Additional exercises

1 Additional exercises

1.1 Additional exercise 1

Content: Chapter 2

1. At what simple interest rate must R2 000 be invested to accumulate to R4 640 at the end of 10 years?
   [1] 23,2%.
   [2] 10,0%.
   [3] 13,2%.
   [4] 5,69%.
   [5] None of the alternatives listed above.

2. What was the present value of a loan on 5 May if it accumulates to R4 500 on 16 August of the same year at a simple interest rate of 15% per year?
   [1] R4 315,55.
   [2] R4 318,96.
   [3] R4 317,26.
   [4] R4 688,63.
   [5] None of the alternatives listed above.

3. Lulu wants to buy a television set that costs R3 500. She approaches a bank, which agrees to lend her the money, if she repays the loan in 10 months' time. What size loan should she apply for if the bank charges 13% per year bank discount on short-term loans?
   [1] R3 879,17.
   [2] R3 120,83.
   [3] R3 500,00.
   [4] R3 925,23.
   [5] None of the alternatives listed above.

4. What is the equivalent simple interest rate r in question 3?
   [1] 14,58%.
   [2] 13,00%.
   [3] 12,99%.
   [4] 11,73%.
   [5] None of the alternatives listed above.

5. Joseph owes R500 due in four months' time and R700 due in nine months' time. What single payment in 12 months' time will liquidate these obligations if a simple interest rate of 11% per year is charged on all the amounts?
   - [1] R1 255,92.
   - [2] R1 276,08.
   - [3] R1 228,42.
   - [4] R1 200,00.
   - [5] None of the alternatives listed above.
6. Joseph (mentioned in question 5), has a debt of R500 payable in four months' time and R700 in nine months' time from now, will receive R1 000 from his grandmother two months from now, with which he will immediately pay off against his debts. The single payment in 12 months' time from now that will liquidate his obligations, if a simple interest rate of 11% per year is charged on all the amounts, is
   - [1] R200,00.
   - [2] R190,44.
   - [3] R164,25.
   - [4] R228,42.
   - [5] none of the alternatives listed above.
7. Mario has been given the option of either paying his R2 500 bill now or settling it for R2 730 after four months. If he chooses to pay after four months, the simple interest rate which he would be charged is
   - [1] 0,276%.
   - [2] 2,3%.
   - [3] 25,27%.
   - [4] 27,60%.
   - [5] none of the alternatives listed above.
8. The amount of capital needed to yield R300 simple interest in 18 months' time if the interest rate is 9,5% per year, equals
   - [1] R2 105,26.
   - [2] R2 850,00.
   - [3] R4 275,00.
   - [4] R4 736,84.
   - [5] none of the alternatives listed above.
9. The simple interest rate that is equal to a discount rate of 12% per year for a period of 18 months is
   - [1] 0,12%.
   - [2] 10,34%.
   - [3] 12,00%.
   - [4] 14,63%.
   - [5] none of the alternatives listed above.

The solutions to these questions are to be found on p 20 of this tutorial letter.

1.2 Additional exercise 2

Content: Chapter 3

1. Mrs Naidoo deposits R15 000 into a new savings account. How much money will she have in the bank after three years if interest is compounded monthly at 8% per year?

   [1] R15 302,00

   [2] R19 053,56

   [3] R18 895,68

   [4] R18 204,44

   [5] None of the alternatives listed above.

2. How long will it take R25 000 to accumulate to R30 835,42 if interest is compounded weekly at 10,5% per year?

   [1] 2 weeks

   [2] 24 weeks

   [3] 52 weeks

   [4] 104 weeks

   [5] None of the alternatives listed above.

3. At what yearly rate, compounded semi-annually, should Mary invest R20 000 in order to have R28 000 available after 30 months?

   [1] 13,92%

   [2] 2,26%

   [3] 13,5%

   [4] 6,96%

   [5] None of the alternatives listed above.

4. John deposits R900 into a savings account paying $6^1/_2$% interest per year compounded quarterly. After three and a half years he withdraws R1 000 from the account and deposits it into another account paying 11% interest per year compounded semi-annually. How much is the total amount accrued from both accounts two years after making the second deposit?

   [1] R1 366,67

   [2] R2 138,82

   [3] R1 384,27

   [4] R2 227,85

   [5] None of the alternatives listed above.

5. What is the effective interest rate of a nominal rate of 18,75% per year compounded every three months?

   [1] 19,95%

   [2] 20,11%

   [3] 26,52%

   [4] 18,75%

   [5] None of the alternatives listed above.

6. Ben deposits R8 000 into a savings account on 18 April. Ben's money earns 16,5% interest per year compounded half-yearly credited on 1 June and 1 December every year. How much money will he have in the bank at the end of nine months, if simple interest is used for odd periods and compound interest for the full terms?

   [1] R9 023,90

   [2] R9 020,16

   [3] R8 832,25

   [4] R9 015,91

   [5] None of the alternatives listed above.

7. How much money will Ben (see question 6) have in the bank after nine months, if fractional compounding is used for the full term?

   [1] R9 013,08

   [2] R8 746,95

   [3] R9 005,26

   [4] R9 685,76

   [5] None of the alternatives listed above.

8. Fatima wants to buy a hi-fi set. She has three options when borrowing the R2 500 from the bank:

   (A) 16 % per year compounded monthly

   (B) 19% per year compounded semi-annually

   (C) 17% per year compounded annually

   Use continuous rates to decide which is the better option for Fatima.

   [1] A

   [2] B

   [3] C

   [4] B and C

   [5] None of the alternatives listed above.

9. Nkosi owes James R500 due in two months, R1 000 due in five months, R1 500 due in eight months and R200 due in 18 months. He wants to discharge his obligations by three equal payments, one due in three months, one due in six months and one due in 10 months. Calculate the payments if interest is compounded monthly at 15% per year and the end of 10 months is taken as the comparison date.

   Which answer is correct?

   [1] R1 111,71

   [2] R1 061,54

   [3] R1 074,21

   [4] R1 172,46

   [5] None of the alternatives listed above.

The solutions to these questions are to be found on p 24 of this tutorial letter.

1.3 Additional exercise 3

Content: Chapters 4 and 5

1. On his 35th birthday Ken decides to put away an amount of R400 at the end of every month towards a retirement annuity that earns interest at 10% per year, compounded monthly. The balance of Ken's account on his 50th birthday is

   [1] R165 788,14.

   [2] R6 929 756,31.

   [3] R37 222,98.

   [4] none of the alternatives listed above.

2. What is the present value of the annuity in question 1?

   [1] R165 788,14.

   [2] R6 929 756,31.

   [3] R37 222,98.

   [4] None of the alternatives listed above.

3. Susan decides to rent a television set for 20 weeks because her "soapies" and her husband's cricket viewing coincide. She has enough money in a savings account to cover the cost of renting a television set. She has two options: firstly to pay an amount of R500 upfront or secondly to pay R25 a week for 20 weeks, paid in advance. She will pay either of the two options out of her savings account, which earns interest at 14% per year compounded weekly. The amount of money that she can save if she chooses option 2 is

   [1] R476,45.

   [2] R12,55.

   [3] R500.

   [4] none of the alternatives listed above.

4. A businessman buys R100 000 of equipment on the following terms:

   Interest will be charged at a rate of 12% per year compounded semi-annually, but no payment will be made until two years after purchase. Thereafter equal semi-annual payments will be made for five years. The semi-annual payment is

   [1] R29 970,75.

   [2] R13 586,80.

   [3] R22 343,84.

   [4] R17 153,02.

   [5] none of the alternatives listed above.

Questions 5, 6 and 7 relate to the following situation:

Magopi wants to buy a townhouse that costs R200 000. After making a down payment of 20% of the price of the townhouse, he manages to secure a loan for 20 years from the Nest Tree Bank at an interest rate of 15% compounded monthly.

5. His monthly payment equals

   [1] R1 952,29.

   [2] R2 106,86.

   [3] R2 130,15.

   [4] R2 633,58.

   [5] none of the alternatives listed above.

6. Mogapi's equity in the townhouse after twelve years equals

   [1] R53 244,87.

   [2] R59 624,72.

   [3] R64 530,56

   [4] R82 595,95.

   [5] none of the alternatives listed above.

7. After twelve years the bank decides to adjust the interest rate to 12% per year, compounded monthly. Mogapi's new monthly payment equals

   [1] R1 908,15.

   [2] R2 281,50.

   [3] R2 385,19.

   [4] R2 201,76.

   [5] none of the alternatives listed above.

The solutions to these questions are to be found on p 28 of this tutorial letter.

1.4 Additional exercise 4

Content: Chapter 6

1. An investor must choose between three alternative proposals A, B and C. The initial investment outlay and the cash inflows for each are set out in the table below. His cost of capital is K = 18%. Use the internal rate of return, the net present value and the profitability index respectively to advise him with regard to the three proposals. All funds are in R1 000s.

Year	**Proposal *A***	**Proposal *B***	**Proposal *C***
0	**Investment: 500**	**Investment: 500**	**Investment: 550**
	Cash inflow	Cash inflow	Cash inflow
1	200	200	280
2	280	250	280
3	300	280	280
4	200	300	280
5	180	280	280

2. John Modise has an opportunity to invest in a construction company. The company foresees the following cash flows during the next four years.

Years	**Cash Flow**
0	−200
1	100
2	−200
3	400
4	400

Money can be borrowed at 15% per year while an investment, can earn 18% interest per year in a high risk development. Considering the MIRR criterion, what advice will you give him in connection with his possible investment.

