UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

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FORMAL LOGIC II

Duration 2 Hours

100 Marks

EXAMINERS .
FIRST MRS B CHIMBO
SECOND MRS S VALLABHAPURAPU

Closed book examination.

This paper consists of 12 pages

INSTRUCTIONS:

1 This paper consists of two sections, Section A, 24 marks and Section B, 76 marks The total mark is 100
2 Answer all twelve questions of Section A and questions 2 to 8 of Section B in your answer book
3 Do all rough work in the answer book
4 Number your answers and label your rough work clearly
5 The mark for every question appears in brackets next to the question

ALL THE BEST!

All references to Tarski's World are as described in the prescribed book: Language, Proof and Logic

SECTION A: QUESTION 1
24 marks

This section, consisting of twelve multiple choice questions, should be answered in your examination book (NOT on a multiple choice sheet). In each case, simply write down the number of the question followed by the number of the chosen option, for example if you choose option 4 for question (i) you could write i. 4.

TARSKI WORLD FOR QUESTIONS i, ii AND iii

The Tarski's world given below is used in questions i, ii and iii of Section A The six blocks in the world are indicated as explained below the table An example of an entry is (**a** T, S) which indicates that the block in that location is called **a**, it is a tetrahedron, and it is small

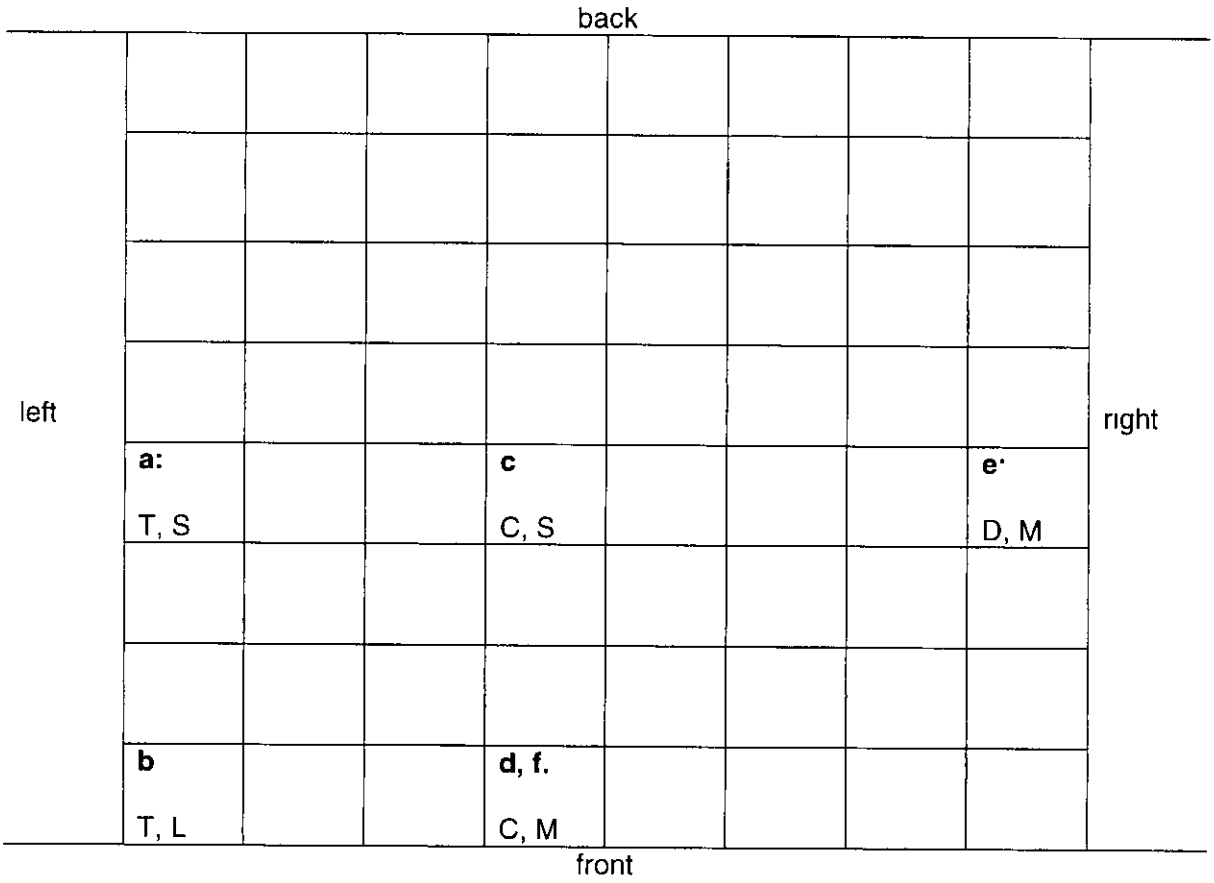

Figure 1: Tarski's World

Shape indicated as follows
C cube
T tetrahedron
D dodecahedron

Size indicated as follows
L large
M medium
S small

Question (i) [2]

Consider the following two sentences in the blocks language The two sentences are followed by four options **Write down the number only of the correct option.**

Sentences

1 1 Same Shape(d, f)

1 2 Cube(b) → ∀x∀ySameShape(x, y)

Options

1 Sentence 1 1 and sentence 1 2 are true in the Tarski world given above
2 Sentence 1 1 is true and sentence 1 2 is false in the Tarski world given above
3 Sentence 1 1 is false and sentence 1 2 is true in the Tarski world given above
4 Sentence 1 1 and sentence 1 2 are false in the Tarski world given above

Question (ii) [2]

Consider the following two sentences in the blocks language The two sentences are followed by four options **Write down the number only of the correct option.**

Sentences

2 1 ∀x (Dodec(x) ∧ Small(x))

2 2 (c ≠ a) ∨ ¬ (e ≠ b)

Options

1 Sentence 2 1 and sentence 2 2 are true in the Tarski world given above
2 Sentence 2 1 is true and sentence 2 2 is false in the Tarski world given above
