Tutorial Letter 101/3/2018

SPECIAL RELATIVITY AND RIEMANNIAN GEOMETRY
APM3713

Semesters 1 & 2

Department of Mathematical Sciences

IMPORTANT INFORMATION:

This tutorial letter contains important information about your module and compulsory assignments.

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CONTENTS

1 INTRODUCTION

Dear Student

Welcome to the APM3713 module in the Department of Mathematical Sciences at Unisa. We trust that you will find this module both interesting and rewarding.

Some of this tutorial matter may not be available when you register. Tutorial matter that is not available when you register will be posted to you as soon as possible, but is also available on myUnisa.

You must be registered on myUnisa (http://my.unisa.ac.za) to be able to submit assignments online, gain access to the library functions and various learning resources, download study material, "chat" to your lecturers and fellow students about your studies and the challenges you encounter, and participate in online discussion forums. myUnisa provides additional opportunities to take part in activities and discussions of relevance to your module topics, assignments, marks and examinations.

A tutorial letter is our way of communicating with you about teaching, learning and assessment. You will receive a number of tutorial letters during the course of the module. This particular tutorial letter contains important information about the scheme of work, resources and assignments for this module as well as the admission requirements for the examination. We urge you to read this and subsequent tutorial letters carefully and to keep it at hand when working through the study material, preparing and submitting the assignments, preparing for the examination and addressing queries that you may have about the course (course content, textbook, worked examples and exercises, theorems and their applications in your assignments, tutorial and textbook problems, etc.) to your APM3713 lecturers.

2 PURPOSE AND OUTCOMES

2.1 Purpose

Qualifying students will have a solid understanding of special relativity and introductory general relativity. Proficiency in associated mathematical topics such as introductory geometry in non-Euclidian space and tensor algebra will also be covered. Students can engage in the fundamental intermediary knowledge, skills and values which will support further studies and applications in the sector of applied mathematics, in the field of mathematical sciences, as part of a degree in mathematics, applied mathematics or physics. These competencies contribute to the development of scientific knowledge and mathematical understanding in Southern Africa, Africa or globally. Enrolled students in this blended mode are connected to the myUnisa platform on a regular basis throughout the semester.

2.2 Outcomes

2.2.1 Demonstrate the principles of special relativity.

2.2.2 Demonstrate physical laws in the context of special relativity.

2.2.3 Demonstrate the fundamental concepts of non-Euclidean geometry in the context of curved spacetime.

2.2.4 Demonstrate the preliminary concepts involved in the theory of general relativity.

3 LECTURER AND CONTACT DETAILS

3.1 Lecturer

The contact details for the lecturer responsible for this module are

Name:	Muzikayise Edward Sikhonde
Tel No:	(011) 670 9168
E-mail address:	sikhome@unisa.ac.za
Postal address:	The AST1631 Lecturer Department of Mathematical Sciences Private Bag X6 Florida 1709 South Africa

All queries that are not of a purely administrative nature but are about the content of this module should be directed to your lecturer. Please have your study material with you when you contact your lecturer by telephone. If you are unable to reach me, leave a message with the departmental secretary. Provide your name, the time of the telephone call and contact details. If you have problems with questions that you are unable to solve, please send your own attempts so that the lecturer can determine where the fault lies.

Please note: Letters to the lecturer may not be enclosed with or inserted into assignments.

3.2 Department

The contact details for the Department of Mathematical Sciences are:

Departmental Secretary:

(011) 670 9147 (South Africa)

(+27) 11 670 9147 (International)

3.3 University

If you need to contact the University about matters not related to the content of this module, please consult the publication *Study @ Unisa* that you received with your study material. This booklet contains information on how to contact the University (e.g. to whom you can write for different queries, important telephone and fax numbers, addresses and details of the times certain facilities are open). Always have your student number at hand when you contact the University.

4 RESOURCES

4.1 Prescribed textbook

The prescribed textbook can be obtained from the University's official booksellers. If you have difficulty locating your book at these booksellers, please contact the Prescribed Books Section at (012) 429 4152 or e-mail vospresc@unisa.ac.za.

The prescribed textbook for this module is:

Title:	Relativity, Gravitation and Cosmology
Authors:	Robert J.A. Lambourne
Publishers:	Cambridge University Press
ISBN:	978-0-521-13138-4

Please buy the textbook as soon as possible since you have to study from it directly – you cannot do this module without the prescribed textbook.

4.2 Recommended books

There are no recommended books for this module.

4.3 Electronic reserves (e-reserves)

There are no e-Reserves for this module.

4.4 Library services and resources information

For brief information go to http://www.unisa.ac.za/brochures/studies

For detailed information, go to http://www.unisa.ac.za/library. For research support and services of personal librarians, click on ”Research support”.

The library has compiled a number of library guides:

- finding recommended reading in the print collection and e-reserves – http://libguides.unisa.ac.za/request/undergrad
- request material – http://libguides.unisa.ac.za/request/request
- postgraduate information services – http://libguides.unisa.ac.za/request/postgrad

- finding, obtaining and using library resources and tools to assist in doing research – http://libguides.unisa.ac.za/Research_Skills
- how to contact the Library/finding us on social media/frequently asked questions – http://libguides.unisa.ac.za/ask

5 STUDENT SUPPORT SERVICES

For information on the various student support services available at Unisa (e.g. student counseling, tutorial classes, language support), please consult the publication *Study @ Unisa* that you received with your study material.

6 STUDY PLAN

The following table provides an outline of the chapters and ideal dates of completion, and other study activities.

To be completed	Semester 1	Semester 2
Sections 1.1 to 1.3 by	12 February 2018	30 July 2018
Sections 1.4 to 2.2.3 by	26 February 2018	13 August 2018
Sections 2.2.4 to 2.3.5(Exl: 2.3.1 to 2.3.4) by	12 March 2018	27 August 2018
Chapters 1 and 2 by	26 March 2018	10 September 2018
Chapter 3 by	10 April 2018	25 September 2018
Chapter 4 by	24 April 2018	09 October 2018
Chapter 3 and 4 by	08 May 2018	23 October 2018

6.1 The syllabus for APM3713

In this course we will cover Chapters 1 to 4 of the textbook. Sections 2.3.2 to 2.3.4 on electromagnetism will not be examined directly as I realize that not all of the students have a physics background and the laws of electromagnetism are more complex and less intuitive than that of mechanics. However, it will be to your advantage to read through these sections. The syllabus is a combination of different types of knowledge topics, (concepts, processes, contexts) skills and values, and includes the following topics: **Special relativity:** Events, frames of reference, postulates of special relativity, Galilean transformations, Lorentz transformations, intervals, time dilation, length contraction, simultaneity, relativistic Doppler effect, velocity transformation, Minkowski spacetime, Minkowski diagrams, lightcones, causality, the twin paradox, **Relativistic mechanics:** Invariants, relativistic momentum, relativistic kinetic energy, total relativistic energy, mass energy, four-momentum, the energy-momentum relation, four-force, general four-vectors, four-tensors, **Riemannian geometry:** Line elements, curved surfaces, metrics, connections, parallel transport, geodesics, concept of curvature, Gaussian curvature, curvature of spacetime and **General relativity:** Principle of equivalence, principle of general covariance, principle of consistency, energy-momentum tensor, Einstein tensor, Einstein field equations, geodesic motion, cosmological constant.

6.2 Assignment structure

Read this section carefully!

There are 7 assignments for this module and you will be required to work consistently and continuously. During this course, you will complete two kinds of assignments. Assignments 1, 2, 3, 5 and 6 are in the form of multiple choice questions. Assignments 4 and 7 are written assignments.

6.2.1 Multiple choice assignments

You are expected to complete these assignments as you are working through the textbook. These questions are generally simpler than the ones that you will encounter in the exam, but it is important that you do these problems to ensure that you understand the work that you have covered. This way any misconceptions you may have will be corrected before you move on to the next part of the work. Full solutions and explanations to these assignments will be posted on myUnisa a few days after the due date. It is important that you keep your rough work that you used to solve these assignments so that you can see where you went wrong in you reasoning if necessary. The multiple choice assignments will all contribute equally to your semester mark and make up a total of 30% of your semester mark (6% each).