The solutions to these questions are to be found on p 30 of this tutorial letter.

1.5 Additional exercise 5

Content: Chapter 7

1. Consider the following bond: Bond XXX:

Coupon rate (half yearly)	14,7% per year
Redemption date	1 January 2028
Yield to maturity	13,5% per year
Settlement date	18 April 2013

   Calculate the all-in price, the accrued interest and the clean price.

2. Calculate the all-in price, the accrued interest and the clean price for the bond in question one on the settlement date 24 June 2013.

3. Consider the following bond:

Bond AAA	
Coupon	11%
Redemption date	1 March 2026
Yield to maturity	13,75%
Settlement date	28 September 2011

   The bond is sold at

   [1] discount.

   [2] premium.

   [3] par.

   [4] none of the alternatives listed above.

4. Consider the following bond: BBB

Coupon rate	7,5%
Redemption date	1 April 2028
Yield to maturity	14%
Settlement date	28 September 2015

   The clean price is given by

   [1] R38,56812%

   [2] R61,03717%

   [3] R71,91128%

   [4] none of the alternatives listed above.

The solutions to these questions are to be found on p 32 of this tutorial letter.

1.6 Additional exercise 6

Content: Typical exam questions

Question 1

Fifteen months from now Jenny has to pay Jonas R5 000. She decides to pay him back earlier. If a simple interest rate of 13% per annum is applicable, then the amount that Jenny will have to pay Jonas seven months from now equals

[1] R4 566,67
[2] R4 601,23
[3] R4 627,24
[4] R4 655,94
[5] none of the above

Question 2

If R17 500 accumulates to R22 000 after 53 months, the continuous compounding interest rate equals

[1] 5,181%
[2] 5,193%
[3] 5,318%
[4] 5,822%
[5] none of the above

Questions 3 and 4 relate to the following situation:
Dube invested an amount of money at j% interest per annum, compounded half yearly. After four years the accumulated amount of R2 844,20 was reinvested at 1,5% per annum more than previously while the compounding period became quarterly.

Question 3

If, after a further three years, the accumulated amount was R3 881,49, then the original interest rate equals

[1] 9%
[2] 10,5%
[3] 11,37%
[4] 12%
[5] none of the above

Question 4

The original amount invested equals

[1] R1 784,49
[2] R1 827,48
[3] R1 888,79
[4] R2 000,00
[5] none of the above

Question 5

The settlement date of Bond E534 with a coupon rate of 14,7% per annum and a yield to maturity of 13,5% per annum is 18 April 2013. If the equation of the price on the next coupon date is

$$P = da_{\overline{n}|z} + 15{,}04289$$

and the all-in price equation is

$$P(18/04/2013) = 107{,}55174(1{,}0675)^{-74/182},$$

then the maturity date will be

[1] 18 April 2027
[2] 1 July 2027
[3] 18 October 2027
[4] 1 January 2028
[5] none of the above

Question 6

Francois's friend James has decided to open a rugby jersey shop next to Francois's rugby ball stall. Starting on 1 January he invested a monthly amount into an account earning 9,4% interest per annum compounded monthly. If he had R250 000 in this account at the end of September of the same year, his monthly deposit equalled approximately

[1] R23 123,46
[2] R24 131,36
[3] R26 709,50
[4] R26 918,73
[5] none of the above

Question 7

The equation for the present value of Bond ABC on 8/12/2015 is

$$P(8/12/2015) = \tfrac{5}{2}a_{\overline{6}|0{,}12\div 2} + 100(1 + \tfrac{0{,}12}{2})^{-6}$$

with

$$f = \frac{33}{184}$$

and the accrued interest equal to R2,06849%.
The clean price of Bond ABC equals

[1] R79,86029%
[2] R82,33420%
[3] R84,40269%
[4] R86,41118%
[5] none of the above

Questions 8 and 9 relate to the following situation:
Clever Chris took out an endowment policy with an annual payment of R6 500 that increases each year by R1 700.

Question 8

If money is worth 10% per annum, then after 20 years the policy is worth

[1] R200 068,74
[2] R459 257,94
[3] R874 574,99
[4] R1 005 962,49
[5] none of the above

Question 9

The amount of interest earned by the policy equals approximately

[1] R162 300
[2] R453 000
[3] R390 000
[4] R552 960
[5] none of the above

Question 10

Adriana's investment with an initial outlay of R225 000 returns a constant cash flow of R36 000 per annum for 15 years. The internal rate of return on the investment equals

[1] 6,01%

[2] 6,25%

[3] 13,65%

[4] 14,46%

[5] none of the above

Questions 11 and 12 relate to the following situation:
The following table supplies data of the average interest rates and corresponding number of policies sold.

Average interest rate (x)	*Number of policies sold* (y)
13,00%	*200*
13,25%	*250*
14,00%	*300*
14,50%	*450*
15,00%	*450*
16,00%	*300*
17,00%	*200*
17,50%	*150*
18,00%	*120*

Question 11

The arithmetic mean of the average interest rate equals

[1] 13,825%

[2] 15,00%

[3] 15,36%

[4] 15,5%

[5] none of the above

Question 12

The relationship between the average interest rates and number of policies sold can be represented by the regression line

[1] $y = -5\,000 + 400x$

[2] $y = -3{,}67 + 15{,}67x$

[3] $y = 17{,}26 - 0{,}00705x$

[4] $y = 718{,}35 - 29{,}26x$

[5] none of the above

Questions 13 and 14 relate to the following situation:
When Caitlyn was born her grandma decided that she would like to give Caitlyn R100 000 on her 21st birthday. For 10 years grandma could manage, every six months, to pay an amount into an account earning 9,25% per annum, compounded half yearly. After 10 years when the interest rate changed to 8% per annum, compounded monthly, grandma stopped paying money into the account but left the money there.

Question 13

The balance in the account when the interest rate changed to 8% per annum, compounded monthly, equals

[1] R36 984,49

[2] R41 599,60

[3] R42 888,29

[4] R53 191,49

[5] none of the above

Question 14

The half-yearly payment into the account equals

[1] R1 242,00

[2] R1 308,77

[3] R1 349,31

[4] R1 673,46

[5] none of the above

The solutions to these questions are to be found on p 36 of this tutorial letter.

1.7 Additional exercise 7

Content: Typical exam questions

Question 1

Ipi invested R16 000 in an account on 21 February. It will accumulate to R16 570,08 on 7 December of the same year. The simple interest rate applicable is

[1] 3,44%
[2] 3,56%
[3] 4,3%
[4] 4,4%
[5] 4,5%

Question 2

The continuous compounding rate for an effective rate of 16,13% is

[1] 16,13%
[2] 17,5%
[3] 19,12%
[4] 21,076%
[5] none of the above

Question 3

Three years ago Jack borrowed R7 000 from Jill at 11% per year compounded quarterly. Eighteen months ago he borrowed another R9 000 at 9% per year compounded monthly. The amount that Jack will owe Jill three years from now equals

[1] R26 654,00
[2] R26 896,73
[3] R26 936,17
[4] R30 682,02
[5] none of the above

Questions 4 and 5 relate to the following situation:
Ian wants to open a bicycle shop at the Bi-a-Ride complex. He estimates that he will have R250 000 available on 16 June from an investment made on 7 September of the previous year, earning 7,65% interest per year compounded on the 1st day of each month.

Question 4

If simple interest is used for odd periods and compound interest for the full terms, then the amount that Ian deposited on 7 September equals

[1] R235 677,98
[2] R235 727,23
[3] R264 776,03
[4] R265 192,37
[5] none of the above

Question 5

If fractional compounding is used for the whole period, then the amount that Ian deposited on 7 September equals

[1] R235 635,69
[2] R235 679,96
[3] R236 048,56
[4] R264 776,03
[5] R265 190,14

Questions 6 and 7 relate to the following situation:
Paul took out an endowment policy. Its initial annual payment of R6 000 will increase annually by R1 500. The policy matures in 30 years. Its expected annual interest rate is 9,70%.

Question 6

The amount that Paul can expect to receive after 30 years equals

[1] R932 583,78
[2] R1 165 729,72
[3] R2 872 232,69
[4] R3 336 150,21
[5] none of the above

Question 7

The total amount paid for the policy equals

[1] R120 575,78
[2] R223 500,00
[3] R225 000,00
[4] R832 500,00
[5] none of the above

Question 8

An investment with an initial outlay of R500 000 generates five successive annual cash inflows of R75 000, R190 000, R40 000, R150 000 and R180 000 respectively. The internal rate of return (IRR) equals

[1] 7,78%
[2] 9,48%
[3] 21,3%
[4] 27,0%
[5] none of the above

Questions 9, 10 and 11 relate to the following situation:
The following is an extract from the amortisation schedule of the home loan of John Pele:

Month	Outstanding principal at month beginning	Interest due at month end	Monthly payment	Principal repaid	Principal outstanding at month end
147	R8 155,83	A	R2 080,54	R2 014,27	R6 141,56
148	R6 141,56	R49,90	R2 080,54	R2 030,64	B
149	B	R33,40	R2 080,54	R2 047,14	R2 063,78
150	R2 063,78	R16,77	R2 080,54	R2 063,77	R0

Question 9

The value of A equals

[1] R41,65
[2] R49,50
[3] R66,27
[4] R166,33
[5] R167,86

Question 10

The value of B equals

[1] R4 061,02

[2] R4 077,79

[3] R4 094,21

[4] R4 110,92

[5] R4 127,68

Question 11

If the interest rate has never changed, the original amount of John Pele's home loan was (rounded to the nearest thousand rand)

[1] R21 000,00

[2] R180 000,00

[3] R310 000,00

[4] R312 000,00

[5] R606 000,00

Questions 12 and 13 relate to the following situation:
Consider Bond ABC.