3 Sentence 2 1 is false and sentence 2 2 is true in the Tarski world given above
4 Sentence 2 1 and sentence 2 2 are false in the Tarski world given above

Question (iii) [2]

Consider the following two sentences in the blocks language The two sentences are followed by four options **Write down the number only of the correct option.**

Sentences

3 1 SameRow(a,c) $\vee$ SameShape(d,b)

3 2 RightOf(d,c) $\wedge$ FrontOf(d,e)

Options

1 Sentence 3 1 and sentence 3 2 are true in the Tarski world given above
2 Sentence 3 1 is true and sentence 3 2 is false in the Tarski world given above
3 Sentence 3 1 is false and sentence 3 2 is true in the Tarski world given above
4 Sentence 3 1 and sentence 3.2 are false in the Tarski world given above

TABLE 1 FOR QUESTIONS (iv) AND (v)

English	FOL	Comment
Names		
John	john	The name of a person
Predicates		
x is a pet	Pet(x)	
x is a patient	Patient(x)	
x owned y	Owned(x, y)	
x fed y at time t y	Fed(x, y)	
x is admitted in the hospital	Hospital(x)	
x is involved in an accident	Accident(x)	

Question (iv) [2]

Consider the FOL sentence below and then write down the number of the option indicating a correct English translation

FOL sentence

$$\exists x(Pet(x) \wedge owned(john,x) \rightarrow Fed(john,x,t))$$

Options

1 John owned the pet which was fed by him at time t
2 John owns the pet
3 John fed the pet at time t
4 All the pets were fed by John

Question (v) [2]

Consider the English sentence below and then write down the number of the option indicating a correct FOL translation

English sentence:

All the patients who are involved in an accident are admitted in the hospital

Options

1 $\exists x[(Hospital(x) \rightarrow (Accident(x) \wedge Hospital(x))]$
2 $\forall x[(Hospital(x) \rightarrow Accident(x)) \wedge Hospital(x)]$
3 $\forall x[(Patient(x) \wedge Accident(x)) \rightarrow Hospital(x)]$
4 $\forall x[(Patient(x) \wedge Hospital(x)) \rightarrow Accident(x)]$

Question (vi) [2]

Consider the FOL sentence below and write down the number of the option giving a correct Disjunctive Normal Form (DNF) of the sentence **(Do not guess but derive the DNF by rough work.)**

$$\neg(\neg A \vee B) \vee \neg(B \wedge \neg C) \vee (D \vee \neg A)$$

Options

1 $(A \wedge \neg B) \vee (\neg B \vee C) \vee D \vee A$
2 $(A \wedge \neg B) \vee (\neg B \vee C) \vee D \wedge \neg A$
3 $(A \wedge \neg B) \vee \neg B \vee \neg C) \vee D \vee \neg A$
4 $(\neg A \vee B) \vee (B \wedge \neg C) \vee (D \vee \neg A)$

The following partially constructed truth table is used in questions vii, viii and ix. The block language is used for all the sentences.