6.2.2 Written assignments

There are two written assignments, one on Chapters 1 and 2, and the other on Chapters 3 and 4. The questions in these assignments are similar to the questions that you will get in the exam, so it would be wise to put some effort into them. These assignments require much more work than the multiple choice assignments, so start working on them early enough. Don't start on them only after the due date of the previous assignment, work on these throughout the semester, as you cover the relevant work. Since these assignments require more work and have more complex questions, they count more towards your semester mark that the multiple choice assignments. The written assignments contribute 70% (35% each) to your semester mark.

See the brochure *Study @ Unisa* for general time management and planning skills.

7 PRACTICAL WORK AND WORK-INTEGRATED LEARNING

There are no practicals for this module.

8 ASSESSMENT

8.1 Assessment criteria

Specific outcome 1: Demonstrate the principles of special relativity.

Assessment criteria

The discussion of the postulates of special relativity shows an understanding of underlining physical concepts.

- The derivation of the Lorentz transformations is comprehensive.
- The applications of Lorentz transformations on spacetime coordinates indicate the consequences accurately.
- The drawings and interpretations of light cones of events that occur in space-time are accurate.
- The explanation of the concept of space-time separation shows understanding.
- The twin paradox is described completely and resolved completely.

Specific outcome 2: Demonstrate physical laws in the context of special relativity.

Assessment criteria

- The explanation of the concept of invariance of physical laws is clear and correct.
- The verification of whether a quantity is invariant (or not) is complete.
- Problems related to relativistic mechanics are solved accurately and logically.
- The explanation of the concept of rest energy is sufficient.
- Calculations with four-vectors are notationally precise and correct.
- Calculations involving basic tensor operations are notationally precise and correct.

Specific outcome 3: Demonstrate the fundamental concepts of non-Euclidean geometry in the context of curved spacetime.

Assessment criteria

- The explanation of what is meant by the concepts of geometry of a curved space is thorough.
- The basic principles of differential geometry are applied correctly.
- Problems using the metric of a surface are performed correctly and logically.
- Calculations involving geodesics on surfaces are done satisfactory and correctly.
- The discussion and interpretation of the concept of curvature when applied to spaces of higher dimensions and spacetime is comprehensive and logical.

Specific outcome 4: Demonstrate the preliminary concepts involved in the theory of general relativity.

Assessment criteria

- The principles of general relativity is formulated and described in a way that shows understanding.
- Algebraic calculations with tensors are notationally precise and correct.
- Covariant derivatives are calculated correctly.
- The explanation of what the energy-momentum and Einstein tensors are comprehensive.
- Calculations with the energy-momentum tensor are performed correctly.
- The calculations performed with the Einstein tensor are correct.
- The application of the Einstein field equations is correct.
- The explanation of the concept of geodesic motion is clear and correct.

8.2 Assessment plan

In each semester there are 5 multiple choice and 2 written assignments for APM3713. All of the assignments count towards your semester mark. Please make sure that you **answer the questions for the semester for which you are registered**. If you answer questions from the wrong semester, your solutions will not be marked and you will get zero marks for the assignments.

Mathematics is a subject that can only be properly mastered by solving problems. This is the reason that there are so many assignments and you are encouraged to solve each problem as best you can. Only selected questions from the written assignments will be marked to count towards your semester mark. Solutions to all the assignment questions will be sent out to students and posted on myUnisa.

In addition to the questions that you have to do for the assignments, it will be advantageous to solve extra problems in preparation for the examination. The textbook has many in-text exercises with solutions at the back. Attempt to solve these yourself before looking at the solutions. The process of thinking about the problem and attempting it will help you a lot more than reading through the answer. You can also use the assignments from the semester that you are not registered for as practice questions.

Assignments will be assessed not only on the mathematical correctness of your work, but also on whether you use mathematical notation and language to communicate your ideas clearly.

Your first attempt at a solution will most likely be "in rough", but the work you submit for marking should be your best version. An assignment should not just be **neat**, your presentation should also be **logical**. An example of sloppy presentation is simply to write down a series of statements without any indication of the connections amongst them (such as implications, etc). For example, consider:

$$x^2 + 4x - 21 = 0; \quad (x + 7)(x - 3) = 0; \quad x = -7, \ x = 3.$$

This is actually three separate statements! One way to rewrite this is as follows:

"By factorizing the left hand side of the equation $x^2 + 4x - 21 = 0$, we get $(x + 7)(x - 3) = 0$, and it follows that $x = -7$ or $x = 3$."

All these points will be taken into consideration when marking assignments.

When marking the assignments, constructive comments will be made on your work, which will then be returned to you. The assignments and the comments on these assignments constitute an important part of your learning and should help you to be better prepared for the next assignment and the examination.

If you need help with problems you must include your attempt at a solution so I can see where you are going wrong. You need to specify your problem clearly.

To be admitted to the examination you need to submit one assignment on or before the compulsory date.

The assignments do not contribute equal weight to your semester mark. The written assignments contribute a total of 70% (35% each) to the semester mark and the multiple choice assignments will make up the other 30% (6% each). Your semester mark for APM3713 counts 30% and your exam mark 70% of your final mark.

A final mark of at least 50% is required to pass the module. If a student does not pass the module then a final mark of at least 40% is required to permit the student access to the supplementary examination. The final mark is composed as follows:

Year mark	
Assignment 01:	6%
Assignment 02:	6%
Assignment 03:	6%
Assignment 04:	35%
Assignment 05:	6%
Assignment 06:	6%
Assignment 07:	35%

$\longrightarrow$

Final mark	
Year mark:	30%
Exam mark:	70%

8.3 Assignment numbers

8.3.1 General assignment numbers

The assignments for this module are Assignment 01, Assignment 02 , etc.

8.3.2 Unique assignment numbers

Please note that each assignment has a unique assignment number which must be written on the cover of your assignment.

8.4 Due dates for assignments

The dates for submission of assignments are:

Semester 1

Assignment Number	Due Dates
01	12 February 2018
02	26 February 2018
03	12 March 2018
04	26 March 2018
05	10 April 2018
06	24 April 2018
07	26 April 2018

Semester 2

Assignment Number	Due Dates
01	30 July 2018
02	13 August 2018
03	27 August 2018
04	10 September 2018
05	25 September 2018
06	28 September 2018
07	04 October 2018

8.5 Submission of assignments

You may submit written assignments either by post or electronically via myUnisa. Assignments may **not** be submitted by fax or e-mail.

For detailed information on assignments, please refer to the *Study @ Unisa* brochure which you received with your study package.

Please make a copy of your assignment before you submit!

To submit an assignment via myUnisa:

- Go to myUnisa.
- Log in with your student number and password.
- Select the module.
- Click on "Assessment Info" in the menu on the left-hand side of the screen.
- Click on the assignment number you wish to submit.
- Follow the instructions.

8.6 The Assignments

Please make sure that you submit the correct assignments for the 1st semester, 2nd semester or year module for which you have registered. For each assignment there is a **fixed closing date**, the date at which the assignment must reach the University. When appropriate, solutions for each assignment will be dispatched, as Tutorial Letter 201 (solutions to Assignment 01) and Tutorial Letter 202 (solutions to Assignment 02) etc., a few days after the closing date. They will also be made available on myUnisa. Late assignments **will not** be marked!

Note that Assignment 01 is the compulsory assignment for admission to the examination and must reach us by the due date.

8.7 Other assessment methods

There are no other assessment methods for this module.

8.8 The examinations

During the relevant semester, the Examination Section will provide you with information regarding the examination in general, examination venues, examination dates and examination times. For general information and requirements as far as examinations are concerned, see the brochure *Study @ Unisa* which you receive with your study material.

Registered for . . .	Examination period	Supplementary examination period
1st semester module	May/June 2018	October/November 2018
2nd semester module	October/November 2018	May/June 2019

9 FREQUENTLY ASKED QUESTIONS

The *Study @ Unisa* brochure contains an A–Z guide of the most relevant study information.

10 SOURCES CONSULTED

None

11 IN CLOSING

We hope that you will enjoy APM3713 and we wish you all the best in your studies at Unisa!

12 ADDENDUM

ADDENDUM A: ASSIGNMENTS - FIRST SEMESTER

ASSIGNMENT 01
Based on Sections 1.1 to 1.3 of the prescribed textbook
Fixed Due Date: 12 February 2018
Only for first semester
UNIQUE ASSIGNMENT NUMBER: **632139**

This is a multiple choice assignment, so you must submit your answers either via myUnisa or on a mark reading sheet. Please consult the booklet *Study @ Unisa* before using myUnisa or completing the mark reading sheet.