Coupon rate:	*9,91% per year*
Yield to maturity:	*7,47% per year*
Settlement date:	*17 June 2013*
Maturity date:	*11 January 2042*

Question 12

The all-in-price equals

[1] R127,85047%

[2] R128,00061%

[3] R128,47362%

[4] R128,62450%

[5] R132,93158%

Question 13

The clean price of Bond ABC equals

[1] R127,34899%
[2] R127,97288%
[3] R128,50209%
[4] R128,66892%
[5] R129,27612%

Questions 14 and 15 relate to the following situation:
The following table supplies data of the inflation rate and the corresponding prime lending rate during the same time period.

Inflation rate (%)(x)	*Prime lending rate (%)(y)*
3,3	*5,2*
6,2	*8,0*
11,0	*10,8*
9,1	*7,9*
5,8	*6,8*
6,5	*6,9*
7,6	*9,0*

Question 14

The linear relationship between the inflation rate and the prime lending rate can be represented by the regression line

[1] $y = 3{,}17477 + 0{,}65407x$
[2] $y = 0{,}65407 + 3{,}17477x$
[3] $y = -2{,}76656 + 1{,}26128x$
[4] $y = 1{,}26128 - 2{,}76656x$
[5] $y = 2{,}28372 + 0{,}88372x$

Question 15

The correlation coefficient equals

[1] −0,908
[2] +0,495
[3] +0,546
[4] +0,908
[5] none of the above

The solutions to these questions are to be found on p 40 of this tutorial letter.

Solutions

2 Solutions: additional exercises

2.1 Solution: additional exercise 1

1. Simple interest: $I = Prt$

$$\begin{aligned} I &= \text{interest earned} = \text{R4 640} - \text{R2 000} = \text{R2 640} \\ P &= \text{principal amount} = \text{R2 000} \\ r &= \text{simple interest rate} = ? \\ t &= \text{period of investment} = 10 \text{ years} \end{aligned}$$

$$\begin{aligned} I &= Prt \\ 2\,640 &= 2\,000 \times r \times 10 \\ \tfrac{2640}{2000 \times 10} &= r \\ r &= 0{,}132 \\ r &= 13{,}2\%. \end{aligned}$$

OR

$$\begin{aligned} S &= P\,(1 + rt) \\ 4\,640 &= 2\,000\,(1 + r \times 10) \\ \tfrac{4\,640}{2\,000} &= 1 + 10r \\ 10r &= \tfrac{4\,640}{2\,000} - 1 \\ 10r &= 1{,}32 \\ r &= 0{,}132 \\ r &= 13{,}2\%. \end{aligned}$$

R2 000 must be invested at an interest rate of 13,2% to accumulate to R4 640 at the end of ten years.

2. Simple interest: $S = P(1 + rt)$

$$\begin{aligned} S &= \text{accumulated amount} = \text{R4 500} \\ P &= \text{principal or present value} = ? \\ r &= \text{simple interest rate} = 0{,}15 \\ t &= \text{period of loan} = \text{from 5 May tot 16 August} \end{aligned}$$

Period	**Number of days**
5 - 31 May	27 (including 5th)
1 - 30 June	30
1 - 31 July	31
1 - 16 August	15 (excluding 16th)
	103 days

OR

Use the number of each day of the year table. Day number 228 (16 August) minus 125 (5 May) equals 103.

$$\begin{aligned} S &= P(1 + rt) \\ P &= \tfrac{S}{1+rt} \\ &= \tfrac{4500}{1+0{,}15\times\frac{103}{365}} \\ &= 4\,317{,}26. \end{aligned}$$

The present value is R4 317,26.

3. Discount: $P = S(1 - dt)$.

$$\begin{aligned} P &= \text{present value} = \text{money she will receive} = \text{R3 500} \\ S &= \text{future value} = \text{money she applies for} = ? \\ d &= \text{discount rate} = 0{,}13 \\ t &= \text{period of loan} = 10 \text{ months} = \tfrac{10}{12}\text{years.} \end{aligned}$$

$$\begin{aligned} P &= S(1-dt) \\ S &= \tfrac{P}{1-dt} \\ S &= \tfrac{3\ 500}{(1-0{,}13\times\frac{10}{12})} \\ &= 3\,925{,}23 \end{aligned}$$

She has to apply for a loan of R3 925,23.

4.

$$\begin{aligned} I &= 3\,925{,}23 - 3\,500 \\ &= 425{,}23 \end{aligned}$$

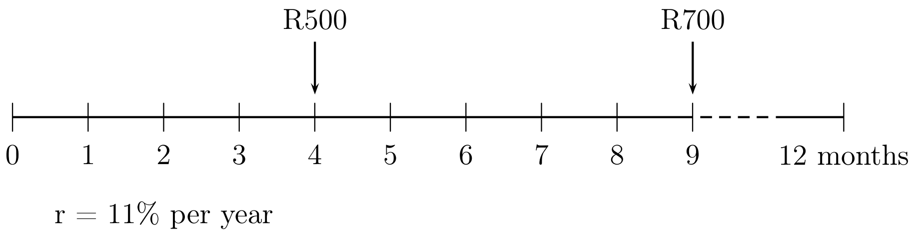
The interest paid in question 3 is R425,23.

Thus the interest rate equivalent to the above interest can be calculated using $I = Prt$.

$$\begin{aligned} I &= Prt \\ 425{,}23 &= 3\,500 \times r \times \tfrac{10}{12} \\ \tfrac{425{,}23}{3500} \times \tfrac{12}{10} &= r \\ r &= 0{,}1458 \end{aligned} \qquad \text{OR} \qquad \begin{aligned} r &= \tfrac{d}{1-dt} \\ &= \tfrac{0{,}13}{1-0{,}13} \times \tfrac{10}{12} \\ &= 0{,}1458. \end{aligned}$$

The equivalent interest rate is 14,58%.

5.



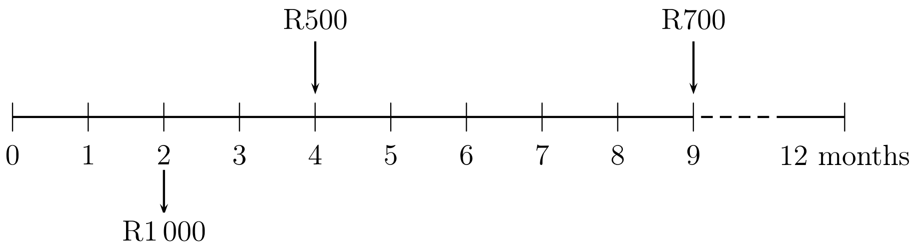
What he owes he must pay back. But because of the time value of money we have to calculate the value of all his obligations and payments on the same date namely month 12. The R500 must move eight months forward and the R700, three months forward.

R500's value at month 12: $500(1 + 0{,}11 \times \frac{8}{12}) = 536{,}67$.

R700's value at month 12: $700(1 + 0{,}11 \times \frac{3}{12}) = 719{,}25$.

Thus he owes R1 255,92 $(536{,}67 + 719{,}25)$ at month 12. He has to make a single payment of R1 255,92 in 12 months' time.

6.



What he owes he must pay back. But because of the time value of money we have to calculate the value of all his obligations and payments on the same date, that is at month 12.

R500 must be moved eight months forward:

$$500\left(1+0{,}11\times\frac{8}{12}\right)=536{,}67$$

R700 must be moved three months forward:

$$700\left(1+0{,}11\times\frac{3}{12}\right)=719{,}25$$

R1 000 must be moved 10 months forward:

$$1\ 000\left(1+0{,}11\times\frac{10}{12}\right)=1\ 091{,}67$$

Thus the amount that he still has to pay at month 12:

Obligations − payments made is R164,25 (536,67 + 719,25 − 1 091,67)

7.

$$\begin{aligned}
I &= Prt\\
\text{Use } I &= 2\,730 - 2\,500 = 230,\\
P &= 2\,500 \text{ and}\\
t &= \tfrac{4}{12}.\\
\text{From } I &= Prt \quad \text{it follows that}\\
230 &= 2\,500\times\tfrac{4}{12}\times r\\
r &= \tfrac{230\times 12}{2\,500\times 4}\\
&= 27{,}6\%.
\end{aligned}$$

Mario will pay 27,60% simple interest.

8.

$$\begin{aligned} I &= \text{Pr}\,t \\ \text{with} \quad I &= 300, \\ r &= 0{,}095 \quad \text{and} \\ t &= \tfrac{18}{12}. \end{aligned}$$

$$\begin{aligned} \text{Now} \quad 300 &= P \times 0{,}095 \times \tfrac{18}{12} \\ P &= \frac{300 \times 12}{0{,}095 \times 18} \\ &= 2\,105{,}26. \end{aligned}$$

The amount needed is R2 105,26.

