UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

COS2601

October/November 2015

THEORETICAL COMPUTER SCIENCE II

Duration 2 Hours

100 Marks

EXAMINERS
FIRST MRS D BECKER
SECOND MR CL PILKINGTON

Closed book examination

This examination question paper consists of 6 pages

Instructions

1 Answer all questions

2 All rough work must be done in your answer book

3. The mark for each question is given in brackets next to the question.

4 Unless otherwise specified, all languages in the questions are defined over the alphabet $\Sigma = \{a\ b\}$

ALL THE BEST!!

SECTION 1
REGULAR EXPRESSIONS AND LANGUAGES
[20 marks]

QUESTION 1 [10]

(a) Consider the language S* where S = {*a bb bab aba*}.

(i) Is *aababbbbabab* a word in S*? Justify your answer
(ii) Provide all the words with 4 characters belonging to S*
(iii) Does any word in S* have 11 characters with an odd total number of *b*'s and an even total number of *a*'s? Justify your answer. (6)

(b) Provide a set S and a set T such that S $\not\subseteq$ T, and such that S* ∩ T* contains four elements with four characters (2)

(c) Consider the language T* where T = {*aa ab ba bb*} Give a description in words of the language T* (2)

QUESTION 2 [10]

(a) Give a regular expression which generates the language of all words containing no single *a*-substring. (Words such as *a* or *aba* or *aababbaaa* etc are not in the language but words such as *aa* or *baaa* etc are in the language.) (4)

(b) Give a regular expression which generates the language of all words containing at least one *a*-substring and at least two occurrences of a *bb*-substring (Words such as *a, bbbb, abbba* or *aababbaaa* etc are not in the language but words such as *abbbb, aaabbbbb or aaabaabbaaabb* etc are in the language) (4)

(c) Consider the following regular expression
a*((b + bb)*aa*)*(b + bb)*aaaa*(b + bb)*a*((b + bb)*aa*)*
Does the regular expression generate *all* the words with at least the *aaa*-substring in them? Justify your answer (2)

SECTION 2
RECURSIVE AND INDUCTIVE PRINCIPLES
[20 marks]

QUESTION 3 [10]

A recursive definition for the language ODDatleast2As defined over the alphabet $\Sigma = \{a \ \ b\}$ should be compiled ODDatleast2As consists of all words

- that are of odd length, and
- that contain at least two *a*'s

Provide

(a) an appropriate universal set, (1)
(b) the generator(s) of ODDatleast2As, (1)
(c) an appropriate function on the universal set, and then (1)
(d) use these concepts to write down a recursive definition for the language ODDatleast2As (7)

QUESTION 4 [10]

(a) Provide a recursive definition of the set P of all integers greater than 0, (1)
(b) formulate the associated induction principle, and then (2)
(c) apply this induction principle to prove that for all n > 0

$$\sum_{j=1}^{n} 3^{(j-1)} = \frac{3^n - 1}{2}$$

(7)

SECTION 3
REGULAR LANGUAGE ACCEPTORS
[20 marks]

QUESTION 5 [10]

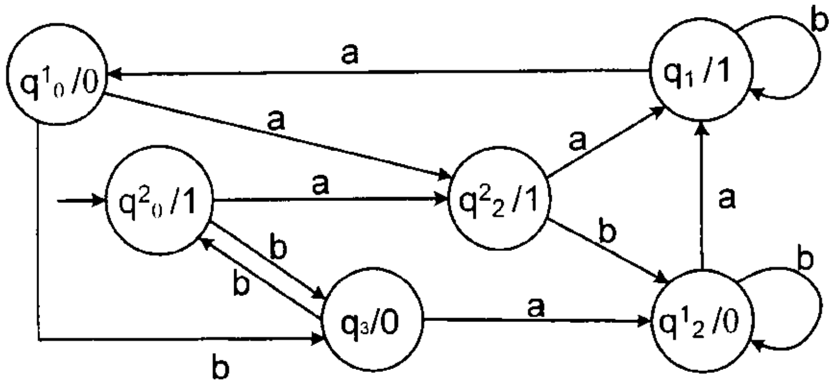
Build an FA (finite automaton) that accepts the language L consisting of all words that

- have *abb* as a substring in all words, and
- do not end on a *b*-substring

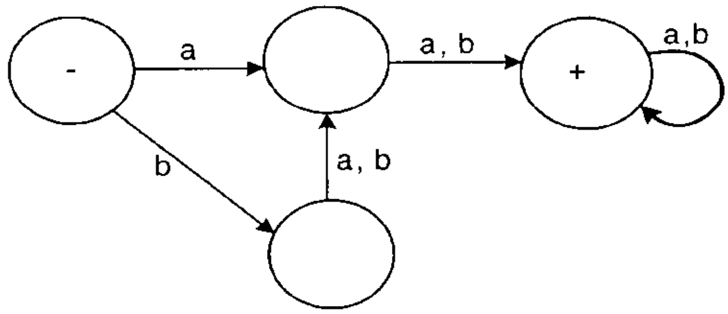
L is defined over the alphabet $\Sigma = \{a \ \ b\}$ (10)

QUESTION 6 [10]

(a) Convert the following Mealy machine to a Moore machine (7)