Answer **ALL** of the MULTIPLE CHOICE questions below;

1. What is the Lorentz factor (γ) when the relative speed between two coordinate frames is 60 % the speed of light?

   (1) 1.58
   (2) 0.8
   (3) 0.64
   (4) 1.25
   (5) 1.56

2. Alice travels past Bob in a spaceship at a speed of $0.6c$. Take S to be the coordinate frame where Bob is stationary and S' to be the coordinate frame that moves with the Alice. Using the standard configuration, Alice is traveling in the positive x (and x') direction. Bob sets off an explosion and in his frame it explodes at time $t = 5$ s at the point $x = 10^6$ km in his frame. Use the Lorentz transformation equations to determine the coordinate x' that Alice measures in her frame for the explosion.

   For the position of the explosion, Alice measures x' equal to

   (1) 10^6 km
   (2) -1.12×10^9 m
   (3) 10^8 m
   (4) 1.25×10^5 km
   (5) 0 m

3. The sum

$$x' = \sum_{\nu=0}^{3} \Lambda^{\mu}{}_{\nu} x^{\nu}$$

   written out in full is

(1) $x' = \Lambda^{\mu}{}_{\nu} x^{\nu} + \Lambda^{\mu}{}_{\nu} x^{\nu} + \Lambda^{\mu}{}_{\nu} x^{\nu} + \Lambda^{\mu}{}_{\nu} x^{\nu}$

(2) $x' = \Lambda^{\mu}{}_{0} x^{0} + \Lambda^{\mu}{}_{1} x^{1} + \Lambda^{\mu}{}_{2} x^{2} + \Lambda^{\mu}{}_{3} x^{3}$

(3) $x' = \Lambda^{0}{}_{0} x^{0} + \Lambda^{1}{}_{1} x^{1} + \Lambda^{2}{}_{2} x^{2} + \Lambda^{3}{}_{3} x^{3}$

(4) $x' = 3\Lambda^{\mu}{}_{\nu} x^{\nu}$

(5) $x' = \Lambda^{\mu}{}_{\nu} x^{\nu}$

4. A spaceship is moving at such a speed past the Earth that the people on Earth measure its length to be one third of its proper length. How fast is the spaceship moving relative to the Earth?

   (1) $2.67 \times 10^{8}\,\mathrm{ms}^{-1}$
   (2) $-2.67 \times 10^{8}\,\mathrm{ms}^{-1}$
   (3) $8 \times 10^{16}\,\mathrm{ms}^{-1}$
   (4) $8.49 \times 10^{8}\,\mathrm{ms}^{-1}$
   (5) $2.83 \times 10^{8}\,\mathrm{ms}^{-1}$

5. A spaceship moving toward the Earth at a speed of $0.75c$ reports back by transmitting on a frequency (measured in the spaceship rest frame) of 100 MHz. To what frequency must Earth receivers be tuned to receive these signals?

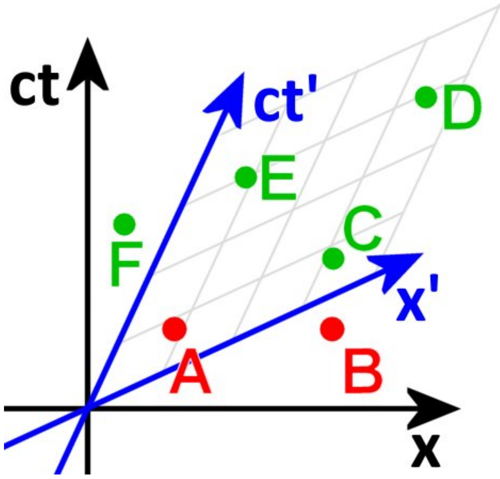
   (1) 264.58 MHz
   (2) 37.80 MHz
   (3) 700 MHz
   (4) 529.15 MHz
   (5) 118.32 MHz

ASSIGNMENT 02
Based on Sections 1.4 to 2.2.3 of the prescribed textbook
Fixed Due Date: 26 February 2018
Only for first semester
UNIQUE ASSIGNMENT NUMBER: **581416**

This is a multiple choice assignment, so you must submit your answers either via myUnisa or on a mark reading sheet. Please consult the booklet *Study @ Unisa* before using myUnisa or completing the mark reading sheet.

Answer **ALL** of the MULTIPLE CHOICE questions below;

An observer in S measures two simultaneous events, Event A and Event B. Another observer is at rest in frame S', which moves in the standard configuration with respect to S at a speed comparable to the speed of light. The situation is indicated in the Minkowski diagram below. Refer to the diagram to answer questions 1 to 3.



1. Would an observer in S' also observe Events A and B to be simultaneous?

   (1) Yes.
   (2) No, he would observe Event A before Event B.
   (3) No, he would observe Event B before Event A.
   (4) It depends on the exact relative speed of S and S'.
   (5) It depends on the exact time measurement that the observer in S measures for the two events.

2. According to the observer in S', which event indicated on the diagram will occur at the same position as Event B?

   (1) A
   (2) C
   (3) D
   (4) F
   (5) There is no such event indicated on the diagram.

3. According to the observer in S, which event indicated on the diagram will occur at the same time as Event F?

   (1) C
   (2) D
   (3) E
   (4) More than one of the above
   (5) None of the above

4. What is the mass energy of a 1 kg lump of metal?

   (1) 3×10^{8} joules
   (2) 9×10^{8} joules

(3) 3×10^{16} joules

(4) 9×10^{16} joules

(5) 9×10^{19} joules

5. At what speed is the relativistic momentum of a particle twice as great as the result obtained from the nonrelativistic expression?

   (1) $0.75c$

   (2) $0.87c$

   (3) $0.50c$

   (4) $2c$

   (5) $0.55c$

6. A proton and neutron collide in an elastic collision. Before the collision, the neutron is stationary and the proton has momentum $\mathbf{p}_p = (-0.3, 0.1, -0.5)$ MeV$/c$ and the proton's momentum after the collision is $(0.2, -0.2, 0.3)$ MeV$/c$. What is the neutron's momentum after the collision?

   (1) $(-0.1, -0.1, -0.2)$ MeV$/c$

   (2) $(0.1, 0.1, 0.2)$ MeV$/c$

   (3) $(0, 0, 0)$ MeV$/c$

   (4) $(0.5, -0.3, 0.8)$ MeV$/c$

   (5) $(-0.5, 0.3, -0.8)$ MeV$/c$

7. A proton (mass $m_p = 938.3\,\text{MeV}/c^2$) that is moving at $0.2c$. What is its kinetic energy?

   (1) 19.35 MeV

   (2) 957.65 MeV

   (3) 18.77 MeV

   (4) 273.04 MeV

   (5) 39.10 MeV

ASSIGNMENT 03

Based on Sections 2.2.4 to 2.3.5(Excl: 2.3.1 to 2.3.4) of the prescribed textbook

Fixed Due Date: 12 March 2018

Only for first semester

UNIQUE ASSIGNMENT NUMBER: **621316**

This is a multiple choice assignment, so you must submit your answers either via myUnisa or on a mark reading sheet. Please consult the booklet *Study @ Unisa* before using myUnisa or completing the mark reading sheet.

Answer **ALL** of the MULTIPLE CHOICE questions below;

1. Consider the following in Minkowski spacetime

$$A_\mu = \sum_\nu \eta_{\mu\nu} A^\nu$$

How many equations does this represent?

(1) 1
(2) 2
(3) 4
(4) 6
(5) 16

2. The four velocity is given by

$$[U^\mu] = \left(U^0,\ U^1,\ U^2,\ U^3\right) = (c\gamma,\ \gamma\mathbf{v})\ .$$

Consider the following quantities from the equation:

a) $[U^\mu]$
b) U^2
c) $\left(U^0,\ U^1,\ U^2,\ U^3\right)$
d) γ
e) $(c\gamma,\ \gamma\mathbf{v})$
f) $\gamma\mathbf{v}$

Which of the following statements are true?