9.

$$\begin{aligned} D &= Sdt \\ \text{with} \quad S &= 100, \\ d &= 0{,}12 \quad \text{and} \\ \text{t} &= \frac{18}{12}. \end{aligned}$$

$$\begin{aligned} \text{Now} \quad D &= 100 \times 0{,}12 \times \frac{18}{12} \\ &= 18{,}00 \end{aligned}$$

In order to determine the applicable interest rate we use

$$\begin{aligned} I &= Prt \\ \text{with} \quad I &= 18, \\ P &= (100 - 18) \quad \text{and} \\ \text{t} &= \frac{18}{12}. \end{aligned}$$

$$\begin{aligned} \text{Therefore} \quad r &= \frac{18 \times 12}{(100 - 18) \times 18} \\ &= 14{,}63. \end{aligned}$$

OR

$$\begin{aligned} r &= \frac{d}{1 - dt} \\ &= \frac{0{,}12}{1 - 0{,}12} \times \frac{18}{12} \\ &= 0{,}1463. \end{aligned}$$

The applicable interest rate is 14,63%.

2.2 Solution: additional exercise 2

1. Compound interest: $S = P\left(1+\frac{j_m}{m}\right)^{tm}$

$$\begin{aligned} S &= \text{Future value} = ? \\ P &= \text{Present value} = \text{R15 000} \\ j_m &= \text{interest rate per year} = 0{,}08 \\ m &= \text{number of compounding periods per year} = 12 \\ \text{t} &= \text{term of investment} = 3 \text{ years.} \end{aligned}$$

$$\begin{aligned} S &= P\left(1+\frac{j_m}{m}\right)^{tm} \\ &= 15\,000\left(1+\frac{0{,}08}{12}\right)^{3\times 12} \\ &= 19\,053{,}56 \end{aligned}$$

She will have R19 053,56 in the bank after three years.

2. Compound interest:

$$\begin{aligned} S &= \text{R30 835,42} \\ P &= \text{R25 000,00} \\ j_m &= 0{,}105. \\ m &= 52 \\ t &= ? \end{aligned}$$

$$\begin{aligned} S &= P\left(1+\frac{j_m}{m}\right)^{tm} \\ 30\,835{,}42 &= 25\,000\left(1+\frac{0{,}105}{52}\right)^{52t} \\ \frac{30\,835{,}42}{25\,000} &= \left(1+\frac{0{,}105}{52}\right)^{52t} \\ \ln\left(\frac{30\,835{,}42}{25\,000}\right) &= 52t\ \ln\left(1+\frac{0{,}105}{52}\right) \\ \frac{\ln\left(\frac{30\,835{,}42}{25\,000}\right)}{\ln\left(1+\frac{0{,}105}{52}\right)} &= 52t \\ 52t &= 104 \\ t &= 2 \end{aligned}$$

It will take $2 \times 52 = 104$ weeks for R25 000 to accumulate to R30 835,42.

3. Compound interest: $S = P\left(1+\frac{j_m}{m}\right)^{tm}$.

$$\begin{aligned} S &= \text{R28 000} \\ P &= \text{R20 000} \\ j_m &= \text{interest rate per year} = ? \\ m &= \text{number of compounding periods per year} = 2 \\ t &= \text{term of investment} = 30 \text{ months} = \text{two and a half years} \end{aligned}$$

$$
\begin{aligned}
S &= P\left(1+\tfrac{j_m}{m}\right)^{tm} \\
28\,000 &= 20\,000\left(1+\tfrac{j_m}{2}\right)^{2{,}5\times 2} \\
\tfrac{28\,000}{20\,000} &= \left(1+\tfrac{j_m}{2}\right)^{5} \\
\left(\tfrac{28\,000}{20\,000}\right)^{\left(\frac{1}{5}\right)} &= 1+\tfrac{j_m}{2} \\
\left(\tfrac{28\,000}{20\,000}\right)^{\left(\frac{1}{5}\right)}-1 &= \tfrac{j_m}{2} \\
j_m &= \left(\left(\tfrac{28\,000}{20\,000}\right)^{\left(\frac{1}{5}\right)}-1\right)\times 2 \\
&= 13{,}92\%
\end{aligned}
$$

Mary should invest the R20 000 at a yearly rate of 13,92% compounded semi-annually.

4. First we calculate the value of R900 after three and a half years.

   With $P = 900$, $j_m = 0{,}065$, $m = 4$ and $t = 3{,}5$.

$$900\left(1+\frac{0{,}065}{4}\right)^{4\times 3{,}5} = \text{R}1\,127{,}85$$

   Of the R1 127,85, R1 000 must be invested for two years at 11% compounded semi-annually and R127,85 (1 127,85 − 1 000) must be invested for two years at $6\frac{1}{2}\%$ compounded quarterly.

   After two years R1 000 will accumulate to:

$$
\begin{aligned}
S &= P\left(1+\tfrac{j_m}{m}\right)^{tm} \\
&= 1\,000\left(1+\tfrac{0{,}11}{2}\right)^{2\times 2} \\
&= 1\,000\left(1+\tfrac{0{,}11}{2}\right)^{4} \\
&= 1\,238{,}82
\end{aligned}
$$

   After two years R127,85 will accumulate to:

$$
\begin{aligned}
S &= 127{,}85\left(1+\tfrac{0{,}065}{4}\right)^{4\times 2} \\
&= 127{,}85\left(1+\tfrac{0{,}065}{4}\right)^{8} \\
&= 145{,}45
\end{aligned}
$$

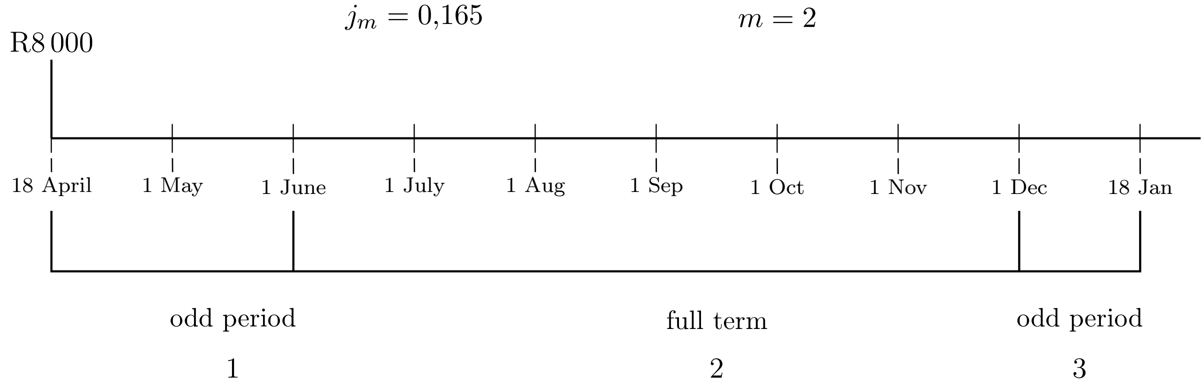
   His total accrued amount after two years of making the second deposit is R1 384,27 (1 238,82 + 145,45).

5. Effective interest rate: $J_{eff} = 100\left[\left(1+\frac{j_m}{m}\right)^{m}-1\right]$

   j_m = nominal rate = 0,1875
   m = number of times the interest is calculated per year $=\frac{12}{3}=4$

$$
\begin{aligned}
J_{eff} &= 100[(1+\tfrac{0{,}1875}{4})^4-1] \\
&= 20{,}11\%
\end{aligned}
$$

6.



Period 1: odd period of 13 + 31 = 44 days (Day number 152 (1 June) minus 108 (18 April) = 44)

Period 2: one full term = one half year (1 June - 1 Dec)

Period 3: odd period of 31 + 17 = 48 days

Value of R8 000 on 1 June:

$$\begin{aligned} S_1 &= P(1+rt) \\ &= 8\,000\left(1+\tfrac{44}{365}\times 0{,}165\right) \end{aligned}$$

Value of R8 000 on 1 December: ($t = \frac{6}{12}$ and $m = 2$)

$$\begin{aligned} S_2 &= S_1\left(1+\frac{j_m}{m}\right)^{tm} \\ &= (\text{value on 1 June}) \times \left(1+\frac{0{,}165}{2}\right)^{\left(\frac{6}{12}\times 2\right)} \end{aligned}$$

Value of R8 000 on 18 January:

$$\begin{aligned} S_3 &= S_2(1+rt) \\ &= (\text{value on 1 December}) \times \left(1+\tfrac{48}{365}\times 0{,}165\right) \end{aligned}$$

Thus value of R8 000 on 18 January:

$$\begin{aligned} S &= 8\,000\left(1+\tfrac{44}{365}\times 0{,}165\right)\left(1+\frac{0{,}165}{2}\right)\left(1+\tfrac{48}{365}\times 0{,}165\right) \\ &= 9\,023{,}90 \end{aligned}$$

He will have R9 023,90 in the bank at the end of nine months.

7. $S = P\left(1+\frac{j_m}{m}\right)^{tm}$

$$\begin{aligned} j_m &= \text{interest rate} = 0{,}165 \\ m &= \text{number of compounding periods per year} = 2 \\ t &= \text{term of investment} \\ &= \text{one compounding period of six months} \\ & \text{plus the number of odd days express as a fraction of a year} \\ &= \left(\tfrac{1}{2}+\tfrac{44+48}{365}\right) \end{aligned}$$

$$
\begin{aligned}
S &= 8000\left(1+\frac{0{,}165}{2}\right)^{\left(\frac{6}{12}+\frac{44+48}{365}\right)\times\frac{2}{1}} \\
&= \text{R}9\,013{,}08
\end{aligned}
$$

He will have R9 013,08 in the bank after nine months if fractional compounding is used.

