COS 2601

OCT/NOV

2016

Question 1

a) abbababbba = w can be formed by the language S^* it can be broken down as follows

$(ab)(bab)(bba) = w$

b) $S \not\subseteq T$ and $T \subseteq S$ but $S^* = T^*$

$S \Rightarrow \{a \;\; aa\}$, $T = \{a \;\; aaa\}$

c) $S = \{\Lambda, a, b\}$

d) $S = \mathbf{S} = \{\ \}$

e) ~~$S^* = \{$ abba~~

$S^* = \{$ abab, baba, bbbb, abba, aabb, bbaa, aaaa $\}$

f) ~~$2^3 = 8$~~ Then number of 4 letter words $= 4^3$

$= \underline{\underline{64}}$

Question 2

a) $(aa + bb)(a+b)^*(aa+bb)$

bi) $w =$ bbabbabba can be formed by the language such that
$w = (bb)(abb)(abba)$

ii) $w =$ bbbbabbaabbbb cannot be formed by the language since there is no way of producing the last bbbb substring

c) No it does not, the language cannot generate the word $w =$ abab

d) Λ cannot be generated by $a^* b^* (a+b) a^* b^*$

Question 3

a) $\Sigma^{*} = \{a\ b\}^{*}$

b) $\{aa\ ba\ bb\} \in$ EvennotAB

c) CONCAT

d) if $w \in$ EvennotAB ~~H~~ and w does not end with an "a" then:

CONCAT (w, aa), CONCAT(w, ba), CONCAT $(w, bb) \in$ Even NotAB

~~con~~
If $w \in$ Even not AB and begins with "b" or ends with "a" then

CONCAT (w, aa), ~~CONCAT(w,~~ $\in$ EvenNotAB

CONCAT (bb, w), $\in$ EvenNotAB

If $w \in$ EvenNotAB and does not begin with a "b" then:

CONCAT (aa, w), ~~ba, w~~ CONCAT(ba, w), ConCAT$(bb, w) \in$ EvenNotAB

Question 4

a) i) $4 \in P$.

ii) If $x \in P$ then so is $x+1 \in P$

iii) The only elements in set P above are only those provided by rule one and rule 2 above.

b) Prove that for every integer, if $P(k)$ is true (induction hypothesis) then $P(k+1)$ is true.

c) hypothesis

$P(n) = \{n! > 2^n \mid n > 3\}$

Test for $n = 4 \in P$

$3! = 3 \times 2 \times 1 = 6 > 2^2 = 4$

$\therefore n! > 2^n$ for $n = 4$.

assume $k \in P$ ie $k! > 2^k$, prove that

$(k+1) \in P$ ie $(k+1)! > 2^{(k+1)}$

$LHS = (k+1)! = (1 \times 2 \times 3 \times \cdots \times k)(k+1)$

$= k!(k+1) > 2^k(k+1)$

$> 2^k \cdot 2^1$

$> 2^{k+1} = RHS$

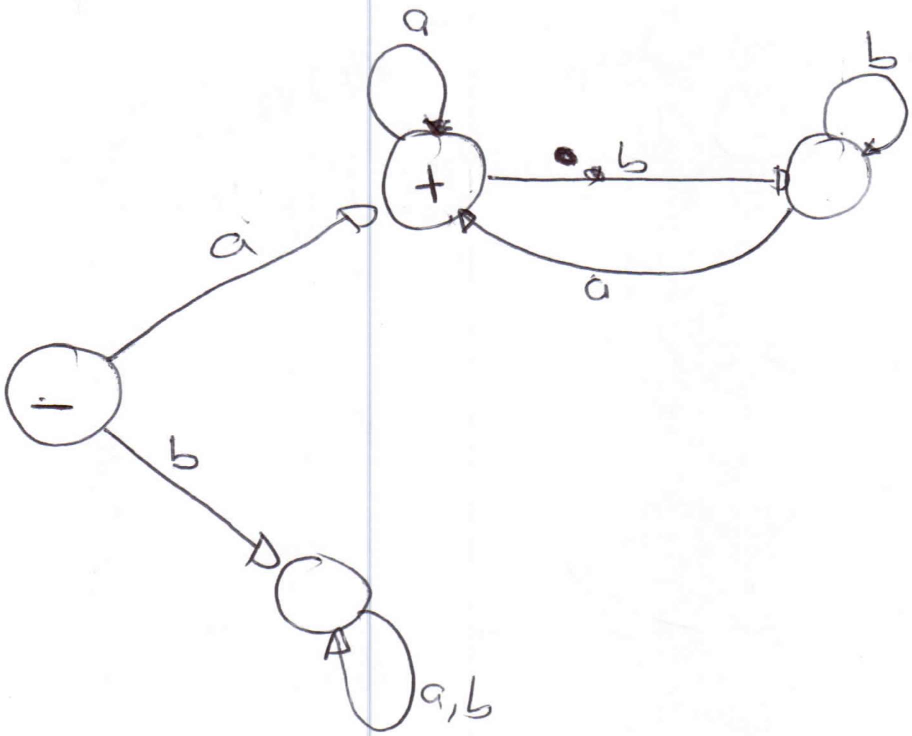
The hypothesis that $n! > 2^n$ for all $n > 3$ is therefore true.