(1) a and c are tensors, b is a vector and d is a scalar
(2) c and e are tensors, f is a vector and b is a scalar
(3) a and b are tensors, f is a vector and d is a scalar
(4) a and e are tensors, d is a vector and f is a scalar
(5) c and f are tensors, e is a vector and b is a scalar

3. The total energy of a photon is 0.745 MeV. What is its momentum?

(1) 1.342 MeV/c
(2) 0.863 MeV/c
(3) 0.210 MeV/c
(4) 0.745 MeV/c
(5) 0.679 MeV

4. An electron (mass m_e = 0.511 MeV/c^2) is moving along the x-axis of an inertial reference frame S with speed $v = 0.8c$, momentum 0.682 MeV/c and total energy 0.852 MeV. What is its momentum as measured in an inertial frame S' that is moving in the standard configuration with speed $0.6c$ relative to S?

(1) $0.682\,\text{MeV}/c$

(2) $0.593\,\text{MeV}/c$

(3) $0.285\,\text{MeV}/c$

(4) $0.214\,\text{MeV}/c$

(5) $0.046\,\text{MeV}/c$

5. A proton (mass $m_p = 938.3\,\text{MeV}/c^2$) is moving with speed $0.4c$ along the x-axis relative to the laboratory frame. What is the value of the first component of the four-momentum P^0 for the proton?

(1) 1210 MeV/c

(2) 1117 MeV/c

(3) 1023 MeV/c

(4) 409.1 MeV/c

(5) 84.45 MeV/c

6. Using equation (2.110) in the textbook, how would a covariant tensor of rank 1 A_ν transform in general?

(1) $A'_\mu = \sum_{\nu=0}^{3} \frac{\partial x'^\nu}{\partial x^\mu} A_\mu$

(2) $A'_\mu = \sum_{\nu=0}^{3} \frac{\partial x^\nu}{\partial x'^\mu} A_\nu$

(3) $A'_\mu = \sum_{\nu=0}^{3} \frac{\partial x'^\mu}{\partial x^\nu} A^\nu$

(4) $A'_\mu = \sum_{\nu=0}^{3} \frac{\partial x^\mu}{\partial x'^\nu} A_\nu$

(5) $A'_\mu = \sum_{\nu=0}^{3} \frac{\partial x^\nu}{\partial x'^\mu} A_\mu$

ASSIGNMENT 04
Based on Chapters 1 and 2 of the prescribed textbook
Fixed Due Date: 26 March 2018
Only for first semester
UNIQUE ASSIGNMENT NUMBER: **811645**

1. As part of its search for extrasolar planets, NASA discovers a planet that appears to be very much like Earth orbiting a star 40 lightyears from our Solar System. An expedition is planned to send astronauts to the planet. NASA would like the astronauts to age no more than 30 years during the journey. In this problem, neglect any issues related to the acceleration of the astronauts' spaceship. (Hint: A lightyear is the distance traveled by light in one year, which is just c multiplied by one year, or 9.46×10^{12} km. In many problems it is simpler to write it as $1c \cdot$ year, since c often cancels out.)

   a) At what velocity must the astronauts' spaceship travel in Earth's reference frame so that the astronauts age 30 years during the journey?

   b) According to the astronauts in the spaceship, what will be the distance of their journey?

   c) Exactly half way to the planet, two of the astronauts get homesick and set off in a space module to return to Earth. According to the astronauts who remain on the spaceship, the module travels at a velocity of $5c/6$ in the direction toward Earth. Find the total amount of time that the two astronauts will have been away according to people on Earth.

2. Tshepo travels to work on the Gautrain every day. He travels between the Hatfield and Park stations, which is a 62 km distance, one way. Suppose the train travels at a constant speed of $160\,\text{kmh}^{-1}$ every trip to and from work, and that he works 250 day per year. How many years would it take for him to be $1\,\mu\text{s} = 10^{-6}\,\text{s}$ younger due to travelling on the Gautrain? (Hints: For speeds that are $V \ll c$, it is necessary to use the binomial theorem when calculating γ. Also, leave all your equations in symbolic form right until the end for the best accuracy.)

3. Maxwell's wave equation for an electric field propagating in the x-direction is

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2},$$

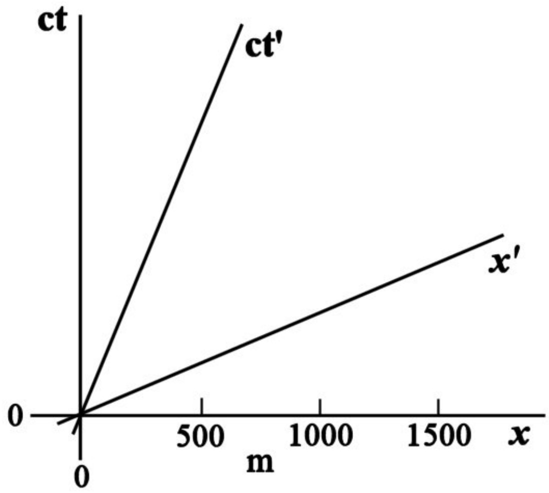
   where $E\,(x,\ t)$ is the amplitude of the electric field. Show that this equation is *not* invariant under a Galilean transformation to a reference frame moving with relative speed v along the$x$-axis.

4. In frame S, event B occurs $2\,\mu$s (2×10^{-6} s) after event A and at $x_B = 1.5$ km from event A. Take event A to occur at time $t_A = 0$ and position $x_A = 0$ in frame S.

   a) How fast must an observer in frame S' be moving along the positive x-axis so that events A and B occur simultaneously in his frame?

   b) Is it possible for event B to precede event A for some observer?

   c) Roughly copy the Minkowski diagram below for frames S and S'. Indicate events A and B on your diagram. If you answered "yes" to part (b) indicate the axes ct'' and x'' of an inertial frame S'' for which event B occurs before event A. If you answered "no" to part (b), use the diagram to explain why.



d) Compute the spacetime separation $(\Delta s)^2$ between the events.

e) Are the two events causally related? Explain your answer.

5. Consider a particle with mass $m = 10^{-25}$ kg that is moving at a constant velocity described by the vector $\mathbf{v} = (0.3c,\ 0.7c\,,\ -0.4c)$ relative to an observer in S.

   a) What is the contravariant four-momentum $[P^\mu]$ of the particle?

   b) Assuming Minkowski spacetime, determine the covariant counterpart of the four-momentum $\left[P_\mu\right]$.

6. A particle is measured in an inertial frame S to have a total energy of $E = 5\,\text{GeV}$ ($1\,\text{GeV} = 10^9\,\text{eV}$) and momentum of $p = 3\,\text{GeV}/c$.

   a) What is the mass of the particle, in GeV/c^2?

   b) What is the speed of the particle?

   c) What is the energy E' of the particle in another inertial frame S' in which the particle's momentum is $p' = 4\,\text{GeV}/c$?

   d) What is the kinetic energy of the particle in S'?

   e) What is the maximum momentum this particle can have, according to the limits set by special relativity?

ASSIGNMENT 05
Based on Chapter 3 of the prescribed textbook
Fixed Due Date: 10 April 2018
Only for first semester
UNIQUE ASSIGNMENT NUMBER: **638485**

This is a multiple choice assignment, so you must submit your answers either via myUnisa or on a mark reading sheet. Please consult the booklet *Study @ Unisa* before using myUnisa or completing the mark reading sheet.

Answer **ALL** of the MULTIPLE CHOICE questions below;

1. The line element for a certain two dimensional Riemann space is given by

$$dl^2 = d\bar{x}^2 + 2\cos\theta d\bar{x}d\bar{y} + d\bar{y}^2 \,.$$

What is the metric tensor of this space?

(1) $\begin{pmatrix} 1 & \cos\theta \\ \cos\theta & 1 \end{pmatrix}$

(2) $\begin{pmatrix} \cos\theta & 1 \\ 1 & \cos\theta \end{pmatrix}$

(3) $\begin{pmatrix} 1 & 2\cos\theta \\ 2\cos\theta & 1 \end{pmatrix}$

(4) $\begin{pmatrix} 1 & 2\cos\theta \\ 0 & 1 \end{pmatrix}$

(5) $\begin{pmatrix} 1 & 0 \\ 2\cos\theta & 1 \end{pmatrix}$

2. The sum

$$\sum_{i,j=1}^{3} g_{ij}g^{ij}$$

is equal to...