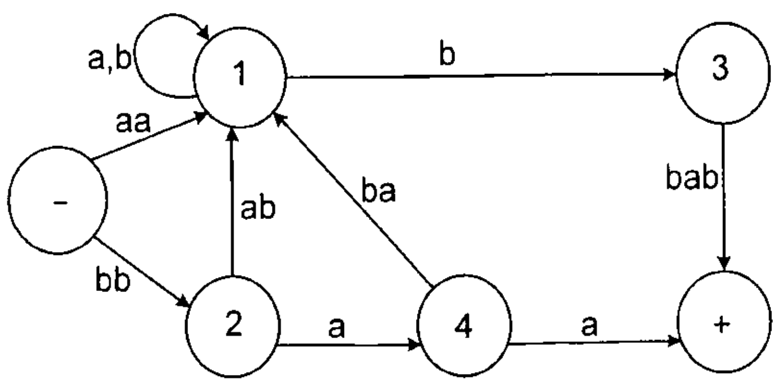
(b) Build a TG (transition graph) with 2 states that accepts exactly the same language as the following FA (3)



SECTION 4
APPLICATION OF ALGORITHMS WITHIN KLEENE'S THEOREM
[20 marks]

QUESTION 7 [10]

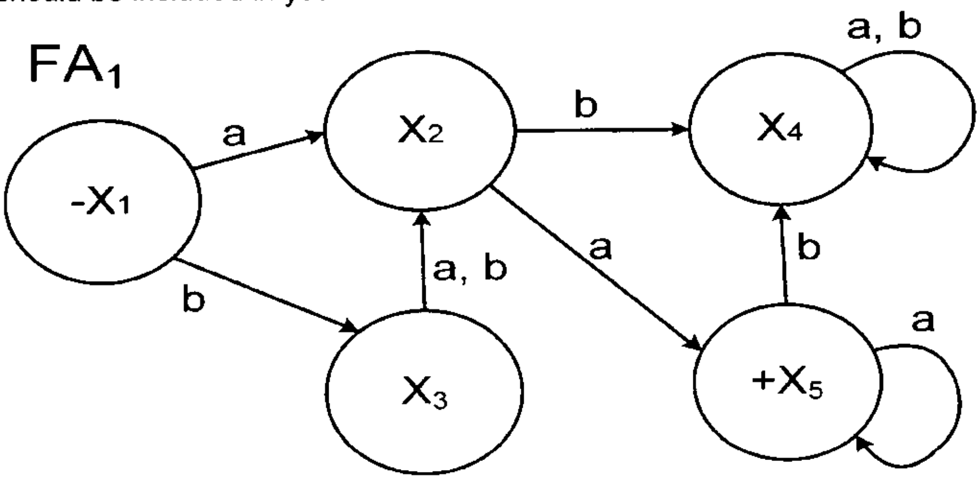
By applying Kleene's theorem, find a regular expression that generates the language accepted by the following TG
Show all the steps – full marks will not be allocated when only a correct final answer is given. (10)

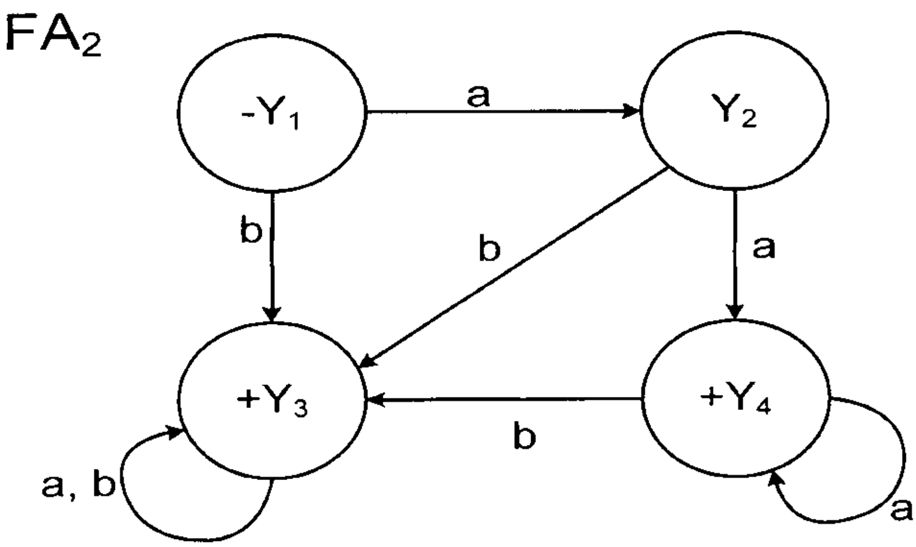


QUESTION 8 [10]

Let L_1 be the language defined by the regular expression $\mathbf{r}_1$, and L_2 be the language defined by the regular expression $\mathbf{r}_2$ Consider FA_1 that accepts all the words of L_1, and FA_2 that accepts all the words of L_2 By applying Kleene's theorem, build another FA that accepts all the words of the language $L_1 \cup L_2$ Do not formulate regular expressions as part of your solution

A table should be included in your answer (10)





SECTION 5
NON-REGULAR LANGUAGES
[10 marks]

QUESTION 9 [10]

Use the Pumping Lemma **with length** to prove that the following language is non-regular

$L = \{ aba^nb^n, n \geq 0 \}$ (10)

SECTION 6
DECIDABILITY
[10 marks]

QUESTION 10 [10]

(a) Let FA_1 accept L_1 and let FA_2 accept L_2 Briefly describe an effective procedure that can be used to decide whether the two given FA's accept the same language (i e whether L_1 and L_2 are equivalent) (5)

(b) Apply the blue paint procedure to determine whether or not the following FA accepts any words (show all the steps) (5)