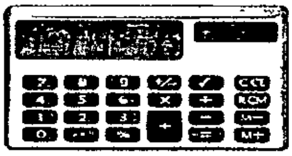
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA university of south africa

APM3713
SECOND PAPER

May/June 2017

SPECIAL RELATIVITY AND RIEMANNIAN GEOMETRY

Duration 2 Hours

100 Marks

EXAMINERS

FIRST	MISS PL SKELTON
SECOND	PROF DP SMITS

Use of a non-programmable pocket calculator is permissible

Closed book examination

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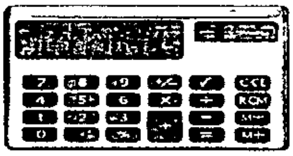
[TURN OVER]

UNIVERSITY EXAMINATIONS



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This paper consists of 6 pages.

Some potentially useful formulae can be found on page 5.

[TURN OVER]

QUESTION 1

An advanced spacecraft travels past the Earth and Mars in a straight line at speed $v = 0.8c$ at a time when the distance between the two planets is 2.4×10^{11} m. This distance is measured in the fixed frame of reference, in which the Sun, Earth and Mars are taken to be at rest. (Neglect the relative motion of the planets for this question.)

a) According to the people on Earth, how long does it take for the spacecraft to go from Earth to Mars? (3)

b) According to the astronauts on the spacecraft, how fast is Mars approaching them? (1)

c) According to the astronauts on the spacecraft, how far apart are Earth and Mars? (5)

d) According to the astronauts, how long does the journey from Earth to Mars take? (3)

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QUESTION 2

The spacetime coordinates of two events as measured in an inertial frame S are as follows:

Event A	$x_A = a$	$t_A = a/(2c)$	$y_A = z_A = 0$
Event B	$x_B = 2a$	$t_B = 5a/(6c)$	$y_B = z_B = 0$

where a is some constant. There is an inertial frame S', in the standard configuration with S, in which these two events are simultaneous.

a) Draw a Minkowski diagram to indicate these two events in the S frame. Also indicate the x' and ct' axes on your diagram. (6)

b) Use the Lorentz transformation equations to find the speed of the S' frame relative to S. (3)

c) At what time do these events occur in the S' frame? (6)

d) Calculate the spacetime separation between these two events. (3)

e) Is the spacetime separation between the two events time-like, space-like or light-like? (1)

f) Is there a frame where Event A caused Event B? (1)

[TURN OVER]

[20]

QUESTION 3

A subatomic particle called a pion (π) decays into a muon (μ) that moves in the positive x-direction and a neutrino (ν) that moves in the negative x-direction The reaction can be written as

$$\pi \to \mu + \nu$$

For this question, assume that the neutrino is massless, that is $m_\nu = 0$, and that the pion was at rest in the laboratory reference frame just before it decayed Give your answers to the questions below in terms of the masses of the pion m_π and muon m_μ

a) Find the the momentum and energy of all three particles in the laboratory frame (13)

b) Write down the four-momenta of the three particles in the laboratory frame (6)

c) Show that the four-momentum of the system was conserved (4)

d) Suppose the muon has a speed of $V = c\left(m_\pi^2 - m_\mu^2\right) / \left(m_\pi^2 + m_\mu^2\right)$ What is the Lorentz factor between the laboratory frame and the muon's rest frame? (4)

e) What is the energy of the neutrino in the muon's rest frame? (5)

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QUESTION 4

Consider the paraboloid with parametrization

$$\begin{aligned} x(u, v) &= u\cos v \\ y(u, v) &= u\sin v \\ z(u, v) &= u^2 \end{aligned}$$

a) Find the line element of the surface (10)

b) What is the metric tensor and the dual metric tensor? (4)

[TURN OVER]

c) Taking $x^1 = u$ and $x^2 = v$, the only non-zero Christoffel coefficients of the surface are

$$\begin{aligned} \Gamma^1_{11} &= \frac{4u}{(1+4u^2)} \\ \Gamma^1_{22} &= \frac{-u}{(1+4u^2)} \\ \Gamma^2_{12} = \Gamma^2_{21} &= \frac{1}{u} \end{aligned}$$

Find the component R^1_{212} of the Riemann curvature tensor (6)

[20]