Question 5

a) a

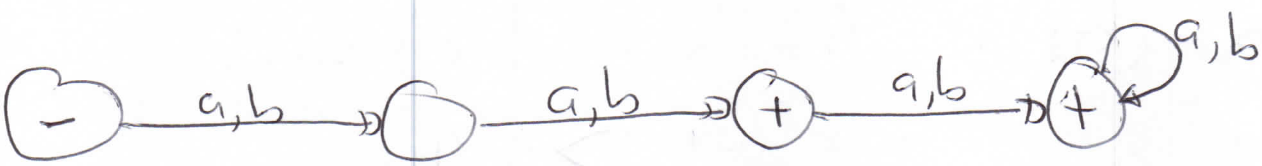
b) ab

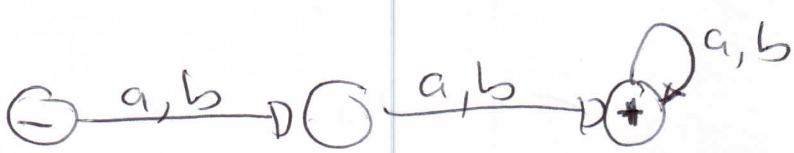
c)



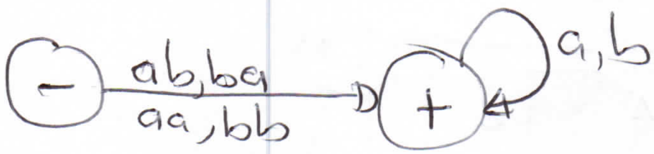
Question 6

a





ii)



b) i)

input		a	b	a	a
state	q_0	q_1	q_2	q_2	q_2
output	1	0	1	1	1

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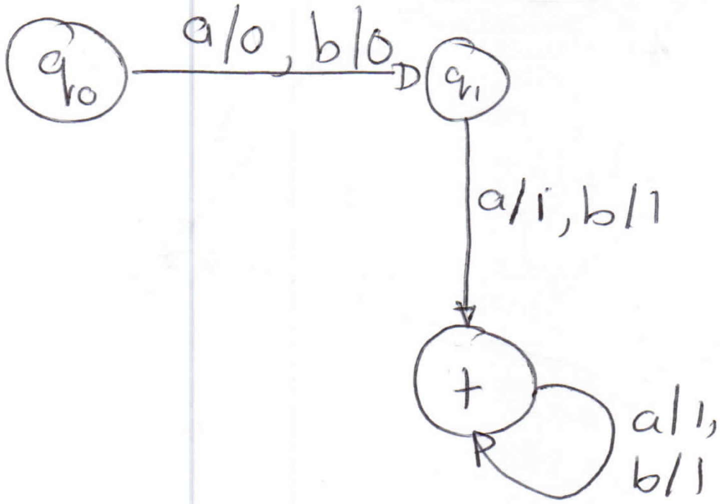
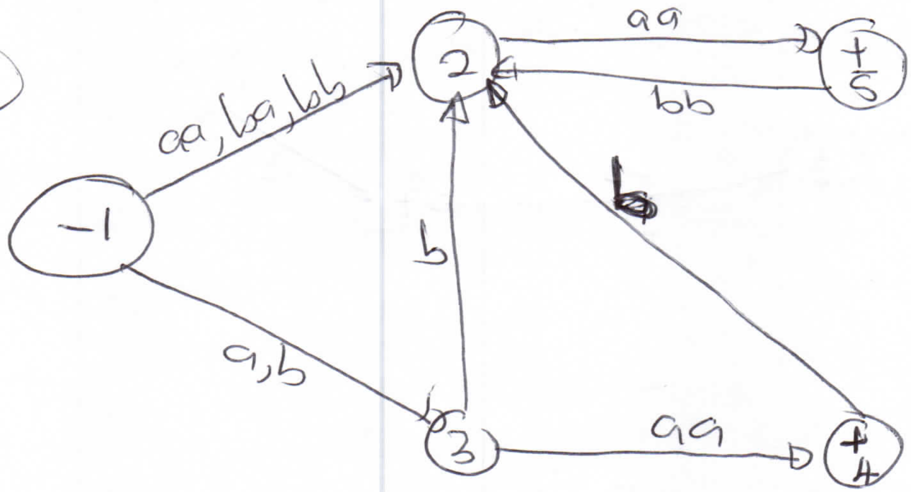
ii)

