UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

COS2601

May/June 2015

THEORETICAL COMPUTER SCIENCE II

Duration 2 Hours

100 Marks

EXAMINERS
FIRST MR CL PILKINGTON
SECOND MRS D BECKER

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue

This paper consists of 6 pages

Instructions

1 Answer all questions

2 All rough work must be done in your answer book

3 The mark for each question is given in brackets next to the question

4 Unless otherwise specified, all languages in the questions are defined over the alphabet $\sum = \{a,b\}$

ALL THE BEST!!

SECTION 1
REGULAR EXPRESSIONS AND LANGUAGES
[20 marks]

QUESTION 1 [10]

(a) Let S = {*ab aba baa bab*} For each of the following strings, state whether it is a word in S* or not Justify your answer

(i) *babababab*
(ii) *abaabbba* (4)

(b) Provide a set S and a set T such that
S ∩ T = {*a aa*} and
T ⊆ S but S ⊄ T and
S* = T* (2)

(c) Let S={*ab ba*} and let T={*aa ab ba bb*}

(i) Is S* ⊆ T*? Justify your answer
(ii) Is (S* ∩ T*) = (S ∩ T)*? Justify your answer (4)

QUESTION 2 [10]

(a) Give a regular expression which generates the language of all words containing an *aa* substring and having only three *a*'s in total in each word (3)

(b) Give a regular expression which generates all words which begin and end with *aa* or *bb* and contain at least 4 letters in each word (3)

(c) Consider the following regular expression
a*((**b** + **bb**)**aa***)*(Λ + **b** + **bb**)**aaaa***(Λ + **b** + **bb**)
Can the regular expression generate the word *abbaaabaabbaaab*? Justify your answer (4)

SECTION 2
RECURSIVE AND INDUCTIVE PRINCIPLES
[20 marks]

QUESTION 3 [10]

A recursive definition for the language EVENAAnotendB defined over the alphabet Σ = {*a b*} should be compiled EVENAAnotendB consists of all words

- that are of even length,
- that contain the *aa* substring, and
- that do **not** end with a *b*

[TURN OVER]

Provide

(a) an appropriate universal set, (1)
(b) the generator(s) of EVENAAnotendB, (1)
(c) an appropriate function on the universal set, and then (1)
(d) use these concepts to write down a recursive definition for the language EVENAAnotendB (7)

QUESTION 4 [10]

(a) Provide a recursive definition of the set P of all integers greater than 0, (1)
(b) formulate the associated induction principle, and then (2)
(c) apply this induction principle to prove that $n! \leq n^n$
where $n! = 1 \quad 2 \quad 3 \quad (n-1) \quad n = (n-1)! \quad n$ (7)

SECTION 3
REGULAR LANGUAGE ACCEPTORS
[20 marks]

QUESTION 5 [10]

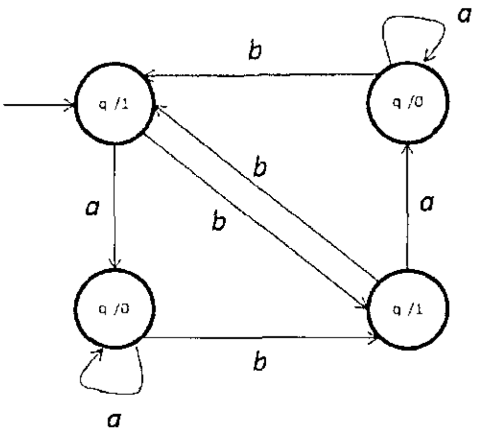
Build an FA (finite automaton) that accepts the language L consisting of all words that

- begin with the *aa* or *bb* substring,
- end with an *a* substring, and
- have an odd number of letters

L is defined over the alphabet $\Sigma = \{a \quad b\}$ (10)

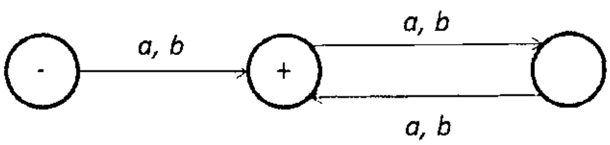
QUESTION 6 [10]

(a) Consider the following Moore machine



(i) Let the input to this Moore machine be *ababba* What is the output of the machine? (3)
(ii) Convert the given Moore machine to a Mealy machine (4)

(b) Build a TG (transition graph) with 2 states that accepts exactly the same language as the following FA (3)

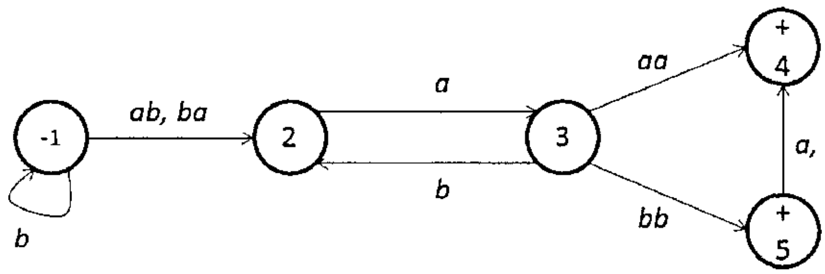


SECTION 4
APPLICATION OF ALGORITHMS WITHIN KLEENE'S THEOREM
[20 marks]

QUESTION 7 [10]

By applying Kleene's theorem, find a regular expression that generates the language accepted by the following TG

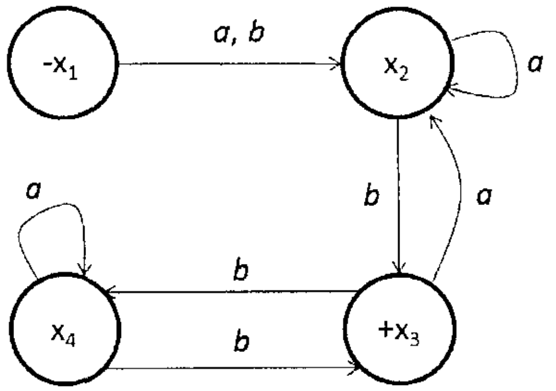
Show all the steps – full marks will not be allocated when only a correct final answer is given (10)



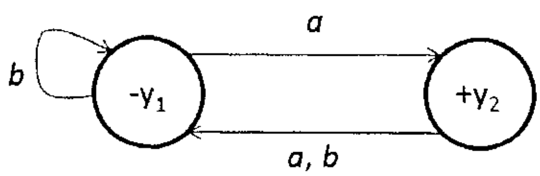
QUESTION 8 [10]

Let L_1 be the language defined by the regular expression $\mathbf{r}_1$, and L_2 be the language defined by the regular expression $\mathbf{r}_2$ Consider FA_1 that accepts all the words of L_1, and FA_2 that accepts all the words of L_2 By applying Kleene's theorem, build another FA that accepts all the words of the language L_1L_2 defined by r_1r_2 Do not formulate regular expressions as part of your solution
A table should be included in your answer (10)

FA₁



FA₂



SECTION 5
NON-REGULAR LANGUAGES
[10 marks]

QUESTION 9 [10]

Use the Pumping Lemma **with length** to prove that the following language is non-regular
$L = \{ a^2b^n a^n, n > 0 \}$ (10)

SECTION 6
DECIDABILITY
[10 marks]

QUESTION 10 [10]

(a) Briefly define a decision procedure (2)

(b) Describe an effective decision procedure that can be used to decide whether a given FA accepts a finite or an infinite language (4)

(c) Using the decision procedure described in (b) above, determine whether the FA below accepts a finite or an infinite language (4)