pretty(x)
$\exists x \neg$ Pretty(x)

QUESTION 7 **[6]**

Draw a Tarski world serving as counterexample to show that no proof is possible for the argument below

$$\begin{array}{|l} \forall x\ \exists y\ (\text{Small}(x) \rightarrow \text{LeftOf}(x, y)) \\ \hline\hline \forall x\ \forall y\ (\text{LeftOf}(x, y) \rightarrow \neg\ \text{Small}(y)) \end{array}$$

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