(1) 0

(2) 1

(3) 2

(4) 3

(5) 4

3. Equation 2.70 in the textbook is written for four dimensional Minkowski space and gives a rule to determine the covariant form of a vector if the metric and contravariant form is known. This same equation written for a general two dimensional space is

$$A_j = \sum_{i=1}^{2} g_{ij}A^i \,.$$

Use this to determine the covariant form of $\left[A^i\right]$ in two dimensional space described by the surface of a paraboloid. The metric tensor for this space is

$$\left[g_{ij}\right] = \begin{pmatrix} 1 + a^2r^2 & 0 \\ 0 & r^2 \end{pmatrix}$$

and let

$$\left[A^i\right] = \begin{pmatrix} 1 \\ a^2 \end{pmatrix} \,.$$

(1) $[A_i] = \begin{pmatrix} a^2 \\ 1 \end{pmatrix}$

(2) $[A_i] = \begin{pmatrix} 1 + a^2r^2 \\ a^2r^2 \end{pmatrix}$

(3) $[A_i] = \begin{pmatrix} a^2 + a^4r^2 \\ r^2 \end{pmatrix}$

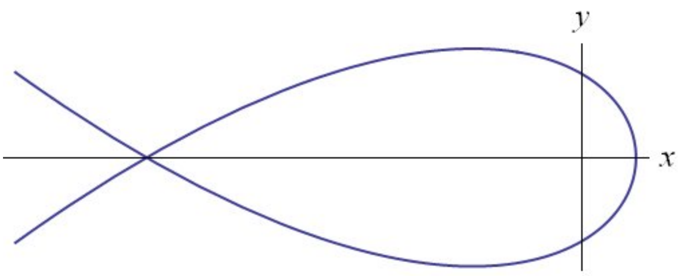
(4) $[A_i] = \begin{pmatrix} a^2 \\ 1 + a^2r^2 \end{pmatrix}$

(5) $[A_i] = \begin{pmatrix} 1 + a^2r^2 + a^2 \\ 1 + a^2r^2 \end{pmatrix}$

4. The Tschirnhausen Cubic is a curve with parametric equations

$$\begin{aligned} x &= a\left(1 - 3t^2\right) \\ y &= at\left(3 - t^2\right) \end{aligned}$$

An example of this curve is shown below for $a > 0$.



What is the general equation for the curvature of the Tschirnhausen Cubic?

(1) $\left[2\left(3t^2 - 1\right)\right]\left[3a\left(t^2+1\right)^3\right]^{-1}$

(2) $2\left[3a\left(t^2+1\right)^2\right]^{-1}$

(3) $-3\left[2\left(t^2-9\right)^{3/2}\right]^{-1}$

(4) $\left[2\left(1+t^2\right)\right]\left[-3a\left(1-6t^2+t^4\right)^{3/2}\right]^{-1}$

(5) $\left(1+t^2\right)\left[-3\left(1-3t^2+t^4\right)^{3/2}\right]^{-1}$

5. Consider a surface with a metric tensor

$$g_{ij} = \begin{pmatrix} \left(1+u^2+v^2\right)^2 & 0 \\ 0 & \left(1+u^2+v^2\right)^2 \end{pmatrix}$$

where $x^1 = u$ and $x^2 = v$. What is the value of the connection coefficient $\Gamma^2{}_{12}$?

(1) $4u\left(1+u^2+v^2\right)^3$

(2) $(v)\,/\left(1+u^2+v^2\right)^2$

(3) $(4u)\,/\left(1+u^2+v^2\right)$

(4) $(2v)\,/\left(1+u^2+v^2\right)$

(5) $(2u)\,/\left(1+u^2+v^2\right)$

ASSIGNMENT 06
Based on Chapter 4 of the prescribed textbook
Fixed Due Date: 24 April 2018
Only for first semester
UNIQUE ASSIGNMENT NUMBER: **828011**

This is a multiple choice assignment, so you must submit your answers either via myUnisa or on a mark reading sheet. Please consult the booklet *Study @ Unisa* before using myUnisa or completing the mark reading sheet.

Answer **ALL** of the MULTIPLE CHOICE questions below;

1. The transformation equations for transforming a contravariant tensor of rank one from polar to Cartesian coordinates are

$$\begin{aligned} A'^1 &= A^1\cos\theta - A^2 r\sin\theta \\ A'^2 &= A^1\sin\theta + A^2 r\cos\theta \end{aligned}$$

   where $x^i = (r,\,\theta)$ (derived in Exercise 4.2 in the textbook). Use these to transform the tensor described by $\left[A^i\right] = (1/\cos\theta,\, r)$ to Cartesian coordinates $x'^i = (x,\,y)$. What is the value of A'^1?

   (1) $\cos^2\theta\left(1+r^2\right)$

   (2) $\tan\theta + r^2\cos\theta$

   (3) $1 - r^2\sin\theta$

   (4) $\cos\theta - r\sin\theta$

   (5) $r\cos\theta - r\tan\theta$

2. Which of the following tensor expressions is incorrect?

   (1) $A^i = \sum_j g^{ij}A_j = \sum_{j,k} g^{ij}g_{jk}A^k$

   (2) $\bar{A}^i{}_{kl} = \sum_{p,r,s} \dfrac{\partial\bar{x}^i}{\partial x^p}\dfrac{\partial x^r}{\partial\bar{x}^k}\dfrac{\partial x^s}{\partial\bar{x}^l}A^p_{rs}$

(3) $\Gamma_{\alpha\beta\gamma} = \frac{1}{2}\left(\frac{\partial g_{\alpha\gamma}}{\partial x^{\beta}} + \frac{\partial g_{\beta\alpha}}{\partial x^{\gamma}} - \frac{\partial g_{\beta\gamma}}{\partial x^{\alpha}}\right)$

(4) $\sum_{i=1}^{3} \delta^{i}{}_{i} = 1$

(5) $\Gamma^{i}{}_{jk} = \Gamma^{i}{}_{kj}$

3. Which of the following expressions is correct?

(1) $R^{\alpha}{}_{\beta}{}^{\gamma}{}_{\delta} = \sum_{\gamma} g^{\eta\gamma} R^{\alpha}{}_{\beta\gamma\delta}$

(2) $R^{\alpha}{}_{\beta}{}^{\gamma}{}_{\delta} = \sum_{\gamma} g_{\eta\gamma} R^{\alpha}{}_{\beta\gamma\delta}$

(3) $R^{\alpha}{}_{\beta}{}^{\gamma}{}_{\delta} = \sum_{\eta} g_{\eta\gamma} R^{\alpha}{}_{\beta\gamma\delta}$

(4) $R^{\alpha}{}_{\beta}{}^{\gamma}{}_{\delta} = \sum_{\eta} g^{\eta\gamma} R^{\alpha}{}_{\beta\gamma\delta}$

(5) $R^{\alpha}{}_{\beta}{}^{\gamma}{}_{\delta} = \sum_{\alpha} g^{\alpha\gamma} R^{\alpha}{}_{\beta\gamma\delta}$

4. How many equations does the following expression represent?

$$T^{\mu\nu} = \left(\rho + p/c^{2}\right) U^{\mu} U^{\nu} - p g^{\mu\nu}$$

(1) 1

(2) 2

(3) 4

(4) 8

(5) 16

ASSIGNMENT 07

Based on Chapters 3 and 4 of the prescribed textbook

Fixed Due Date: 26 April 2018

Only for first semester

UNIQUE ASSIGNMENT NUMBER: **724533**

1. A paraboloid can be parametrized as

$$\begin{aligned} x(u, v) &= au\cos v \\ y(u, v) &= au\sin v \\ z(u, v) &= u^{2} \end{aligned}$$

(a) Find the line element for the surface.

(b) What is the metric tensor and the dual metric tensor?

(c) Determine the values of all the Christoffel coefficients of the surface.

(d) What is the value of the Riemann curvature tensor?

(e) What is the Ricci tensor for the surface?

(f) What is the curvature scalar R for the surface?

(g) What is the Gaussian curvature of the surface?

(h) Is the surface Euclidean? Explain your answer.

(i) Suppose that the surface is filled with non-interacting particles, or dust. Use the two dimensional version of the energy-momentum tensor for dust and Einstein's field equation to find an expression for the Einstein constant κ for this surface.

