OBJECTIVE:

1. Able to calculate the present value of annuities
2. Able to calculate the future value of annuities
3. Able to complete an amortisation schedule

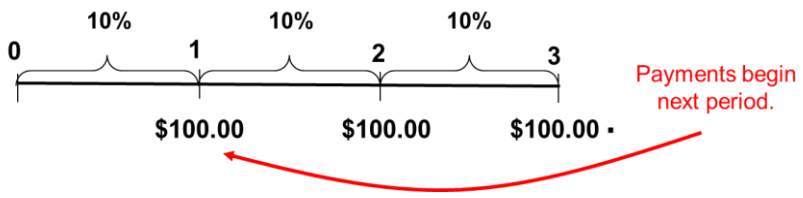
TARGET: QMI1500 and BNU1501, any other modules using Annuities and amortisation.

ANNUITIES

Present Value

Future Value

- An **annuity** is
  - A series of constant payments
  - That occur at regular intervals and
  - Do not continue forever.
- **NOTE**:
  - The first payment occurs next period (not now).
- The constant cash flows of a 3-year annuity of R100.00 per year at 10% can be represented on a time line:



- Present Value of an Annuity

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ &= R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \end{aligned}$$

  - ‘Borrowing’ Problems (Loans)
    - How much can I borrow (PV / P)?
    - How much are my payments (PMT / R)?
    - What interest rate am I getting (I/Y / i)?
    - How long will it take (N / n)?
- Future Value of an Annuity

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

  - ‘Savings’ Problems (Retirement Fund)
    - How much will I have (FV / S)?
    - How much are my payments (PMT / R)?
    - What interest rate am I getting (I/Y / i)?
    - How long will it take (N / n)?

Using Financial Calculator

- Present value

$$P = Ra_{\overline{n}|i}$$
$$= R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

  * Calculate the present value of R1 600 quarterly payments for five years at an interest rate of 20% per year, compounded quarterly.

$$P = 1\,600a_{\overline{5\times4}|0,20\div4}$$
$$= 19\,939{,}54.$$

The present value is R19 939,54.

Key in as

[2ndF] [CA]

[2ndF] [P/Y] 4 [ENT]

[ON/C]

[+/−] 1 600 [PMT]

5 [2ndF] [×P/Y] [N]

20 [I/Y]

[COMP] [PV]

The answer is 19 939,54.

EXERCISE:

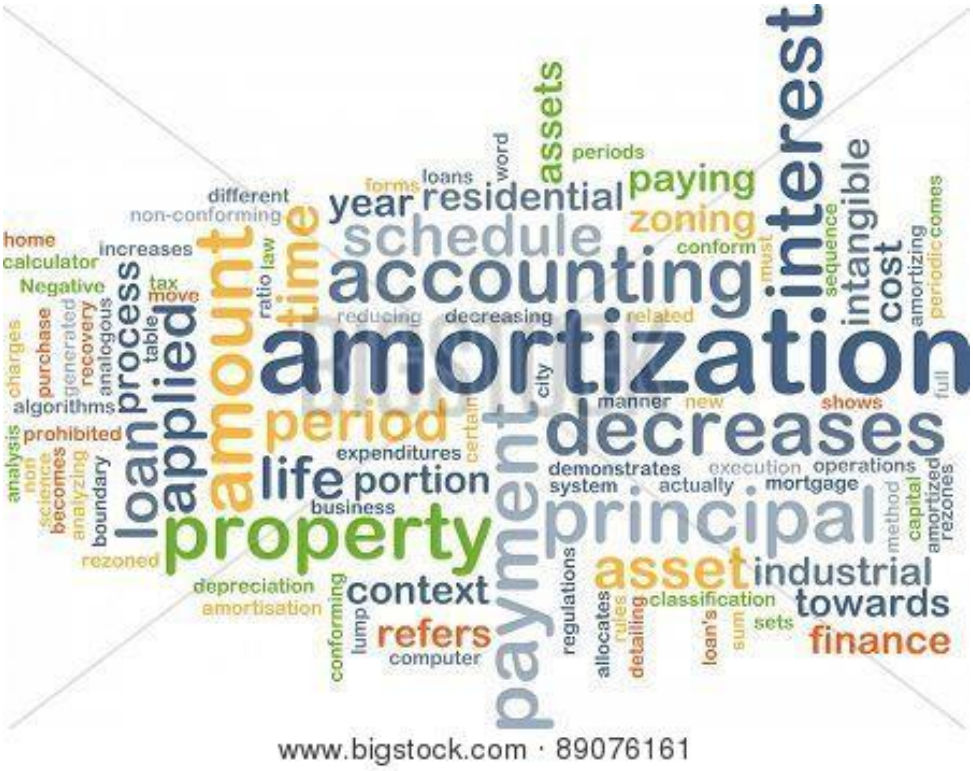
Future Value of an Annuity

- How much will you have if you save R100.00 per year for 25 years at 8%?
  - **R7,310.59**

- How much will you have if you save R1000.00 per year for 5 years at 7%?
  - **R5,750.74**
- How much will you have if you save R1.00 per year for 50 years at 10%?
  - **R1,163.91**

Present Value of an Annuity

- What is the present value of R100.00 per year for 25 years at 8%?
  - **R1,067.48**
- What is the present value of R1000.00 per year for 5 years at 7%?
  - **R4,100.20**
- What is the present value of R1.00 per year for 50 years at 10%?
  - **R9.91**



- Amortisation is:
  - a simple way to evenly spread out costs over a period of time.
  - we amortize items such as loans, rent /mortgages, annual subscriptions and intangible assets.