8. Continuous rate: $c = mln\left(1+\frac{j_m}{m}\right)$.

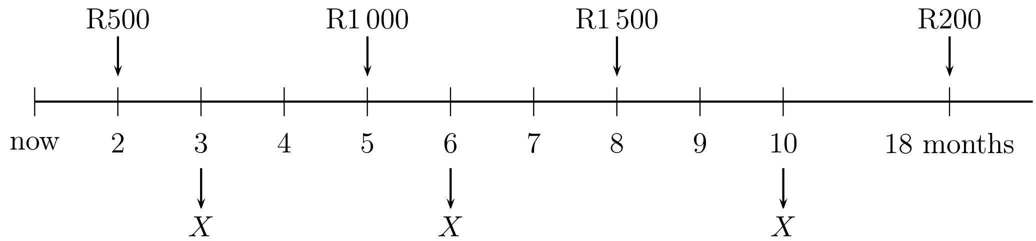
Option A: $c = 12\ln\left(1+\frac{0{,}16}{12}\right) = 15{,}89\%.$

Option B: $c = 2\ln\left(1+\frac{0{,}19}{2}\right) = 18{,}15\%.$

Option C: $c = 1\ln\left(1+\frac{0{,}17}{1}\right) = 15{,}70\%.$

The best option for Fatima is the one with the lowest interest rate, thus option C.

9.



Because of the time value of money all the moneys must be taken to the same date namely the comparison date, month 10.

The values of the obligations at the end of month 10:

For R500: $500 \times \left[1+\frac{0{,}15}{12}\right]^{8}$ $\quad t=\frac{8}{12}$ and $m=12$.

For R1 000: $1\,000 \times \left[1+\frac{0{,}15}{12}\right]^{5}$ $\quad t=\frac{5}{12}$ and $m=12$.

For R1 500: $1\,500 \times \left[1+\frac{0{,}15}{12}\right]^{2}$ $\quad t=\frac{2}{12}$ and $m=12$.

For R200: $200 \times \left[1+\frac{0{,}15}{12}\right]^{-8}$ $\quad t=\frac{-8}{12}$ and $m=12$.

Thus the total obligation at the end of month 10 is:

$$
\begin{aligned}
\text{Obligations} &= 500\left[1+\frac{0{,}15}{12}\right]^{8}+1\,000\left[1+\frac{0{,}15}{12}\right]^{5}+1\,500\left[1+\frac{0{,}15}{12}\right]^{2}+200\left[1+\frac{0{,}15}{12}\right]^{-8} \\
&= 3\,335{,}14
\end{aligned}
$$

The values of the payments at the end of month 10 are:

First payment: $X\left[1+\frac{0{,}15}{12}\right]^7$.

Second payment: $X\left[1+\frac{0{,}15}{12}\right]^4$.

Third payment: X.

Take out the common factor X in all the terms:

$$\begin{aligned}
\text{Payments} &= \text{Obligations}\\
X\left[1+\tfrac{0{,}15}{12}\right]^7 + X\left[1+\tfrac{0{,}15}{12}\right]^4 + X &= 3\,335{,}14 \quad \text{Take out the common factor } X\text{:}\\
X\left[\left(1+\tfrac{0{,}15}{12}\right)^7 + \left(1+\tfrac{0{,}15}{12}\right)^4 + 1\right] &= 3\,335{,}14\\
X &= \frac{3\,335{,}14}{\left(1+\frac{0{,}15}{12}\right)^7+\left(1+\frac{0{,}15}{12}\right)^4+1}\\
&= \frac{3\,335{,}14}{3{,}14180}\\
X &= 1\,061{,}54
\end{aligned}$$

Nkosi has to make three payments of R1 061,54 each.

2.3 Solution: additional exercise 3

1. Future value of an annuity: $S = Rs_{\overline{n}|i}$

   R: payments made = R400

   n: term of payments = 15 years = 15 × 12 months = 180 months

   i: interest rate = 0,10 ÷ 12 (NB Don't round this off.)

$$\begin{aligned}
S &= 400s_{\overline{15\times12}|0{,}10\div12}\\
&= 165\,788{,}14
\end{aligned}$$

   Peter's account balance on his 50th birthday is R165 788,14.

2. Present value of an annuity : $P = Ra_{\overline{n}|i}$

   R: payments made = R400

   n: term of payments = 15 years = 15 × 12 months = 180 months

   i: interest rate = 0,10 ÷ 12

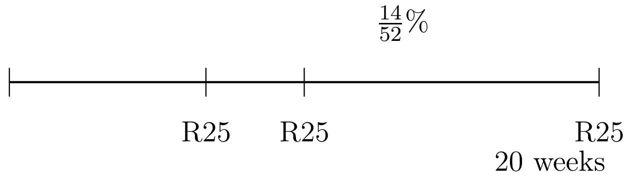
$$\begin{aligned}
P &= 400a_{\overline{15\times12}|0{,}10\div12}\\
&= 37\ 222{,}98
\end{aligned}$$

OR

$$\begin{aligned}
S &= P\left(1+\frac{j_m}{m}\right)^{tm}\\
165\,788{,}14 &= P\left(1+\frac{0{,}10}{12}\right)^{15\times12}\\
P &= 37\,222{,}98
\end{aligned}$$

   The present value of the annuity is R37 222,98.

3.



PV of first option: R500

PV of second option:

$$\begin{aligned} P &= (1+i)\ Ra_{\overline{n}|i} \\ &= \left(1+\frac{0{,}14}{52}\right) 25a_{\overline{20}|0{,}14\div 52} \\ &= 487{,}45 \end{aligned}$$

PV of first option − PV of second option

$$\begin{aligned} &= 500 - 487{,}45 \\ &= 12{,}55 \end{aligned}$$

She can save R12,55 if she chooses option 2.

4. Value of his obligation after two years, thus four half-year periods:

$$\begin{aligned} S &= P\left(1+\frac{j_m}{m}\right)^{tm} \\ &= 100\,000\left(1+\frac{0{,}12}{2}\right)^{2\times 2} \\ &= 126\,247{,}70 \end{aligned}$$

The new value of the loan is now R126 247,70.

Value of semi-annual payments for the next five years, thus $5 \times 2 = 10$ half year periods:

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ 126\ 247{,}70 &= Ra_{\overline{5\times 2}|0{,}12\div 2} \\ &= 17\,153{,}02 \end{aligned}$$

The semi-annual payment is R17 153,02.

5. First work out the amount of the loan for which Mogapi must apply.

20% of R200 000 = R40 000. The amount of the loan is R160 000.

OR

80% of R200 000 equals R160 000.

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ \text{with}\quad P &= 160\,000 \\ n &= 20\times 12 \\ i &= 0{,}15\div 12 \\ R &= ? \end{aligned}$$

$$\begin{aligned} 160\,000 &= Ra_{\overline{20\times 12}|0{,}15\div 12} \\ R &= 2\,106{,}86 \end{aligned}$$

The monthly payments are R2 106,86.

6.

$$\begin{array}{rrcl} & P & = & Ra_{\overline{n}|i} \\ \text{with} & R & = & 2\ 102{,}86 \\ & n & = & (20-12)\times 12 \\ & i & = & 0{,}15 \div 12 \\ & P & = & ? \\ \\ & P & = & 2\ 106{,}86a_{\overline{8\times 12}|0{,}15\div 12} \\ & & = & 117\,404{,}05 \end{array}$$

Mogapi's equity in his townhouse after twelve years equals R82 595,95 $(160\,000 - 117404{,}05 + 40000)$

7.

$$\begin{array}{rrcl} & P & = & Ra_{\overline{n}|i} \\ \text{with} & P & = & 117\ 404{,}05 \\ & n & = & 8\times 12 \\ & i & = & 0{,}12 \div 12 \\ & R & = & ? \\ \\ & 117\,404{,}05 & = & Ra_{\overline{8\times 12}|0{,}12\div 12} \\ & & = & 1\,908{,}15 \end{array}$$

The payments are R1 908,15.

2.4 Solution: additional exercise 4

1. **Internal rate of return:**

$$\text{A: } f(I) = \tfrac{200}{1+I} + \tfrac{280}{(1+I)^2} + \tfrac{300}{(1+I)^3} + \tfrac{200}{(1+I)^4} + \tfrac{180}{(1+I)^5} - 500$$

IRR = 37,7%

$$\text{B: } f(I) = \tfrac{200}{1+I} + \tfrac{250}{(1+I)^2} + \tfrac{280}{(1+I)^3} + \tfrac{300}{(1+I)^4} + \tfrac{280}{(1+I)^5} - 500$$

IRR=40,3%

$$\text{C: } f(I) = \tfrac{280}{1+I} + \tfrac{280}{(1+I)^2} + \tfrac{280}{(1+I)^3} + \tfrac{280}{(1+I)^4} + \tfrac{280}{(1+I)^5} - 550$$

Use IRR = 42,1%

Because $I > K = 18\%$ all three proposals are acceptable.

As C has the highest internal rate of return, we advise to choose C.

Net present value

A: $N = \frac{200}{1{,}18} + \frac{280}{1,18^2} + \frac{300}{1,18^3} + \frac{200}{1,18^4} + \frac{180}{1,18^5} - 500 = 235$

B: $N = \frac{200}{1{,}18} + \frac{250}{1,18^2} + \frac{280}{1,18^3} + \frac{300}{1,18^4} + \frac{280}{1,18^5} - 500 = 297$

C: $P = 280a_{\overline{5}|0{,}18} = 876$

$NPV = 876 - 550 = 326$

Choose C because it has the highest NPV.