QUESTION 5

a) Suppose that $R_{iklm} = K\,(g_{il}g_{km} - g_{im}g_{kl})$, where K is a constant, on some four dimensional Riemannian space Show that for the curvature scalar we have $R = -12K$ (9)

b) Suppose that in a two-dimensional Euclidean space with coordinates x^μ ($\mu = 1, 2$) the coordinates x^1 and x^2 correspond to the polar coordinates r and θ Also suppose that the coordinates x'^μ correspond to the usual Cartesian coordinates x and y

If A^μ is a general tensor component in the (r, θ) coordinates, and A'^μ is the corresponding tensor component in the (x, y) coordinates, find the transformation that expresses A'^μ in terms of the A^μ for each value of μ (7)

[16]

Total: [100]

FORMULA SHEET

Speed of light in a vacuum: $c = 3 \times 10^8\,\mathrm{ms}^{-1}$
Mass of electron: $m_e = 0\ 511\,\mathrm{MeV}/c^2 = 9\ 11 \times 10^{-31}\,\mathrm{kg}$
Mass of proton. $m_p = 938\ 3\,\mathrm{MeV}/c^2 = 1\ 67 \times 10^{-27}\,\mathrm{kg}$
$1\,\mathrm{eV} = 1\ 60 \times 10^{-19}\,\mathrm{J}$
$1\,\mathrm{MeV} = 10^6\,\mathrm{eV}$

$$\begin{aligned} t' &= t \\ x' &= x - Vt \\ y' &= y \\ z' &= z \end{aligned}$$

$$\begin{aligned} E' &= \gamma\,(E - Vp_x) \\ p'_x &= \gamma\left(p_x - \frac{VE}{c^2}\right) \\ p'_Y &= p_y \\ p'_z &= p_z \end{aligned}$$

$$E^2 = p^2c^2 + m^2c^4$$

$$\begin{aligned} t' &= \gamma\left(t - \left(V/c^2\right)x\right) \\ x' &= \gamma\,(x - Vt) \\ y' &= y \\ z' &= z \end{aligned}$$

$$\begin{aligned} F'^0 &= \gamma\left(F^0 - VF^1/c\right) \\ F'^1 &= \gamma\left(F^1 - VF^0/c\right) \\ F'^2 &= F^2 \\ F'^3 &= F^3 \end{aligned}$$

$$f_{rec} = f_{em}\sqrt{\frac{c-V}{c+V}}$$

$$L_C\,(\mathrm{P},\,\mathrm{Q}) = \int_P^Q \left[\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2\right]^{1/2} du$$

$$\begin{aligned} v'_x &= \frac{v_x - V}{1 - v_xV/c^2} \\ v'_y &= \frac{v_y}{\gamma\,(1 - v_xV/c^2)} \\ v'_z &= \frac{v_z}{\gamma\,(1 - v_xV/c^2)} \end{aligned}$$

$$\Gamma^i_{jk} = \frac{1}{2}\sum_m g^{im}\left(\frac{\partial g_{mk}}{\partial x^j} + \frac{\partial g_{jm}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^m}\right)$$

[TURN OVER]

$$\frac{d^2x^i}{d\lambda^2} + \sum_{j,k} \Gamma^i{}_{jk} \frac{dx^j}{d\lambda}\frac{dx^k}{d\lambda} = 0$$

$$k = \frac{|xy - yx|}{(x^2 + y^2)^{3/2}}$$

$$A'^\alpha = \sum_\beta \frac{\partial x'^\alpha}{\partial x^\beta} A^\beta$$

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$$R^l{}_{ijk} = \frac{\partial \Gamma^l{}_{ik}}{\partial x^j} + \frac{\partial \Gamma^l{}_{ij}}{\partial x^k} + \sum_m \Gamma^m_{ik}\Gamma^l{}_{mj} - \sum_m \Gamma^m_{ij}\Gamma^l{}_{mk}$$

$$T^{\mu\nu} = \left(\rho + p/c^2\right) U^\mu U^\nu - p g^{\mu\nu}$$

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R$$

$$\nabla_\beta v_\alpha = \frac{\partial v_\alpha}{\partial x^\beta} - \sum_\lambda \Gamma^\lambda_{\alpha\beta} v_\lambda$$

$$\nabla_\beta v^\alpha = \frac{\partial v^\alpha}{\partial x^\beta} + \sum_\lambda \Gamma^\alpha_{\lambda\beta} v^\lambda$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\kappa T_{\mu\nu}$$