	P	Q	R	(P ∨ Q) ∨ R	Q ∧ R		some sentence
1	T	T	T	T	T		
2	T	T	F	F	F		
3	T	F	T	T	F		
4	T	F	F	F	F		
5	F	T	T	T	T		
6	F	T	F	F	F		
7	F	F	T	T	F		
8	F	F	F	F	F		

Question (vii) [2]

Suppose we want to determine if a certain sentences in the blocks language built from the three atomic sentences P, Q, R is a tautological consequence of (P ∨ Q) ∨ R and Q ∧ R We build the complete truth table and then inspect the last column The sentence is a tautological consequence of the two sentences (P ∨ Q) ∨ R and Q ∧ R

Options

1 Only the entry in line 8 is True
2. Only in the entry in line 1 and 5 is True
3 The entry in lines 1 is True and entry in all other lines are False
4 The only entry in line 5 is True

Question (viii) [2]

Suppose we want to determine if a certain sentences in the blocks language built from the three atomic sentences P, Q, R is a logical consequence of (P ∨ Q) ∨ R and Q ∧ R We build the complete truth table and then inspect the last column The sentence (P ∨ Q) ∨ R is a logical consequence of Q ∧ R if

Options

1 Only the entry in line 8 is True
2 Only in the entry in line 1 and 5 is True
3 The entry in lines 1 is True and entry in all other lines are False
4 The only entry in line 5 is True

Question (ix) [2]

Suppose we want to determine if a certain sentences in the blocks language built from the three atomic sentences P, Q, R is a logical consequence of $(P \vee Q) \vee R$ and $Q \wedge R$ We build the complete truth table and then inspect the last column The sentence $(P \vee Q) \vee R$ is a tautologically equivalent to $Q \wedge R$ if

Options

1 All the last two columns are identical
2 At least one column has one true
3 The two columns should have truth values True
4 None of the above

Question (x) [2]

Consider the following FOL sentence and the joint truth table below and then write down the number of the option giving the correct interpretation of the truth table

$$\neg [(P \wedge (\neg Q \vee R)) \leftrightarrow (P \wedge (\neg Q \vee R))]$$

P	Q	R	$\neg Q$	$\neg Q \vee R$	$P \wedge (\neg Q \vee R)$	$(P \wedge (\neg Q \vee R))$
T	T	T	F	T	T	T
T	T	F	F	F	F	F
T	F	T	T	T	T	T
T	F	F	T	T	T	T
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	T	T	F	F
F	F	F	T	T	F	F

Options

1 The sentence is a contradiction
2 The sentence is a tautology
3 The sentence is a contingent
4 The sentence is not well-formed

Question (xi) [2]

Consider the argument below and then write down the number of the option giving the correct evaluation of the argument

All students who study hard get good marks
Jacob is intelligent
∴
Jacob got good marks

Options

1 The argument is both sound and valid
2 The argument is sound but not valid
3 The argument is valid but not sound
4 The argument is neither sound nor valid

Question (xii) [2]

Consider the following argument where the blocks language is used and then write down the number of the option giving the correct evaluation of the argument

SameSize(a, b)
SameCol(b, c) $\wedge$ LeftOf(a, c)
SameShape (b, a)
∴
LeftOf(b, a)

Options

1 The argument is both sound and valid
2 The argument is sound but not valid
3 The argument is valid but not sound
4 The argument is neither sound nor valid

SECTION B
76 marks

Table 2 below lists the names and predicates that should be used in questions 2 and 3 of Section B