2 In a given frame of reference, with coordinates $\left(x^0, x^1, x^2, x^3\right)$, the components of a symmetric second order tensor $[A^{\mu\nu}]$ are given by $A^{03} = -2$, $A^{13} = 1$ and $A^{02} = -1$, with all unspecified components being zero. Find the value of the component A'^{03} of the tensor $[A'^{\mu\nu}]$ in a frame obtained by the coordinate transformation

$$\begin{aligned} x'^0 &= 4x^0 - 13x^1 + 4x^2 - 15x^3 \\ x'^1 &= -13x^0 + 14x^1 - 3x^2 + 12x^3 \\ x'^2 &= x^0 - 4x^1 + 11x^2 - 5x^3 \\ x'^3 &= 8x^0 + 14x^1 - 3x^2 + 17x^3 \end{aligned}$$

3. Show that if the metric g_{ij} is diagonal, then $\Gamma^i_{kl} = 0$ whenever i, k and i are distinct, i.e. whenever $i \neq k \neq l$.

4. If $A_{ab} = A_{ba}$ and $B^{ab} = -B^{ba}$ for all a, b, show that $A_{ab}B^{ab} = 0$.

5. The principle of consistency requires that the laws of general relativity should approximate the laws of Newtonian physics in the Newtonian limit. Show that the relativistic momentum $p = \gamma mv$ reduces to the classical momentum when $v \ll c$.

ADDENDUM B: ASSIGNMENTS - SECOND SEMESTER

ASSIGNMENT 01
Based on Sections 1.1 to 1.3 of the prescribed textbook
Fixed Due Date: 30 July 2018
Only for second semester
UNIQUE ASSIGNMENT NUMBER: **887583**

This is a multiple choice assignment, so you must submit your answers either via myUnisa or on a mark reading sheet. Please consult the booklet *Study @ Unisa* before using myUnisa or completing the mark reading sheet.

Answer **ALL** of the MULTIPLE CHOICE questions below.

1. Alice sees an explosion happening and measures the spacetime coordinates of the explosion to be $(t, x, y, z) = (1.5\,\text{ns}, 2\,\text{m}, 1\,\text{m}, 0\,\text{m})$. Bob is riding past in a train at a constant speed of $V = 0.4c$ in the positive x-direction. Use the Lorentz transformation equations to determine what time Bob measures the explosion taking place. (Hint: 1 ns (nanosecond) = 10^{-9}s).

   (1) $1.5\,\text{ns}$
   (2) $1.407 \times 10^{-9}\,\text{s}$
   (3) $-1.17 \times 10^{-9}\,\text{s}$
   (4) $1.5\,\text{s}$
   (5) $-1.3\,\text{ns}$

2. A group of astronauts take on a mission to travel to a nearby planet. According to the people on Earth, the spaceship takes 100 years to reach its destination, but the astronauts on the spaceship only aged 30 years during the journey. How fast was the spaceship travelling, assuming that it was moving at a constant velocity?

   (1) $0.83c$
   (2) $0.95c$
   (3) $3.48c$
   (4) $0.91c$
   (5) $1.04c$

3. The sum

$$B_\alpha = \sum_{\beta=0}^{3} \eta_{\beta\alpha} B^\beta$$

   written out in full is

   (1) $B_\alpha = 3\eta_{\beta\alpha} B^\beta$
   (2) $B_\alpha = \eta_{\beta\alpha} B^\beta$
   (3) $B_\alpha = \eta_{\beta\alpha} B^\beta + \eta_{\beta\alpha} B^\beta + \eta_{\beta\alpha} B^\beta + \eta_{\beta\alpha} B^\beta$
   (4) $B_\alpha = \eta_{0\alpha} B^0 + \eta_{1\alpha} B^1 + \eta_{2\alpha} B^2 + \eta_{3\alpha} B^3$
   (5) $B_\alpha = \eta_{00} B^0 + \eta_{11} B^1 + \eta_{22} B^2 + \eta_{33} B^3$

4. Two alien races, the Klingons and the Romulans are having a battle in space. During the battle, two of the spaceships approach each other head on at a speed of $0.4c$. The Klingon ship shoots a torpedo in the direction of the Romulan ship. The Klingons measure the torpedo leaving their ship at a speed of $0.6c$. How fast does the Romulans measure the torpedo to be approaching them?

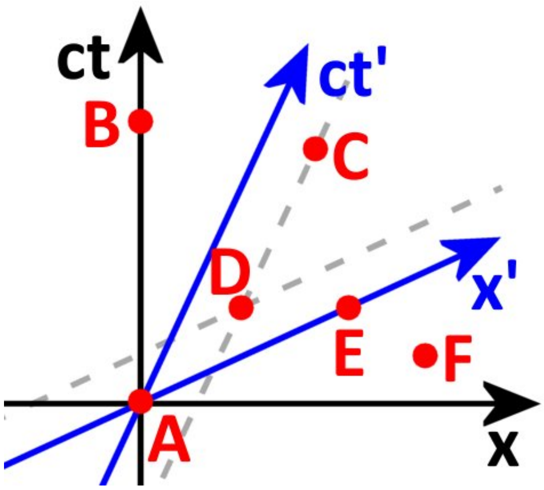
   (1) c
   (2) $0.26c$
   (3) $0.81c$
   (4) $1.32c$
   (5) $0.73c$

ASSIGNMENT 02
Based on Sections 1.4 to 2.2.3 of the prescribed textbook
Fixed Due Date: 13 August 2018
Only for second semester
UNIQUE ASSIGNMENT NUMBER: **831155**

This is a multiple choice assignment, so you must submit your answers either via myUnisa or on a mark reading sheet. Please consult the booklet *Study @ Unisa* before using myUnisa or completing the mark reading sheet.

Answer **ALL** of the MULTIPLE CHOICE questions below.

Consider the spacetime diagram below to answer questions 1 to 3.



1. In the S frame, the following two events occur simultaneously?

   (1) A and B
   (2) A and E
   (3) A and D
   (4) D and C
   (5) D and E

2. To which point would observers in both the S and S' frame assign the same spacetime coordinates?

   (1) A
   (2) B
   (3) C
   (4) E
   (5) None of the points

3. Which of the following statements are true *for all frames*?

   (1) Event A happens before event C
   (2) Events E and F are causally related
   (3) Events A and B occur at the same position
   (4) Events A and B occur at the same time
   (5) Event C caused event F

4. What is the mass energy of a proton with rest mass $m_p = 1.67 \times 10^{-27}$ kg?

   (1) 5.01×10^{-19} joules
   (2) 1.50×10^{-10} joules
   (3) 1.50×10^{44} joules
   (4) 1.67×10^{-27} joules
   (5) 9×10^{16} joules

5. How fast must a body be travelling so that its correct relativistic momentum is 1 % greater than the classical momentum?

   (1) $0.04c$
   (2) $0.42c$
   (3) $0.14c$
   (4) $1.41c$
   (5) $1.35c$

6. A proton and neutron collide in an elastic collision. Before the collision, the neutron is stationary and the proton has momentum $\mathbf{p}_p = (0.4, -0.2, 0.8)$ MeV$/c$ and the proton's momentum after the collision is $(-0.2, -0.5, 0.6)$ MeV$/c$. What is the neutron's momentum after the collision?

   (1) $(0.2, 0.5, -0.6)$ MeV$/c$
   (2) $(0.2, -0.7, 1.4)$ MeV$/c$
   (3) $(0.6, 0.3, 0.2)$ MeV$/C$
   (4) $(0, 0, 0)$ MeV$/c$
   (5) $(-0.6, 0.7, 0.2)$ MeV$/c$

7. A proton (mass $m_p = 938.3\,\text{MeV}/c^2$) is moving with speed $0.4c$ along the x-axis relative to the laboratory frame. What is its kinetic energy?

   (1) 7.6×10^{18} MeV
   (2) 1023 MeV
   (3) 178.7 MeV
   (4) 273 MeV

(5) 85.39 MeV

ASSIGNMENT 03
Based on Sections 2.2.4 to 2.3.5()Excl: 2.3.1 to 2.3.4) of the prescribed textbook
Fixed Due Date: 27 August 2018
Only for second semester
UNIQUE ASSIGNMENT NUMBER: **669266**

This is a multiple choice assignment, so you must submit your answers either via myUnisa or on a mark reading sheet. Please consult the booklet *Study @ Unisa* before using myUnisa or completing the mark reading sheet.

Answer **ALL** of the MULTIPLE CHOICE questions below.

