Minimise $z = 2x_1 - x_2 + x_3$

subject to

$$3x_1 + x_2 + x_3 \leq 60$$
$$x_1 - x_2 + 2x_3 \leq 10$$
$$x_1 + x_2 - x_3 \leq 20$$

and $x_1, x_2, x_3 \geq 0$.

The LP problem in standard form is

Maximise $Z = -z = -2x_1 + x_2 - x_3$

subject to

$$3x_1 + x_2 + x_3 + s_1 = 60$$
$$x_1 - x_2 + 2x_3 + s_2 = 10$$
$$x_1 + x_2 - x_3 + s_3 = 20$$

and $x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$.

Initial simplex tableau

	Z	x_1	x_2	x_3	s_1	s_2	s_3	rhs	BV	θ
R_1	0	3	1	1	1	0	0	60	s_1	$60/1$
R_2	0	1	-1	2	0	1	0	10	s_2	—
R_3	0	1	1	-1	0	0	1	20	s_3	$20/1$ $\rightarrow$
R_0	1	2	-1	1	0	0	0	0	Z	

- The most negative entry in R_0 is -1 that corresponds to x_2. It follows that x_2 is an entering variable.
- $\min \theta = \min\{60, -, 20\} = 20$. The leaving variable is s_3.

- We apply the one zero method to x_2's column and get the following simplex tableau

Tableau 1.

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs	BV	θ
0	2	0	2	1	0	-1	40	s_1	40/2 →
0	2	0	1	0	1	1	30	s_2	30/1
0	1	1	-1	0	0	1	20	x_2	—
1	3	0	0 ↑	0	0	1	20	z	

At this stage we have reached a final simple tableau. An optimal solution is given by

$z = 20$, $x_1 = 0$, $x_2 = 20$ and $x_3 = 0$.

As the nonbasic variable x_3 is zero, that means the LP problem admits ~~of~~ multiple optimal solutions. We reincorporate x_3 in the basis. It follows that

- The entering variable is x_3
- $\min \theta = \min\{20, 30, -\} = 20$. s_1 is the leaving variable.
- We apply the 1-0 method to x_3's column and get what follows:

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs	BV
0	1	0	1	1/2	0	-1/2	20	x_3
0	1	0	0	-1/2	1	3/2	10	s_2
0	2	1	0	1	0	1/2	40	x_2
1	3	0	0	0	0	1	20	z

~~It~~ An alternative optimal solution is given by

$z = 20, \quad x_1 = 0, \quad x_2 = 40, \quad x_3 = 20.$

The general optimal solution is $\zeta = -z = -20$,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \alpha \begin{bmatrix} 0 \\ 20 \\ 0 \end{bmatrix} + (1-\alpha) \begin{bmatrix} 0 \\ 40 \\ 20 \end{bmatrix}, \quad \alpha \in [0, 1].$$

Maximise $z = 3x_1 + x_2$

subject to

$$x_1 + x_2 \geq 3$$
$$2x_1 - x_2 \geq -4$$
$$x_1 + x_2 = 3$$

and $x_1, x_2 \geq 0$.

The LP in standard form is

Maximise $z = 3x_1 + x_2$

subject to

$$\begin{aligned} x_1 + x_2 - s_1 \qquad &= 3 \\ -2x_1 + x_2 \qquad + s_2 &= 4 \\ x_1 + x_2 \qquad &= 3 \end{aligned}$$

and $x_1, x_2, s_1, s_2 \geq 0$.

The LP in augmented form is

Maximise $z = 3x_1 + x_2 - Ma_1 - Ma_3$

subject to

$$\begin{aligned} x_1 + x_2 - s_1 \qquad + a_1 &= 3 \\ -2x_1 + x_2 \qquad + s_2 \qquad &= 4 \\ x_1 + x_2 \qquad + a_3 &= 3 \end{aligned}$$

and $x_1, x_2, s_1, s_2, a_1, a_3 \geq 0$.

Initial simplex tableau

	z	x_1	x_2	s_1	s_2	a_1	a_3	Rhs	BV	θ
R_1	0	1	1	-1	0	1	0	3	a_1	3/1
R_2	0	-2	1	0	1	0	0	4	s_2	—
R_3	0	1	1	0	0	0	1	3	a_3	3/1 →
R_0	1	-3	-1	0	0	M	M	0	z	
R_0'	1	-3-2M ↑	-1-2M	M	0	0	0	-6M	z	

Where $R_0' = R_0 - M(R_1 + R_3)$

- The entering variable is x_1
- $\min \theta = \min\{3, -, 3\} = 3$. The leaving variable is either a_1 or a_3. Let's choose a_3
- We apply the 1-0 method to x_1's column and get the following simplex tableau:

Tableau 1

z	x_1	x_2	s_1	s_2	a_1	a_3	Rhs	BV	θ
0	0	0	-1	0	1	-1	0	a_1	
0	0	3	0	1	0	2	10	s_2	
0	1	1	0	0	0	1	3	x_1	
1	0	2	M	0	0	3+2M	6	z	

There is no more entering variable and the optimal solution is

$z = 6$; $x_1 = 3$; $x_2 = 0$.