English	FOL
Names	
Petro	petro
Jabulani	jabu
"Days and Years"	days
"Third Street"	third
"Enemies or not?"	enemies
Predicates	
x is an actor	Actor(x)
x is a soap opera	Soap(x)
x plays in y	Play(x, y)
x watches y	Watch(x, y)

Table 2

QUESTION 2 **[12]**

Translate the following English sentence into a first-order logic (FOL) sentence, using the names and predicates given in Table 2

a) Unless Petro plays in "Days and Years", Jabulani watches either "Enemies or not?" or "Third Street"

b) If "Third Street" is not a soap opera but "Days and Years" is, Petro or Jabulani watches "Enemies or not?"

c) Petro either plays in "Third Street" or in "Enemies or not?" but not in both

QUESTION 3 **[9]**

Translate the following first-order logic (FOL) sentence into an English sentence, using the names and predicates given in Table 2

a) $(Watch(petro, third) \land Watch(jabu, third)) \leftrightarrow (Play(petro, third) \land Play(jabu, third))$

b) $\exists x (\neg Actor(x) \land Play(x, days) \land \neg Watch(x, days))$

c) $\forall x (Play(x, enemies) \rightarrow \neg Watch(x, enemies))$

QUESTION 4 **[8]**

Question 4(a) **(4)**

Consider the following argument and then answer the questions given below

1 All nurses are friendly
2 All teachers are witty
3 Jacob is not a teacher and he is not a nurse

Jacob is unfriendly and not witty

(i) Is the argument valid? Explain your answer

(ii) Is the argument sound? Explain your answer

Question 4(b) **(4)**

Consider the following argument and then answer the questions given below

1 All piano players are able to play "Rock around the clock"
2 Rutendo is a piano player

Rutendo can play "Rock around the clock"

(i) Is the argument valid? Explain your answer

(ii) Is the argument sound? Explain your answer

QUESTION 5 **[12]**

Show that the following arguments are valid by giving an ***informal proof***, phrased in complete, well-formed English sentences Use proof by cases and/or proof by contradiction

(i)

1 Beauty and Tim are married and live in a flat
2 At 18 00 Beauty is always either at the gym or in the kitchen of the flat
3 Tim is always either at work or with Beauty
4 It is 18 00
5 Beauty is not at the gym
6 The building where Tim works is locked up for the night

Tim is in the kitchen

(ii)

1 Nku or Abe swims and Louka or Sam plays basketball
2 Nku does not swim or Louka does not play basket ball
3 Nku does not swim or Louka does not play basketball
4 Either Abe does not swim or Sam plays basketball

Sam plays basketball

QUESTION 6 **[6]**

Construct a truth table for the following FOL sentence and show that it is a tautology

$$(A \rightarrow (B \wedge C)) \leftrightarrow ((\neg B \vee \neg C) \rightarrow \neg A)$$

QUESTION 7 **[23]**

Give ***formal proofs*** of the arguments below using the rules of natural deduction You may ***not*** use Taut Con or FO Con or Ana Con It is important to number your statements, to indicate subproofs and at each step to give the rule that you are using

Question 7.1 **(7)**

Using the natural deduction rules, give a formal proof that the following three sentences are inconsistent

$\neg P \vee Q$
$\neg P \rightarrow Q$
$\neg Q$

Question 7.2 **(8)**

Using the natural deduction rules, give a formal proof of

Q

from the premises

$(\neg P \rightarrow Q) \wedge (R \rightarrow \neg Q)$
$\neg Q \rightarrow \neg S$
$\neg S \rightarrow (R \wedge \neg P)$

Question 7.3 **(8)**

Using the natural deduction rules, give a formal proof of

$$\forall x\,[P(x) \rightarrow Q(x)] \rightarrow [\forall x P(x) \rightarrow \forall x Q(x)]$$

from no premises

QUESTION 8 **[6]**

Consider the Tarski's World below This world illustrates that $\neg\exists x Tet(x)$ is **NOT** a logical consequence of the premises below

1 $\forall x \forall y$ [LeftOf(x, y) $\rightarrow$ Larger(x, y)] and

2 $\forall x \forall y$ [Smaller(y, x) $\rightarrow$ (Cube(x) $\wedge$ Dodec(y))]

Explain why it is the case?