1. An electron (mass $m_e = 0.511\,\text{MeV/c}^2$) is moving along the x-axis of an inertial reference frame S with speed $v = 0.8c$, momentum 0.682 MeV/c and total energy 0.852 MeV. What is its total energy in an inertial frame S' that is moving in the standard configuration with speed $0.6c$ relative to S?

   (1) 0.852 MeV
   (2) 0.738 MeV
   (3) 0.554 MeV
   (4) 0.511 MeV
   (5) 0.443 MeV

2. For the electron described in the previous question, what is the measured value of the momentum in the S' frame?

   (1) $0.682\,\text{MeV}/c$
   (2) $0.593\,\text{MeV}/c$
   (3) $0.285\,\text{MeV}/c$
   (4) $0.214\,\text{MeV}/c$
   (5) $0.046\,\text{MeV}/c$

3. A proton (mass $m_p = 938.3\,\text{MeV}/c^2$) is moving with speed $0.4c$ along the x-axis relative to the laboratory frame. What is its kinetic energy?

   (1) 7.6×10^{18} MeV
   (2) 1023 MeV
   (3) 178.7 MeV

(4) 273 MeV
(5) 84.45 MeV

4. What is the value of the first element of the four-momentum P^0 for the proton described in the previous question?

(1) 1210 MeV/c
(2) 1117 MeV/c
(3) 1023 MeV/c
(4) 409.1 MeV/c
(5) 84.45 MeV/c

5. Using equation (2.110) in the study guide, how would a contravariant tensor of rank 1 A^ν transform in general?

(1) $A'^\mu = \sum_{\nu=0}^{3} \frac{\partial x'^\nu}{\partial x^\mu} A^\mu$

(2) $A'^\mu = \sum_{\nu=0}^{3} \frac{\partial x^\mu}{\partial x'^\nu} A_\nu$

(3) $A'^\mu = \sum_{\nu=0}^{3} \frac{\partial x^\mu}{\partial x'^\nu} A^\nu$

(4) $A'^\mu = \sum_{\nu=0}^{3} \frac{\partial x'^\mu}{\partial x^\nu} A^{\nu*}$

(5) $A'^\mu = \sum_{\nu=0}^{3} \frac{\partial x^\nu}{\partial x'^\mu} A^\mu$

ASSIGNMENT 04
Based on Chapters 1 and 2 of the prescribed textbook
Fixed Due Date: 10 September 2018
Only for second semester
UNIQUE ASSIGNMENT NUMBER: **830986**

1. Bob, standing at the rear end of a railroad car, shoots an arrow toward the front end of the car. The velocity of the arrow as measured by Bob is $1/5c$. The length of the car as measured by Bob is 150 meters. Alice, standing on the station platform observes all of this as the train passes by her with a velocity of $3/5c$. What values does Alice measure for the following quantities:

(a) The length of the railroad car?

(b) The velocity of the arrow?

(c) The amount of the time the arrow is in the air?

(d) The distance that the arrow travels?

2. Maxwell's wave equation for an electric field propagating in the x-direction is

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2},$$

where $E(x, t)$ is the amplitude of the electric field. Show that this equation is invariant under a Lorentz transformation to a reference frame moving with relative speed v along the x-axis.

3. A physics professor claims in court that the reason he went through the red light ($\lambda = 650\,\text{nm}$) was that, due to his motion, the red color was Doppler shifted to green ($\lambda = 550\,\text{nm}$). How must he have been going for his story to be true? Hint: The relation between frequency f and wavelength λ of light is given by $c = \lambda f$.

4. In the context of special relativity, a contravariant four-vector can be constructed from the charge density ρ and the current density $\mathbf{j}$ as follows $[J^\mu] = \left(c\rho, j_x, j_y, j_z\right)$ where j_x, j_y and j_z are the components of $\mathbf{j}$ in the x, y and z directions, respectively. Hint: To answer the questions below, use the properties of four-vectors. Do not try to solve this using electromagnetism.

   (a) Determine the transformation equations of J^μ to a frame S' that is moving with a constant speed V in the positive x-direction.

   (b) Construct a quantity using the components of J^μ that is a Lorentz invariant in Minkowski spacetime.

   (c) Imagine you are in a reference frame in which $\rho = 2/c$ and $j_x = j_y = j_z = 2$. Determine $[J'^\mu]$ as measured by someone moving at a velocity $V = \sqrt{3/4}c$ along the x-direction with respect to your reference frame.

5. An electron e^- with kinetic energy 1 MeV makes a head-on collision with a positron e^+ that is at rest. (A positron is an antimatter particle that has the same mass as an electron, but opposite charge.) In the collision the two particles annihilate each other and are replaced by two photons γ of equal energy. The reaction can be written as

$$e^- + e^+ \to 2\gamma.$$

Determine the energy, momentum and speed of each photon.

6. In special relativity, the energy, momentum and mass of a particle are all closely related to one another.

   (a) Derive the relation $E^2 = c^2p^2 + m^2c^4$ by starting from the relativistic definitions of E and p, i.e. $E = \gamma mc^2$ and $p = \gamma mv$.

   (b) Use the equation derived in part (a) to show that the mass of a particle can be expressed as

$$m = \frac{c^2p^2 - E_k^2}{2E_kc^2}$$

   where E_k is the kinetic energy of the particle.

ASSIGNMENT 05
Based on Chapter 3 of the prescribed textbook
Fixed Due Date: 25 September 2018
Only for second semester
UNIQUE ASSIGNMENT NUMBER: **821468**

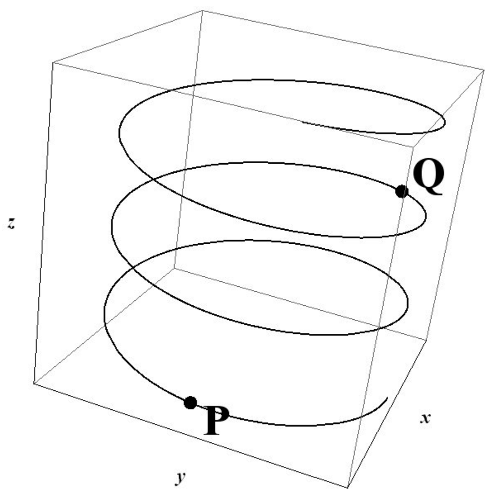
This is a multiple choice assignment, so you must submit your answers either via myUnisa or on a mark reading sheet. Please consult the booklet *Study @ Unisa* before using myUnisa or completing the mark reading sheet.

Answer **ALL** of the MULTIPLE CHOICE questions below.

1. The equation for a length of a curve in an Euclidean plane can easily be generalized to give the length of a curve that exists in Euclidean (normal) three dimensional space. The length of such a space curve is given by

$$L(\mathrm{P}, \mathrm{Q}) = \int_P^Q dl = \int_{u_\mathrm{P}}^{u_\mathrm{Q}} \left(\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2 + \left(\frac{dz}{du}\right)^2 \right)^{1/2} du$$

Consider the circular helix described by $x = \sin u$, $y = \cos u$ and $z = u/10$ with the points P and Q defined by the points where $u_\mathrm{P} = \pi/2$ and $u_\mathrm{Q} = 15\pi/4$ as shown in the figure below.



What is the length of the curve between points P and Q in arbitrary units?