OR

$$\begin{aligned} N &= \frac{280}{1{,}18} + \frac{280}{1{,}18^2} + \frac{280}{1{,}18^3} + \frac{280}{1{,}18^4} + \frac{280}{1{,}18^5} - 550 \\ &= 326 \end{aligned}$$

Profitability index

A: $PI = \frac{235+500}{500} = \frac{735}{500} = 1{,}47.$

B: $PI = \frac{\text{NPV+Outlay}}{\text{Outlay}} = \frac{297+500}{500} = 1{,}594$

C: $PI = \frac{\text{NPV+Outlay}}{\text{Outlay}} = \frac{326+550}{550} = 1{,}593$

Recommendation: Choose B.

2. $\text{MIRR} = (C/PV_{out})^{1/n} - 1$

I: Present value of the cash outflows

$$\begin{aligned} PV_{out} &= 200 + 200(1 + 0{,}15)^{-2} \\ &= 351{,}23 \end{aligned}$$

C: Future value of cash inflows

$$\begin{aligned} C &= 100(1 + 0{,}18)^3 + 400(1 + 0{,}18)^1 + 400 \\ &= 1036{,}30 \end{aligned}$$

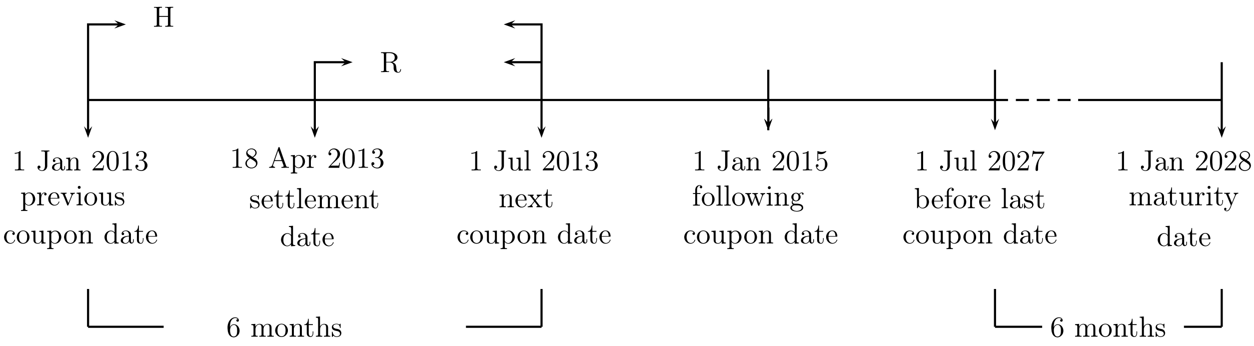
n: number of years $= 4$

$$\begin{aligned} \text{MIRR} &= \left(\frac{C}{PV_{out}}\right)^{\frac{1}{n}} - 1 \\ \text{MIRR} &= \left(\frac{1036{,}30}{351{,}23}\right)^{\left(\frac{1}{4}\right)} - 1 \\ &= 0{,}3106 \\ &= 31{,}06\% \end{aligned}$$

Because 31,06% is larger than the compound interest rate of 18% he is going to earn more by investing in the Construction Company than leaving the money in the bank.

2.5 Solution: additional exercise 5

1.



The number of years = 1/1/2028 − 1/7/2013.

As the months (January and July) aren't the same we must move the next coupon date to the following coupon date. Thus 1/7/2013 becomes 1/1/2015.

$$\begin{aligned} \text{Years} &= (1/1/2028 - 1/1/2015) \\ &= 14 \end{aligned}$$

We must now multiply by two to get the half yearly coupons.

$$\begin{aligned} n &= 14 \times 2 \\ &= 28 \end{aligned}$$

We must now add the period 1/7/2013 to 1/1/2015 that is one coupon.

$$\begin{aligned} n &= 28 + 1 \\ &= 29 \end{aligned}$$

The number of days from the settlement date 18 April, until the next coupon date 1 July, is R: The day number 182 (1 July) minus 108 (18 April) equals 74 thus R = 74.

The number of days in the half year in which the settlement date falls (1/1/2013 to 1/7/2013) is H: The day number 182 (1 July) minus 1 (1 January) equals 181, thus $H = 181$.

The present value of the bond on 1/7/2013 is:

$$\begin{aligned} P &= da_{\overline{n}|z} + 100(1+z)^{-n} \\ &= \tfrac{14{,}7}{2} a_{\overline{29}|0{,}135 \div 2} + 100\left(1 + \tfrac{0{,}135}{2}\right)^{-29} \\ &= 107{,}55174 \end{aligned}$$

As the settlement date is more than ten days from the next coupon date we must add the coupon.

$$\begin{aligned} P(1/7/2013) &= 107{,}55174 + 7{,}35 \\ &= 114{,}90174 \end{aligned}$$

We must now discount the present value of the bond back to the settlement date to obtain the all-in-price.

$$\begin{aligned}\text{All-in price} &= 114{,}90174\left(1+\tfrac{0{,}135}{2}\right)^{-\left(\frac{74}{181}\right)}\\ &= 111{,}87388\end{aligned}$$

The All-in price is R111,87388%.

$$\begin{aligned}\text{Accrued interest} &= \tfrac{H-R}{365}\times c\\ &= \tfrac{181-74}{365}\times 14{,}7\\ &= 4{,}30932\end{aligned}$$

$$\begin{aligned}\text{Clean price} &= \text{All-in price} - \text{accrued interest}\\ &= 111{,}87388 - 4{,}30932\\ &= 107{,}56456\end{aligned}$$

The clean price is R107,56456%.

2. The present value of 1 July 2013 is

   P = R107,55174%. (see question one)

   As the settlement date 24 June 2013 is less than ten days from the next coupon date we don't add the coupon. It is an ex-interest case. We must discount the present value of the bond on 1 July 2013 back to the settlement date of 24 June 2013.

   The number of days (R) from the settlement date to the next coupon date is 182 (1 July) minus 174 (24 June) equals 7.

$$\begin{aligned}\text{All-in price} &= 107{,}55174\left(1+\tfrac{0{,}135}{2}\right)^{-\left(\frac{7}{181}\right)}\\ &= 107{,}28039\end{aligned}$$

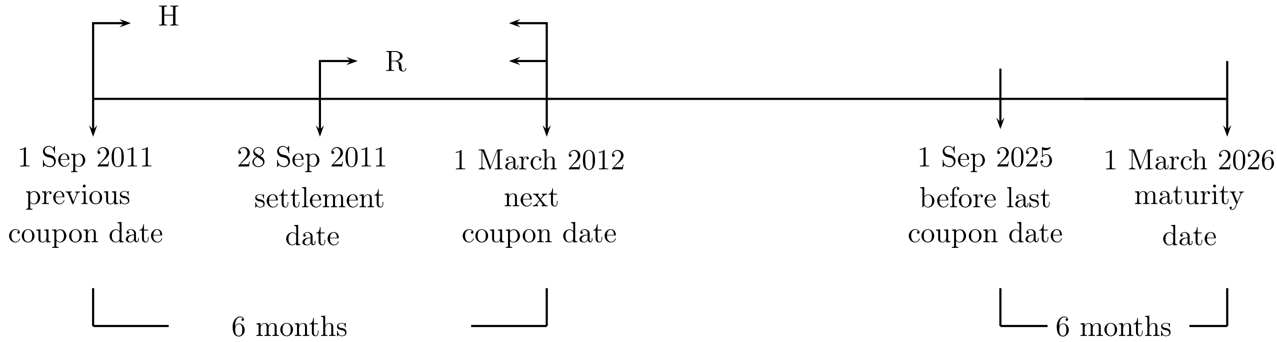
The all-in price is R107,28039%.

$$\begin{aligned}\text{Accrued interest} &= \tfrac{-R}{365}\times c\\ &= \tfrac{-7}{365}\times 14{,}7\\ &= -0{,}28192\end{aligned}$$

$$\begin{aligned}\text{Clean price} &= \text{All-in-price} - \text{accrued interest}\\ &= 107{,}28039 - (-0{,}28192)\\ &= 107{,}56231\end{aligned}$$

The clean price is R107,56231%.