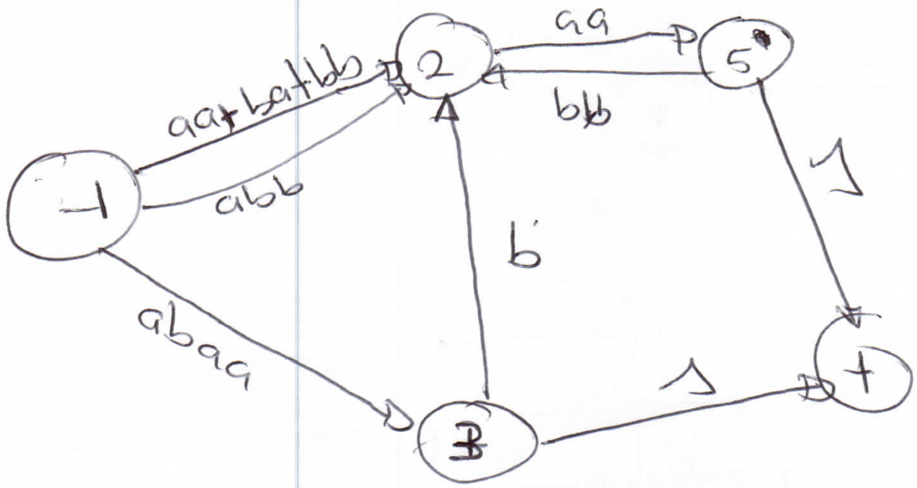


Question 7

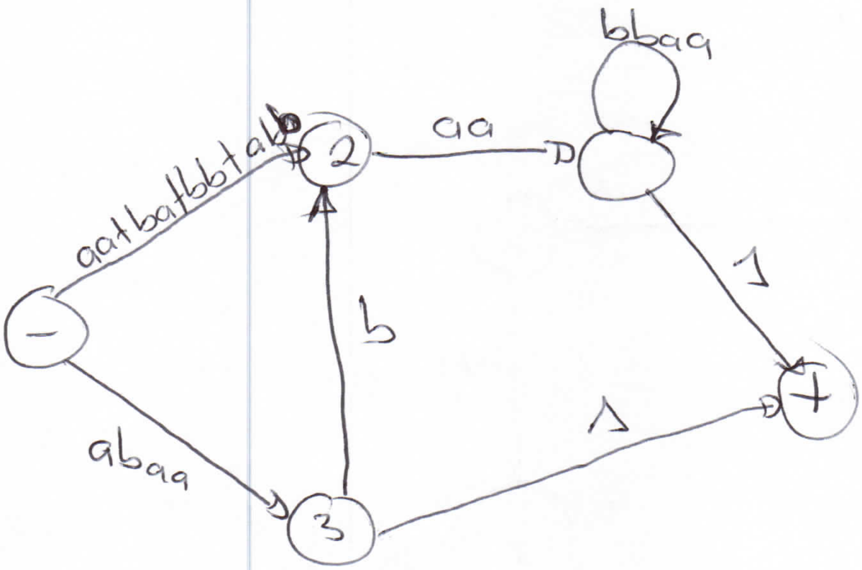
1



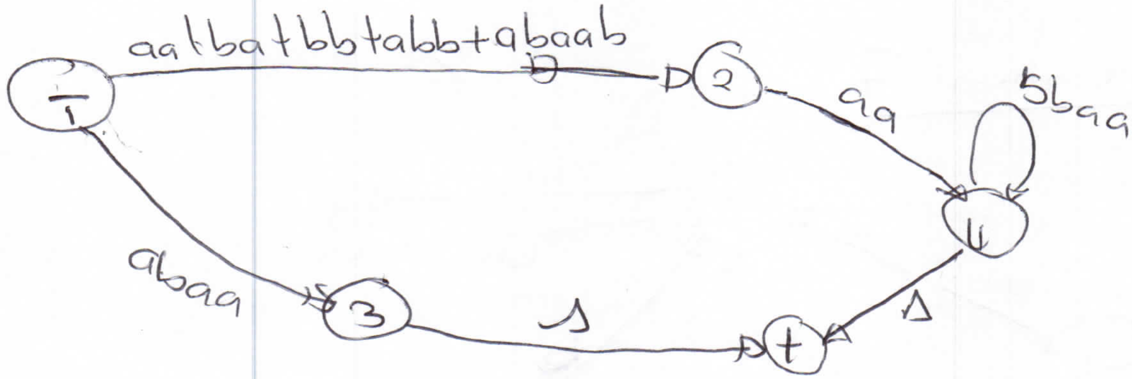
(2)


aa+ba+bb
abb
aa
bb
b
Λ
abaa
Λ
2
5
3
-
+