(1) 18.79
(2) 13.42
(3) 10.26
(4) 10.31
(5) 1.02

2. The line element for a certain two dimensional Riemann space is given by
$$dl^2 = dr^2 + 2r\sin\phi drd\phi + r^2 d\phi^2 \,.$$
What is the metric tensor of this space?
   (1) $\begin{pmatrix} 1 & 2r\sin\phi \\ 2r\sin\phi & r^2 \end{pmatrix}$
   (2) $\begin{pmatrix} 1 & r\sin\phi \\ r\sin\phi & r^2 \end{pmatrix}$
   (3) $\begin{pmatrix} r\sin\phi & 1 \\ 1 & r\sin\phi \end{pmatrix}$
   (4) $\begin{pmatrix} r^2 & 2r\sin\phi \\ 0 & 1 \end{pmatrix}$
   (5) $\begin{pmatrix} 1 & 0 \\ 2r\sin\phi & r^2 \end{pmatrix}$

3. The sum
$$\sum_{i=1}^{3} \delta_{ii}$$
is equal to...
   (1) 0
   (2) 1
   (3) 2
   (4) 3
   (5) 4

4. Equation 2.70 in the textbook is written for four dimensional Minkowski space and gives a rule to determine the covariant form of a vector if the metric and contravariant form is known. This same equation written for a general two dimensional space is
$$A_j = \sum_{i=1}^{2} g_{ij} A^i \,.$$
Use this to determine the covariant form of $\left[A^i\right]$ in two dimensional space described by the surface of a unit sphere. The metric tensor (with $x^1 = \theta$ and $x^2 = \phi$) for this space is
$$\left[g_{ij}\right] = \begin{pmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{pmatrix}$$
and let
$$\left[A^i\right] = \begin{pmatrix} \pi \\ \pi/4 \end{pmatrix} .$$
   (1) $[A_i] = \begin{pmatrix} \pi \\ 1/2 \end{pmatrix}$

(2) $[A_i] = \begin{pmatrix} \pi \\ \pi/\left(4\sqrt{2}\right) \end{pmatrix}$

(3) $[A_i] = \begin{pmatrix} \pi \\ \pi/2 \end{pmatrix}$

(4) $[A_i] = \begin{pmatrix} \pi \\ \pi/8 \end{pmatrix}$

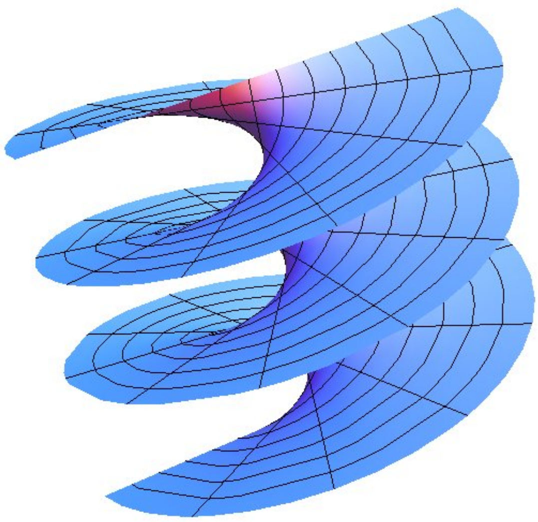
(5) $[A_i] = \begin{pmatrix} \pi \\ \pi/4 \end{pmatrix}$

5. Calculate R^1_{221} of the right helicoid shown below that is parametrized as

$$\begin{aligned} x &= u\cos v \\ y &= u\sin v \\ z &= cv \end{aligned}$$

where c is a constant and $x^1 = u$ and $x^2 = v$ if it is given that the only non-zero Christoffel coefficients for the surface are

$$\begin{aligned} \Gamma^2_{12} &= \Gamma^2_{21} = \frac{u}{u^2 + c^2} \\ \Gamma^1_{22} &= \frac{-u}{u^2 + c^2} \end{aligned}$$



(1) $\left(c^2 - 2u^2\right)\left(u^2 + c^2\right)^{-2}$

(2) $-c^2\left(u^2 + c^2\right)^{-2}$

(3) $c^2\left(u^2 + c^2\right)^{-2}$

(4) $-u\left(c^2 + u + u^2\right)\left(u^2 + c^2\right)^{-2}$

(5) $-u^2\left(u^2 + c^2\right)^{-2}$

ASSIGNMENT 06
Based on Chapters 4 of the prescribed textbook
Fixed Due Date: 28 September 2018
Only for second semester
UNIQUE ASSIGNMENT NUMBER: **820406**

This is a multiple choice assignment, so you must submit your answers either via myUnisa or on a mark reading sheet. Please consult the booklet *Study @ Unisa* before using myUnisa or completing the mark reading sheet.

Answer **ALL** of the MULTIPLE CHOICE questions below.

1. The transformation equations for transforming a contravariant tensor of rank one from polar to Cartesian coordinates are

$$\begin{aligned} A'^1 &= A^1 \cos\theta - A^2 r \sin\theta \\ A'^2 &= A^1 \sin\theta + A^2 r \cos\theta \end{aligned}$$

where $x^i = (r, \theta)$ (derived in Exercise 4.2 in the textbook). Use these to transform the tensor described by $\left[A^i\right] = (1/\cos\theta, r)$ to Cartesian coordinates $x'^i = (x, y)$. What is the value of A'^2?

(1) $\cos^2\theta\left(1 + r^2\right)$
(2) $\tan\theta + r^2\cos\theta$
(3) $1 - r^2\sin\theta$
(4) $\cos\theta - r\sin\theta$
(5) $r\cos\theta - r\tan\theta$

2. Which of the following tensor expressions is incorrect?

(1) $A^i = \sum_j g^{ij}A_j = \sum_{j,k} g^{ij}g_{jk}A^k$
(2) $\bar{A}^i{}_{kl} = \sum_{p,r,s} \frac{\partial \bar{x}^i}{\partial x^p}\frac{\partial x^r}{\partial \bar{x}^k}\frac{\partial x^s}{\partial \bar{x}^l}A^p{}_{rs}$
(3) $\Gamma_{\alpha\beta\gamma} = \frac{1}{2}\left(\frac{\partial g_{\alpha\gamma}}{\partial x^\beta} + \frac{\partial g_{\beta\alpha}}{\partial x^\gamma} - \frac{\partial g_{\beta\gamma}}{\partial x^\alpha}\right)$
(4) $\sum_{i=1}^{3} \delta^i{}_i = 3$
(5) $\Gamma^i{}_{jk} = \Gamma^j{}_{ik}$

3. Which of the following expressions are correct?

(1) $\sum_m \delta_l^m g_{km} = 3g_{km}$

(2) $\sum_m \delta_l^m g_{km} = g^{kl}$

(3) $\sum_m \delta_l^m g_{km} = g^{km}$

(4) $\sum_m \delta_l^m g_{km} = g_{kl}$

(5) $\sum_m \delta_l^m g_{km} = g_{km}$

4. How many equations does the following expression represent?

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\kappa T_{\mu\nu}$$

(1) 1

(2) 2

(3) 4

(4) 8

(5) 16

ASSIGNMENT 07
Based on Chapters 3 and 4 of the prescribed textbook
Fixed Due Date: 04 October 2018
Only for second semester
UNIQUE ASSIGNMENT NUMBER: **689805**

1. A cylindrical surface with radius a can be parametrized as

$$\begin{aligned} x(u, v) &= a\cos v \\ y(u, v) &= a\sin v \\ z(u, v) &= u \end{aligned}$$

(a) Find the line element for the surface.

(b) What is the metric tensor and the dual metric tensor?

(c) Determine the values of all the Christoffel coefficients of the surface.

(d) What is the value of the Riemann curvature tensor?

(e) What is the Ricci tensor for the surface?

(f) What is the curvature scalar R for the surface?

(g) What is the Gaussian curvature of the surface?

(h) Is the surface Euclidean? Explain your answer.

(i) Suppose that the surface is filled with non-interacting particles, or dust. Use the two dimensional version of the energy-momentum tensor for dust and Einstein's field equation to find an expression for the Einstein constant κ for this surface.

2. Show that the contracted Christoffel symbol Γ^i_{ik} is given by

$$\Gamma^i_{ik} = \frac{g^{im}}{2} \frac{\partial g_{mi}}{\partial x^k}$$

3. Verify that if a tensor is symmetric in one frame, it will be symmetric in all coordinate frames. That is, show that if it is given that $X^{ij} = X^{ji}$ in frame S, then it will be true that $\bar{X}^{ij} = \bar{X}^{ji}$ in a coordinate frame $\bar{S}$.

4. Suppose that $R_{iklm} = K\left(g_{il}g_{km} - g_{im}g_{kl}\right)$ on some four dimensional Riemannian space. Show that for the curvature scalar we have $R = -12K$.

5. Two N-dimensional Riemann spaces M and $\bar{M}$ have the metric tensors g_{ij} and $\bar{g}_{ij}$ respectively, and

$$\bar{g}_{ij} = kg_{ij}$$

where k is a constant. What are the relationships between the curvature tensors, Ricci tensors, curvature scalar and Einstein tensors of the two spaces?

ADDENDUM C: Additional Notes

Note that it is crucial for you to understand the content of this module in order to be able to properly do your assignments on your own and solve related problems. For this purpose, video notes will be uploaded under the main website of this module https://my.unisa.ac.za/portal/site/MODULECODE-18-Y1. In order to fully understand the concepts of this module and benefit from this course, you can request video notes from the lecturer about specific topics as well.