3.



$$\begin{aligned} n &= 01/03/2026 - 01/03/2012 \\ &= 14 \times 2 \\ &= 28 \end{aligned}$$

$$\begin{aligned} P\,(01/03/12) &= d a_{\overline{n}|z} + 100\,(1+z)^{-n} \\ &= \tfrac{11}{2} a_{\overline{28}|0{,}1375 \div 2} + 100\left(1 + \tfrac{0{,}1375}{2}\right)^{-28} \\ &= 83{,}10812 \end{aligned}$$

Cum interest thus we add the coupon:

$$\begin{aligned} \text{Price} &= 83{,}10812 + 5{,}5 \\ &= 88{,}60812 \end{aligned}$$

The remaining days from 28/9/2011 to 1/3/2012 is:

Day number 365 (31 December) minus 271 (28 September) plus 60 (1 March).

$$R = 365 - 271 + 60 = 154.$$

The number of days in the half-year in which the settlement date falls is between 1/9/2011 and 1/3/2012. Day number 365 (31 December) minus 244 (1 September) plus 60 (1 March).

$$H = 365 - 244 + 60 = 181.$$

The fraction to be discounted back is thus

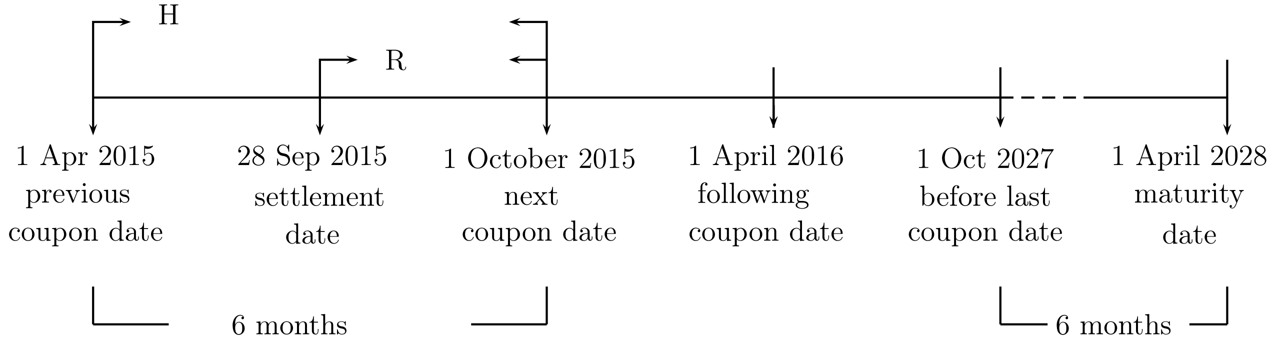
$$\begin{aligned} f &= \tfrac{R}{H} \\ &= \tfrac{154}{181} \end{aligned}$$

The all-in price is:

$$\begin{aligned} P\,(28/09/2011) &= 88{,}60812 \left[1 + \tfrac{0{,}1375}{2}\right]^{-\left(\frac{154}{181}\right)} \\ &= 83{,}73459 \end{aligned}$$

The price of R83,73459% is less than R100%, thus the bond is sold at a discount.

4. Bond BBB:



$$\begin{aligned} \text{Years} &= 2028 - 2016 \\ &= 12 \end{aligned}$$

Multiply by two and add one to get n.

$$\begin{aligned} &= 12 \times 2 \\ &= 24 + 1 \\ &= 25. \end{aligned}$$

$$\begin{aligned} P\,(1/10/2015) &= d a_{\overline{n}|z} + 100\,(1+z)^{-n} \\ &= \tfrac{7{,}5}{2} \times a_{\overline{25}|0{,}14 \div 2} + 100\ \left(1\ +\ \tfrac{0{,}14}{2}\right)^{-25} \\ &= 62{,}12585 \end{aligned}$$

The bond is sold ex-interest.

The remaining days from 28/9/2015 to 1/10/2015 is $R = 3$.

The number of days in the half year 1/4/2015 to 1/10/2015 is:

$$H = 274 \text{ (1 October)} - 91 \text{ (1 April)} = 183.$$

The all-in price is:

$$\begin{aligned} P(28/09/2015) &= 62{,}12585 \left(1 + \tfrac{0{,}14}{2}\right)^{-\left(\frac{3}{183}\right)} \\ &= 62{,}05699 \end{aligned}$$

The all-in price is R62,05699%.

The accrued interest is

$$\begin{aligned} &= \tfrac{-R}{365} \times C \\ &= \tfrac{-3}{365} \times 7{,}5 \\ &= -0{,}06164 \end{aligned}$$

$$\begin{aligned} \text{The clean price} &= \text{All-in-price} - \text{accrued interest} \\ &= 62{,}05699 + 0{,}06164 \\ &= 62{,}11863. \end{aligned}$$

The clean price is R62,11863%.

2.6 Solution: additional exercise 6

1.

$$\begin{aligned} S &= P(1+rt) \\ S &= 5\,000 \\ r &= 13\% \\ t &= \frac{8}{12} \end{aligned}$$

$$\begin{aligned} 5\,000 &= P\left(1 + 0{,}13 \times \frac{8}{12}\right) \\ P &= 4\,601{,}23 \end{aligned}$$

Jenny will pay Jonas R4601,23.

2.

$$\begin{aligned} S &= Pe^{ct} \\ S &= 22\,000 \\ P &= 17\,500 \\ t &= \frac{53}{12} \end{aligned}$$

$$\begin{aligned} 22\,000 &= 17\,500e^{\left(\frac{53}{12}c\right)} \\ e^{\left(\frac{53}{12}\cdot c\right)} &= \frac{22\,000}{17\,500} \\ \frac{53}{12}c\ln e &= \ln\left(\frac{22\,000}{17\,500}\right) \\ c &= \ln\left(\frac{22\,000}{17\,500}\right) \div \frac{53}{12} \\ &= 5{,}181\% \end{aligned}$$

The continuous rate is 5,181%.

3.

$$\begin{aligned} S &= P\left(1+\frac{j_m}{m}\right)^{tn} \\ P &= 2\,844{,}20 \\ S &= 3\,881{,}49 \\ t &= 3 \\ m &= 4 \end{aligned}$$

$$\begin{aligned} 3\,881{,}49 &= 2\,844{,}20\left(1+\frac{j_m}{4}\right)^{3\times 4} \\ j_m &= 10{,}50\% \end{aligned}$$

The original interest rate was 9% (10,5−1,5).

4.

$$\begin{aligned} S &= P\left(1+\frac{j_m}{m}\right)^{tm} \\ S &= 2\,844{,}20 \\ j_m &= 9\% \\ m &= 2 \\ t &= 4 \end{aligned}$$

$$\begin{aligned} 2\,844{,}20 &= P\left(1+\frac{0{,}09}{2}\right)^{4\times 2} \\ P &= 2\,000{,}00 \end{aligned}$$

The original amount invested was R2 000.

5.

$$\begin{aligned} P &= da_{\overline{n}|z} + 100(1+z)^{-n} \\ 15{,}04289 &= 100(1+0{,}0675)^{-n} \\ n &= 29 \end{aligned}$$

The number of years is $14\frac{1}{2}$.

The settlement date must be moved forward for 74 days.

Day number 108 (18 April) plus 74 equals 182 and that is 1 July 2013. This date must be moved forward for $14\frac{1}{2}$ years. The maturity date is therefore 1 January 2028.

6.

$$\begin{aligned} S &= (1+i)Rs_{\overline{n}|i} \\ S &= 250\,000 \\ i &= 9{,}4\% \div 12 \\ n &= 9 \end{aligned}$$

$$\begin{aligned} 250\,000 &= (1+i)Rs_{\overline{9}|0{,}094\div 12} \\ (1+i)R &= 26\,918{,}73 \\ R &= 26\,709{,}50 \end{aligned}$$

Francois' friend James monthly deposits equal R26 709,50.

7.

$$\begin{aligned} P(8/12/2015) &= \frac{5}{2}a_{\overline{6}|0{,}12\div 2} + 100\left(1+\frac{0{,}12}{2}\right)^{-6} \\ &= 82{,}78936 \end{aligned}$$

The price on 8/12/2015 is R82,78936%. To this a coupon must be added because it is a cum interest case. The price is therefore R85,28936% ($2{,}5 + 82{,}78936$).

This amount must be discounted back for 33 days.

$$\begin{aligned} \text{The all-in price} &= 85{,}28936\left(1+\frac{0{,}12}{2}\right)^{-33/184} \\ &= 84{,}40269 \end{aligned}$$

The all-in price is R84,40269%. The accrued interest must be deducted from this amount.

$$\begin{aligned} \text{The clean price} &= 84{,}40269 - 2{,}06849 \\ &= 82{,}33420 \end{aligned}$$

The clean price is R82,33420%.

8.

$$\begin{aligned} S &= \left(R+\frac{Q}{i}\right)s_{\overline{n}|i} - \frac{nQ}{i} \\ R &= 6\,500 \\ Q &= 1\,700 \\ i &= 10\% \\ n &= 20 \end{aligned}$$

$$\begin{aligned} S &= \left(6\,500+\frac{1\,700}{0{,}10}\right)s_{\overline{20}|0{,}10} - \frac{20\times 1\,700}{0{,}10} \\ &= 1\,345\,962{,}49 - 340\,000 \\ &= 1\,005\,962{,}49 \end{aligned}$$

An amount of R1 005 962,48 was received.

9.

$$\begin{aligned}
\text{Amount paid} &= nR + Q\left(\frac{n(n-1)}{2}\right) \\
&= 20 \times 6\,500 + 1\,700\left(\frac{20 \times 19}{2}\right) \\
&= 130\,000 + 323\,000 \\
&= 453\,000
\end{aligned}$$

Interest earned is the amount received for the policy, R1 005 962,49, minus the actual amount paid of R453 000 and that equals R552 962,49.

10.

$$\begin{aligned}
P &= Ra_{\overline{n}|i} \\
P &= 225\,000 \\
R &= 36\,000 \\
n &= 15 \\
\\
225\,000 &= 36\,000a_{\overline{15}|i} \\
i &= 13{,}65\%
\end{aligned}$$

The internal rate of return is 13,65%.

11. The arithmetic mean equals 15,36.

12. The regression line is represented by

$$y = 718{,}35 - 29{,}26x$$

13.

$$\begin{aligned}
S &= P\left(1 + \frac{j_m}{m}\right)^{tm} \\
S &= 100\,000 \\
j_m &= 8\% \\
m &= 12 \\
t &= 11 \\
\\
100\,000 &= P\left(1 + \frac{0{,}08}{12}\right)^{11 \times 12} \\
P &= 41\,599{,}60
\end{aligned}$$

The amount in the account was R41 599,60.

