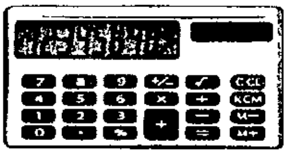
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA university of south africa

APM3713

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SPECIAL RELATIVITY AND RIEMANNIAN GEOMETRY

Duration 2 Hours

100 Marks

EXAMINERS

FIRST	MR ME SIKHONDE
SECOND	PROF DP SMITS

Use of a non-programmable pocket calculator is permissible

Closed book examination

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This paper consists of 5 pages.

Some potentially useful formulae can be found on page 4 and 5

QUESTION 1

As part of its search for extrasolar planets, NASA discovers a planet that appears to be very much like Earth orbiting a star 40 lightyears from our Solar System An expedition is planned to send astronauts to the planet NASA would like the astronauts to age no more than 30 years during the journey In this problem, neglect any issues related to the acceleration of the astronauts' spaceship (Hint A lightyear is the distance traveled by light in one year, which is just c multiplied by one year, or $9\ 46 \times 10^{12}$ km *In many problems it is simpler to write it as* $1c$ year, since c often cancels out)

a) At what velocity must the astronauts' spaceship travel in Earth's reference frame so that the astronauts age 30 years during the journey? (10)

b) According to the astronauts in the spaceship, what will be the distance of their journey? (6)

c) What will the duration of their(astronauts) journey be according to the people on Earth? (4)

[20]

QUESTION 2

a) The space and time coordinates of two events as measured in an inertial frame S are as follows

Event A	$x_A = a$	$t_A = a/(2c)$	$y_A = z_A = 0$
Event B	$x_B = 2a$	$t_B = 5a/(6c)$	$y_B = z_B = 0$

where a is some constant There is an inertial frame S', in the standard configuration with S, in which these two events are simultaneous

i Draw a rough Minkowski diagram of frames S and S' Indicate Events A and B on your diagram (5)

ii Use the Lorentz transformation equations to calculate the speed of the S' frame relative to S (5)

iii At what time do these events occur in the S' frame? (5)

b) Prove that for a general time-like spacetime separation between two events, the two events can never be considered to occur simultaneously (5)

[20]

[TURN OVER]

QUESTION 3

At the Stanford Linear Accelerator, electrons are accelerated to energies of 50 GeV(where 1 GeV = 10^9 eV) Take the electrons' rest mass as 0 511 MeV/c2

a) If this energy were classical kinetic energy, what would be the electrons' speed? (6)

b) Is the above value in (a) of the speed physical? and if not, why? (2)

c) What is the electrons' actual (relativistic) speed? (10)

d) Is the above value in (c) of the speed physical? and if not, why? (2)

[20]

QUESTION 4

The metric of a unit 2-sphere is given by

$$ds^2 = d\theta^2 + \sin^2\theta d\phi^2$$

a) Setting $x^1 = \theta$ and $x^2 = \phi$ What is the metric tensor and the dual metric tensor? Explain how you determine these (5)

b) Given that the only non-zero Christoffel coefficients are $\Gamma^2{}_{21} = \frac{\cos\theta}{\sin\theta}$, and $\Gamma^1{}_{22} = -\frac{\cos\theta}{\sin\theta}$, Calculate $R^2{}_{121}$ for the metric (10)

c) Calculate the Gaussian curvature of the surface (5)

[20]

QUESTION 5

a) Suppose that $R_{\mu\nu\delta\gamma} = K\left(g_{\mu\delta}g_{\nu\gamma} - g_{\mu\gamma}g_{\nu\delta}\right)$ on some four dimensional Riemannian space Show that for the curvature scalar we have $R = \pm 12K$ (10)

b) Show that if $A_{\mu\nu}$ is a symmetric tensor and $B^{\mu\nu}$ is an antisymmetric tensor, then $\sum_{\mu,\nu=0}^{3} A_{\mu\nu}B^{\mu\nu} = 0$ (5)

c) Show that if $G_{\mu\nu} = 0$, then $R_{\mu\nu} = 0$ (5)

[20]

Total: [100]

[TURN OVER]