  - Amortisation schedule is a table indicating the distribution of each payment in regard to interest and principal reduction

Year	(d) Outstanding principal at year beginning	(b) Interest due at year end (simple)	(a) Payment	(c) Principal repaid

(a) Payment: $R = \frac{P}{\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]}$

(b) Interest: I = PRT

(c) Principal repaid: principal repaid = payment − interest due

(d) Outstanding Principal: OP = outstanding principal from previous year − principal repaid from previous year

EXAMPLE

Draw up an amortisation schedule for a loan of R4 000 for three years at 15% per annum compounded half-yearly and repayable in six half-year payments

$P = 4\,000$

$i = \frac{0{,}15}{2} = 0{,}075$ per half year

$n = 3 \times 2 = 6$ half years

The payments are

$$R = 4\,000 \div a_{\overline{6}|0{,}075}$$
$$= 852{,}18.$$

The payments are R852,18.

The amortisation schedule is as follows:

Period (that is half-years)	Outstanding principal at half-year beginning	Interest due at end of half-year	Payment	Principal repaid
1	4 000,00	300,00	852,18	552,18
2	3 447,82	258,59	852,18	593,59
3	2 854,23	214,07	852,18	638,11
4	2 216,12	166,21	852,18	685,97
5	1 530,15	114,76	852,18	737,42
6	792,73	59,45	852,18	792,73
Total		1 113,08	5 113,08	4 000,00

Using Financial Calculator:

Draw up an amortisation schedule for a loan of R5 000 which is repaid in annual payments over five years at an interest rate of 15% per year.

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ 5\,000 &= Ra_{\overline{5}|0,15} \\ R &= 1\,491{,}58 \end{aligned}$$

Key in as

[2ndF] [CA]

[2ndF] [P/Y] 1 [ENT]

[ON/C]

[+/−] 5 000 [PV]

5 [2ndF] [×P/Y] [N]

15 [I/Y]

[COMP] [PMT]

1 491.58 appears on the screen.

Press [AMRT] (fourth row, first key) 1

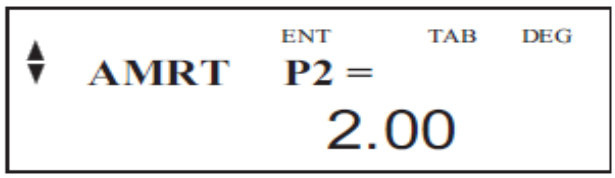
[▼] (Down arrow) 1 [ENT]

Press [▼] BALANCE = −4 258.42 appears on the screen.

Press [▼] $\sum$ PRINCIPAL = 741.58 appears on the screen.

Press [▼] $\sum$ INTEREST = 750.00 appears on the screen.

Press [[▼] 2 [ENT]] twice



appears on the screen.

Press [▼] BALANCE = −3 405.60

Press [▼] $\sum$ PRINCIPAL = 852.82

Press [▼] $\sum$ INTEREST = 638.70

Press [[▼] 3 [ENT]] twice

EXERCISES:

Exercise1:

Questions 19 and 20 are based on the following information:

Mr Nkosi wants to buy his first car from a local car dealer The car costs R150 000 excluding value-added tax (VAT) Mr Nkosi persuaded the sales lady to give 5% discount on the cash price However, he still have to pay 14% VAT after the discount and R1 500 (including VAT) for licensing, registration and roadworthiness of the car

Question 19

Calculate the total cost of the car

[1] R163 950

[2] R162 450

[3] R181 050

[4] R164 074

[5] none of the above

Question 20

Mr Nkosi does not have enough money to pay the total costs He decides to buy a cheaper car such that the total costs are R100 000 He pays R3 000 deposit He then signs a five year contract to make monthly payments to the dealer Interest is charged at 12% per annum compounded monthly Calculate the monthly payments

[1] R1 616,67

[2] R5 224,44

[3] R2 157,71

[4] R3 412,16

[5] none of the above

Exercise2:

Question 22

Sea-Salt Restaurant borrows R100 000 for five years from a bank to upgrade their premises The bank charges interest at 12% per annum compounded monthly over five years What is the equal monthly repayments of Sea-Salt Restaurant to the bank?