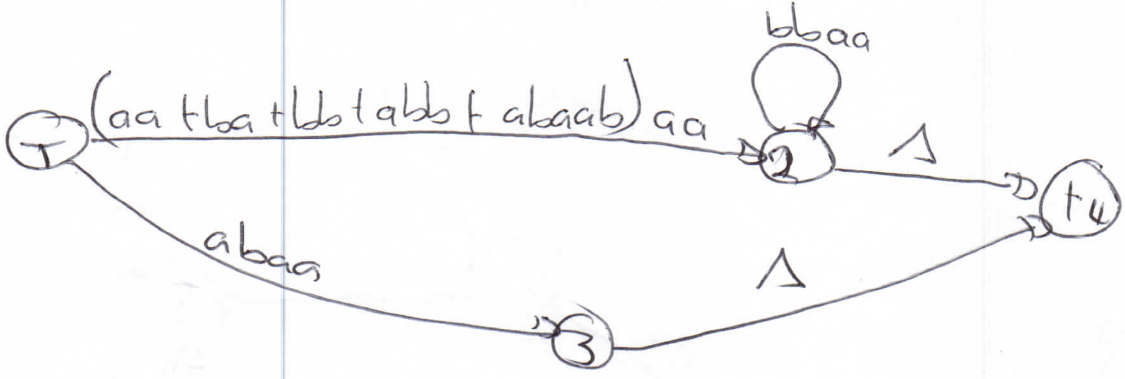
(3)


aa+ba+bb+abb
aa
bbaa
b
Λ
abaa
Λ
2
3
-
+

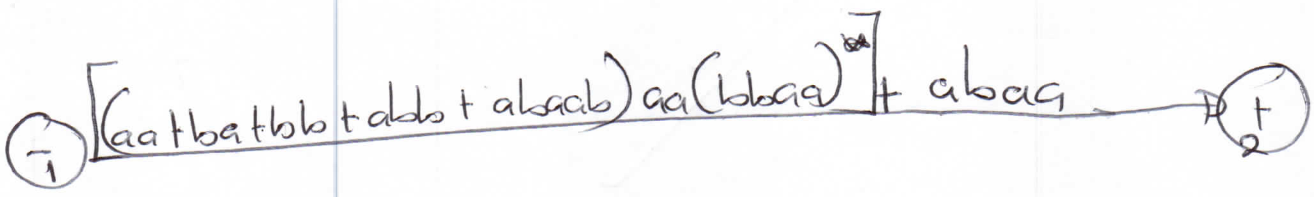
(4)


aa+ba+bb+abb+abaab
aa
bbaa
abaa
Λ
Λ
2
3
4
-
+

(5)