14.

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ S &= 41\,599{,}60 \\ n &= 10 \times 2 \\ i &= 9{,}25\% \div 2 \\ \\ 41\,599{,}60 &= Rs_{\overline{10\times 2}|0{,}0925\div 2} \\ R &= 1\,308{,}77 \end{aligned}$$

Grandma will pay R1 308,77 every six months in this account.

2.7 Solution: additional exercise 7

1.

$$\begin{aligned} S &= P(1+rt) \\ P &= 16\,000 \\ S &= 16\,570{,}08 \\ t &= 341 - 52 = \frac{289}{365} \\ 16\,570{,}08 &= 16\,000\left(1 + r \times \frac{289}{365}\right) \\ r &= \left(\frac{16\,570{,}05}{16\,000} - 1\right) \div \frac{289}{365} \\ &= 0{,}045 \end{aligned}$$

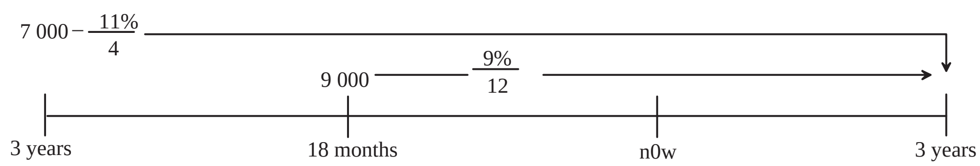
The simple interest rate is 4,5%.

2.

$$\begin{aligned} J_\alpha &= 100(e^c - 1) \\ 16{,}13 &= 100(e^c - 1) \\ 0{,}1613 + 1 &= e^c \\ \ln\, 1{,}1613 &= c \ln\, e \\ c &= 0{,}1495 \end{aligned}$$

The continuous rate is 14,95%.

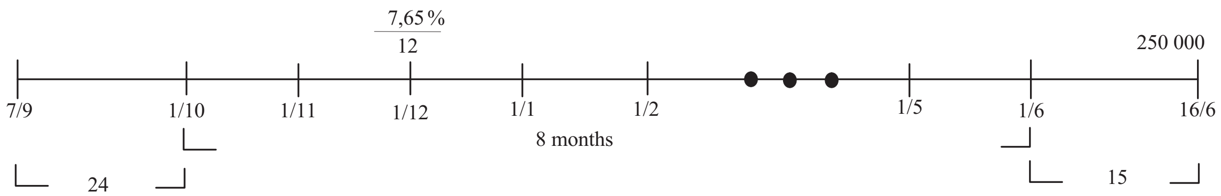
3.



$$
\begin{aligned}
S &= P(1+\tfrac{j_m}{m})^{tm} + P(1+\tfrac{j_m}{m})^{tm} \\
&= 7\,000\left(1+\tfrac{0,11}{4}\right)^{6\times 4} + 9\,000\left(1+\tfrac{0,09}{12}\right)^{4,5\times 12} \\
&= 13\,423,38 + 13\,473,35 \\
&= 26\,896,73
\end{aligned}
$$

The amount that Jack owes Jill is R26 896,73.

4.



$$
\begin{aligned}
S &= P(1+rt)\left(1+\frac{j_m}{m}\right)^{tm}(1+rt) \\
250\,000 &= P\left(1+0,0765\times\frac{15}{365}\right)\left(1+\frac{0,0765}{12}\right)^{\frac{8}{12}\times\frac{12}{1}}\left(1+0,0765\times\frac{24}{365}\right) \\
249\,216,50 &= P\left(1+\frac{0,0765}{12}\right)^{\frac{8}{12}\times\frac{12}{1}}\left(1+0,0765\times\frac{24}{365}\right) \\
236\,863,47 &= P\left(1+0,0765\times\frac{24}{365}\right) \\
P &= 235\,677,98
\end{aligned}
$$

Ian deposit R235 677,98.

5.

$$
\begin{aligned}
S &= P\left(1+\frac{jm}{m}\right)^{tm} \\
250\,000 &= P\left(1+\frac{0,0765}{12}\right)^{\left(\frac{8}{12}+\frac{15+24}{365}\right)\times\frac{12}{1}} \\
P &= 235\,679,96
\end{aligned}
$$

Ian deposit R235 679,96.

6.

$$\begin{aligned}
S &= \left(R+\frac{Q}{i}\right)s_{\overline{n}|i}-\frac{nQ}{i}\\
R &= 6\,000\\
Q &= 1\,500\\
n &= 30\\
i &= 9{,}7\%\\
S &= \left(6\,000+\frac{1\,500}{0{,}097}\right)s_{\overline{30}|0{,}097}-\frac{30\times 1\,500}{0{,}097}\\
&= 3\,336\,150{,}21-463\,917{,}53\\
&= 2\,872\,232{,}69
\end{aligned}$$

Paul can expect to receive R2 872 232,69.

7.

$$\begin{aligned}
\text{Amount paid} &= nR+\frac{Q(n(n-1))}{2}\\
&= 30\times 6\,000\ +\ \frac{1\,500(30\times 29)}{2}\\
&= 180\,000\ +\ 652\,500\\
&= 832\,500
\end{aligned}$$

Paul paid R832 500 for the policy.

8.

$$\begin{aligned}
IRR &= \frac{A}{(1+i)}+\frac{B}{(1+i)^2}+\frac{C}{(1+i)^3}+\frac{D}{(1+i)^4}+\frac{E}{(1+i)^5}-Z\\
&= \frac{75\,000}{(1+i)}+\frac{190\,000}{(1+i)^2}+\frac{40\,000}{(1+i)^3}+\frac{150\,000}{(1+i)^4}+\frac{180\,000}{(1+i)^5}-500\,000\\
&= 0{,}0778
\end{aligned}$$

The internal rate of return (IRR) is 7,78%.

9.

$$\begin{aligned}
A &= 2\,080{,}54-2\,014{,}27\\
&= 66{,}27
\end{aligned}$$

The interest due at month end (A) is R66,27.

10.

$$\begin{aligned}
B &= 6\,141{,}56-2\,030{,}64\\
&= 4\,110{,}92
\end{aligned}$$

The outstanding principal at month beginning (B) is R4 110,92.

11. We must first determine the applicable interest rate.

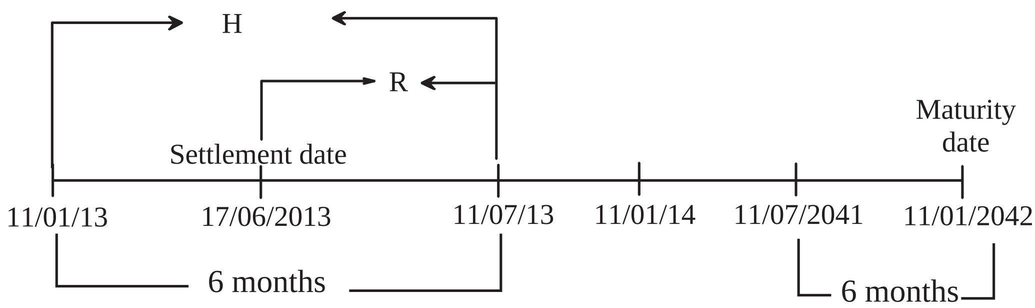
$$\begin{aligned} I &= Prt \\ 16{,}77 &= 2\,063{,}78 \times r \times \frac{1}{12} \\ r &= 0{,}0975 \end{aligned}$$

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ &= 2\,080{,}54s_{\overline{150}|0{,}0975\div 12} \\ &= 180\,000{,}00 \end{aligned}$$

The original amount was R180 000,00.

The answers differs due to rounding off.

12.



$$\begin{aligned} \text{Years} &= 11/01/42 - 11/01/14 \\ &= 28 \end{aligned}$$

We must now multiply with to and add one to get n.

$$\begin{aligned} &= 28 \times 2 + 1 = 57 \\ R &= 192 - 168 = 24 \\ H &= 192 - 11 = 181 \end{aligned}$$

$$\begin{aligned} P(11/07/13) &= da_{\overline{n}|z} + 100(1 + z = z)^{-n} \\ &= \frac{9{,}91}{2}a_{\overline{57}|0{,}0747\div 2} + 100\left(1 + \frac{0{,}0747}{2}\right)^{-57} \\ &= 128{,}62450 + 4{,}955 \quad \text{(cum interest)} \\ &= 133{,}57950 \end{aligned}$$

$$\begin{aligned}\text{All-in price} &= 133{,}57950\left(1+\frac{0{,}0747}{2}\right)^{-\frac{24}{181}}\\ &= 132{,}93158\end{aligned}$$

The All-in price is R132,93158%.

13.

$$\begin{aligned}\text{Accrued interest} &= \frac{H-R}{365}\times C\\ &= \frac{181-24}{365}\times 9{,}91\\ &= 4{,}26266\end{aligned}$$

$$\begin{aligned}\text{Clean price} &= \text{All-in price} - \text{Accrued interest}\\ &= 132{,}93158 - 4{,}26266\\ &= 128{,}66892\end{aligned}$$

The clean price is R128,66892%.

14.

$$y = 3{,}17477 + 0{,}65407x$$

The equation is $y = 0{,}65407x + 3{,}17477$

15.

$$r = 0{,}90828$$

The correlation coefficient is 0,90828.