[1] R1 224,44

[2] R1 212 32

[3] R2 202,42

[4] R2 224,44

[5] none of the above

Exercise3:

Question 27

Dikeledi wants to give her brother a R10 000 gift on his graduation day in four years' time from now Calculate the money she should save on a monthly basis at an annual interest rate of 9,5% compounded monthly in order to have R10 000 in four years The correct answer to two decimal places, is

[1] R251,23

[2] R2 470,51

[3] R172 06

[4] R6 848,85

[5] none of the above

Exercise4:

Question 23

Sarah wants to save an amount of R100 000 over two years for a deposit on a townhouse She wants to make weekly payments into an account which offers an 8% yearly interest rate, compounded weekly Determine the size of her weekly payments

[1] R3 856,06

[2] R4 522,73

[3] R887,40

[4] R1 041,25

Exercise5:

Question 24

Joseph bought a town house in Northriding for R700 000 He paid a deposit of R100 000 and managed to secure a loan for the outstanding amount The term of the loan is 20 years and the applicable fixed interest rate is 7,75% per year, compounded monthly Calculate Joseph's monthly payment, to the nearest cent

[1] R5 746,64

[2] R1 050,69

[3] R2 519,51

[4] R4 925,69

Exercise6:

Questions 23 and 24 are based on the following information:
A loan of R50 000 with an interest rate of 15% per annum compounded monthly is amortised by equal payments for three years

Question 23

Calculate the monthly payments The monthly payments are

[1] R624,00
[2] R12 841,53
[3] R17 598,58
[4] R1 733,27
[5] none of the above

Question 24

Calculate how much interest was paid at the end of the term The interest is

[1] R52 295,08
[2] R22 500,00
[3] R12 397,72
[4] R1 875,00
[5] none of the above

Exercise7:

Question 24

In 1997 Justin bought a three bedroom house for R480 000 He paid a deposit of R150 000 and secured a loan for the outstanding amount The yearly interest rate on the loan was 24%, compounded monthly, and the term was 20 years Determine the outstanding amount at the end of the tenth year if Justin had done all monthly payments in full and the interest rate stayed fixed on 24% per year for the whole period

[1] R439 201,59
[2] R301 951,08
[3] R329 370,97
[4] R0,00

Question 25

Refer to question 24 above
Suppose Justin had paid R8 000 per month into this loan account from the start How long would it take to pay off the loan if the interest rate stayed fixed?

[1] 12,7 years
[2] 3,4 years
[3] 7,3 years
[4] 22,1 years

Exercise8:

Questions 23, 24 and 25 are based on the following information.
Jonty wants to buy a new truck that costs R350 000 The bank lends him 80% of the price of the tru at 11% interest rate per year compounded monthly, with a term of seven years The rest of the mon will be paid by Jonty as a deposit for the truck

Question 23

What is the monthly payment of the loan?

[1] R5 992,85

[2] R4 794,28

[3] R2 227,62

[4] R2 784,52

[5] none of the above

Question 24

Which one of the following is the correct armortisation schedule for the first two months?

[1]

Month	Outstanding principal	Interest due	Monthly payment	Principal repaid
1	350 000,00	3 208,33	5 992,85	2 784,52
2	347 215,48	3 182,81	5 992,85	2 810,04

[2]

Month	Outstanding principal	Interest due	Monthly payment	Principal repaid
1	280 000,00	2 566,67	41 480,05	38 913,38
2	241 086,62	2 209,96	41 480,05	39 270,09

[3]

Month	Outstanding principal	Interest due	Monthly payment	Principal repaid
1	280 000,00	2 566,67	4 794,28	2 227,61
2	277 772,39	2 546,25	4 794,28	2 248,03

[4]

Month	Outstanding principal	Interest due	Monthly payment	Principal repaid
1	350 000,00	3 208,33	51 850,06	48 641,73
2	301 358,27	2 762 45	51 850,06	49 087,61

[5] none of the above

Question 25

After five years the bank adjusts the interest rate to 15% per year, compounded monthly What will the new monthly payment be if the other terms of the loan remains the same?

[1] R6 234,43

[2] R4 608,03

[3] R5 246,38

[4] R4 987,54

[5] none of the above

Exercise9:

Question 24

Joseph bought a town house in Northriding for R700 000 He paid a deposit of R100 000 and managed to secure a loan for the outstanding amount The term of the loan is 20 years and the applicable fixed interest rate is 7,75% per year, compounded monthly Calculate Joseph's monthly payment, to the nearest cent

[1] R5 746,64

[2] R1 050,69

[3] R2 519,51

[4] R4 925,69

Exercise10:

Question 25

Jack needs R20 000, in 2 years time, to buy his brother's car He wants to start investing part of his weekly salary into an account which returns 8,5% interest per year, compounded weekly Calculate the weekly payment he needs to make into this investment account, to the nearest cent

[1] R209,27

[2] R176,58

[3] R767,45

[4] R192,15

Exercise11:

Question 23

Nikiwe moves into her new flat and wants to buy R7 500 worth of furniture She has R5 500 to invest at an interest rate of 10% per year, compounded monthly How long, in years, will it take for her R5 500 to grow to R7 500?

[1] 37 years

[2] 4 years

[3] 38 years

[4] 3 years

[5] None of the above

References:

1. QMI1500 & BNU1501 Study guide and tutorial letters
2. QMI1500 & BNU1501 Past Exam papers