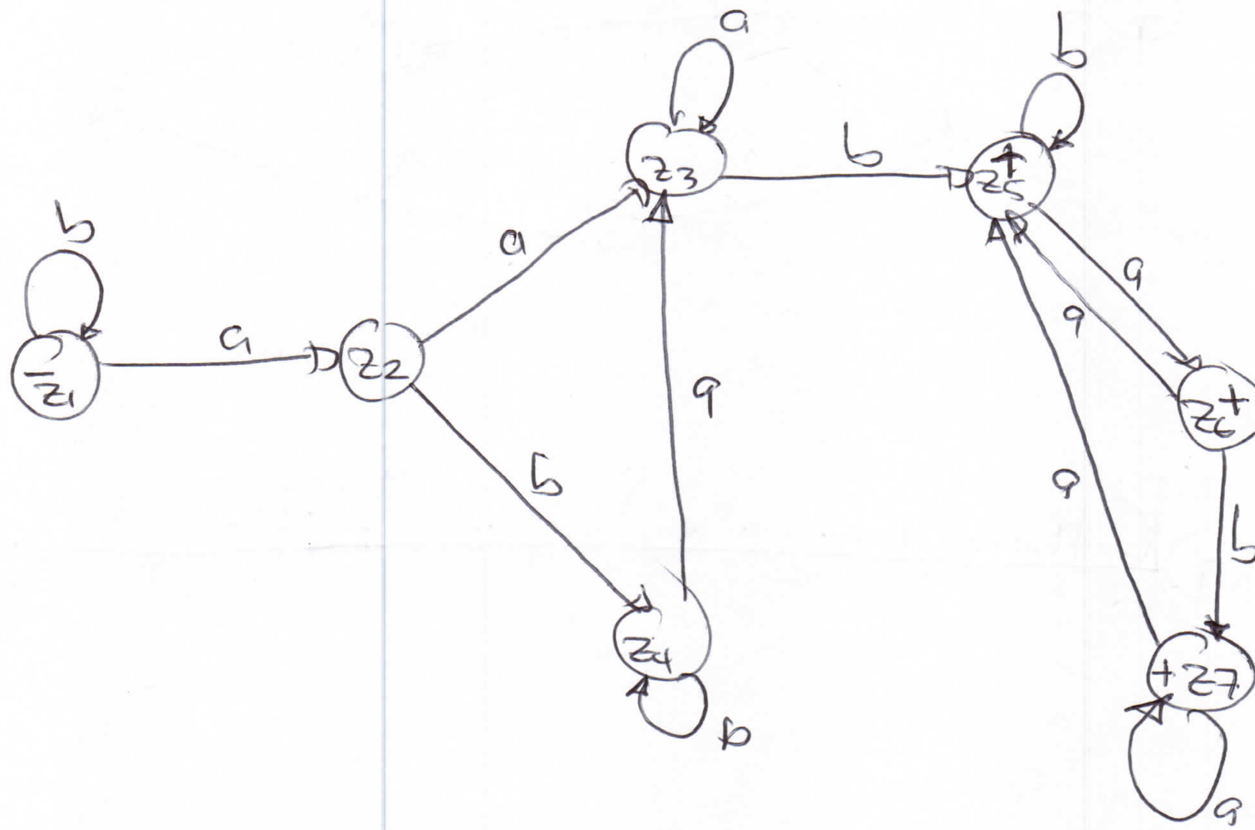


(6)



Question 8

New State	transition a	transition b	Old states
z_1	z_2	z_1	w_1
z_2	z_3	z_4	w_2 or x_1
z_3	z_2	z_5	w_2 or x_2 or x_1
z_4	z_3	z_4	w_1 or x_1 or w_2
$+z_5$	z_6	z_5	w_1 or x_3 or x_1 or w_2
$+z_6$	z_7	z_5	w_1 or x_3 or x_2 or w_2 or x_1
$+z_7$	z_7	z_6	w_2 or x_3 or x_2, x_1



Question 9

$L = \{ a^{n!} b^{n!}, n \geq 0 \}$

assume $w = a^N b^N$ where $N = n!$, $\ell(w) > N$

if L is regular then there is a word w_* such that $w_* = xyz$ and another word $w = xy^n z \in L$.

If y comprises only of a's then when we pump it + $xyyz$ we end up having more a's than b's which is not allowed in L. Similarly, If the middle part comprises only of b's the $xyyz$ will have more b's than a's.

The solution is that the y part must have some positive number of a's and some positive number of b's. This would mean that y contains the substring ab. This in turn means that $xyyz$ will have two pairs of the substring ab but every word in L contains to L, therefore L is not regular.

Question 16

a) Decision procedure is an effective ~~procedure~~ solution to a problem that has a yes or no answer.

b) A problem that has a decision procedure is called decidable.

c) If L_1 and L_2 are two finite automata then to determine if $L_1 = L_2$ a new finite automata of the form $(L_1 \cap L_2') + (L_1' \cap L_2)$ ie all the elements in L_1 that are not in L_2 or vice-versa. If they are ~~not~~ equal the new machine will not accept any words. If the are not equal then the machine will accept a word.

d) If F accepts any input string w such that

$N \leq \text{length}(w) < 2N$ then F accepts an infinite language otherwise it is not. N is the number of states

~~w = bb~~

$w = baa$ is accepted by F therefore it is an infinite language.