FORMULA SHEET

Speed of light in a vacuum: $c = 3 \times 10^8\,\mathrm{ms}^{-1}$
Mass of electron. $m_e = 0\ 511\,\mathrm{MeV}/c^2 = 9\ 11 \times 10^{-31}\,\mathrm{kg}$
Mass of proton. $m_p = 938\ 3\,\mathrm{MeV}/c^2 = 1\ 67 \times 10^{-27}\,\mathrm{kg}$
Mass of pion $m_\pi = 139\ 6\,\mathrm{MeV}/c^2 = 2\ 49 \times 10^{-28}\,\mathrm{kg}$
Mass of muon· $m_\mu = 105\ 7\,\mathrm{MeV}/c^2 = 1\ 89 \times 10^{-28}\,\mathrm{kg}$
$1\,\mathrm{eV} = 1\ 60 \times 10^{-19}\,\mathrm{J}$
$1\,\mathrm{MeV} = 10^6\,\mathrm{eV}$

$$\begin{aligned} t' &= t \\ x' &= x - Vt \\ y' &= y \\ z' &= z \end{aligned}$$

$$\begin{aligned} v'_x &= \frac{v_x - V}{1 - v_x V/c^2} \\ v'_y &= \frac{v_y}{\gamma\left(1 - v_x V/c^2\right)} \\ v'_z &= \frac{v_z}{\gamma\left(1 - v_x V/c^2\right)} \end{aligned}$$

$$\begin{aligned} t' &= \gamma\left(t - \left(V/c^2\right)x\right) \\ x' &= \gamma\left(x - Vt\right) \\ y' &= y \\ z' &= z \end{aligned}$$

$$\begin{aligned} E' &= \gamma\left(E - Vp_x\right) \\ p'_x &= \gamma\left(p_x - \frac{VE}{c^2}\right) \\ p'_y &= p_y \\ p'_z &= p_z \end{aligned}$$

$$E^2 = p^2c^2 + m^2c^4$$

$$f_{rec} = f_{em}\sqrt{\frac{c - V}{c + V}}$$

$$\begin{aligned} F'^0 &= \gamma\left(F^0 - VF^1/c\right) \\ F'^1 &= \gamma\left(F^1 - VF^0/c\right) \\ F'^2 &= F^2 \\ F'^3 &= F^3 \end{aligned}$$

$$\lambda_{em} = \lambda_{rec}\sqrt{\frac{c - V}{c + V}}$$

$$L_C\,(\mathrm{P},\,\mathrm{Q}) = \int_P^Q \left[\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2\right]^{1/2} du$$

$$\begin{aligned} \nabla_\beta v_\alpha &= \frac{\partial v_\alpha}{\partial x^\beta} - \sum_\lambda \Gamma^\lambda_{\alpha\beta} v_\lambda \\ \nabla_\beta v^\alpha &= \frac{\partial v^\alpha}{\partial x^\beta} + \sum_\lambda \Gamma^\alpha_{\lambda\beta} v^\lambda \end{aligned}$$

$$\Gamma^i{}_{jk} = \frac{1}{2}\sum_m g^{im}\left(\frac{\partial g_{mk}}{\partial x^j} + \frac{\partial g_{jm}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^m}\right)$$

$$\begin{aligned} A'^\alpha &= \sum_\beta \frac{\partial x'^\alpha}{\partial x^\beta} A^\beta \\ A'_\alpha &= \sum_\beta \frac{\partial x^\beta}{\partial x'^\alpha} A_\beta \end{aligned}$$

$$\frac{d^2x^i}{d\lambda^2} + \sum_{j,k}\Gamma^i{}_{jk}\frac{dx^j}{d\lambda}\frac{dx^k}{d\lambda} = 0$$

$$K = \frac{R_{1212}}{g}$$

$$k = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{\left(\dot{x}^2 + \dot{y}^2\right)^{3/2}}$$

$$T^{\mu\nu} = \left(\rho + p/c^2\right) U^\mu U^\nu - p g^{\mu\nu}$$

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R$$

$$R^l{}_{ijk} = \frac{\partial \Gamma^l{}_{ik}}{\partial x^j} - \frac{\partial \Gamma^l{}_{ij}}{\partial x^k} + \sum_m \Gamma^m_{ik}\Gamma^l{}_{mj} - \sum_m \Gamma^m_{ij}\Gamma^l{}_{mk}$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\kappa T_{\mu\nu}$$

Examiners:	
First	Mr ME Sikhonde
Second	Prof D P Smits
External	Prof CM Villet (UJ)