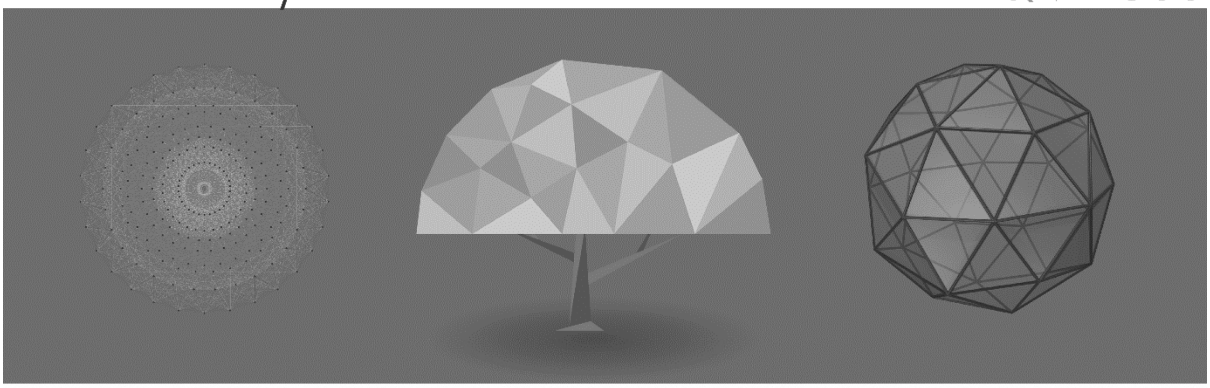
STUDY GUIDE 2 FOR

Elementary Quantitative Methods

QMI1500



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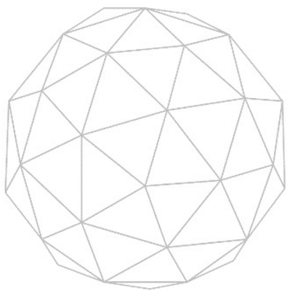

WORKBOOK

The purpose of this workbook is
to give students extra examples
and exercises for all the topics
covered in the study guide.

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This study guide contains the workbook for QMI1500

Contents

COMPONENT 1

Numbers and working with numbers

The following examples and activities will improve your understanding of the basic principles covered in component 1 of the study guide. There is a worksheet for every study unit in component 1. Work through each worksheet after you have studied the relevant study material in the study guide.

1.1 Worksheet 1

Worksheet 1 is based on study unit 1.1: *Priorities and laws of operations*, on pages 2 – 7 of the study guide. Do the activities and exercise before you proceed.

Example 1.1

Study the following:

$$\begin{aligned} 5(7+3) - 32 \div 4 \times 3 &= 5 \times 10 - 32 \div 4 \times 3 \\ &= 50 - 8 \times 3 \\ &= 50 - 24 \\ &= 26. \end{aligned}$$

Activity 1.1

1. Complete the calculation:

$$\begin{aligned} (50 - (3 + 4 \times 6) + 15 \div 5 \times 3) - (36 \div 4 + 2) &= (50 - (3 + 24) + 3 \times 3) - (9 + 2) \\ &= \underline{\qquad\qquad\qquad} \\ &= \underline{\qquad\qquad\qquad} \\ &= \underline{\qquad\qquad\qquad} \\ &= \underline{\qquad\qquad\qquad} \end{aligned}$$

2. Try this one on your own:

$$\begin{aligned} (13 + 12) \times (20 + (8 \times 5 - 10) \div 5) &= \underline{\qquad\qquad\qquad} \\ &= \underline{\qquad\qquad\qquad} \\ &= \underline{\qquad\qquad\qquad} \\ &= \underline{\qquad\qquad\qquad} \\ &= \underline{\qquad\qquad\qquad} \end{aligned}$$

The answer is 650. Did you succeed?

1.2 Worksheet 2

Worksheet 2 is based on study unit 1.2: *Variables*, on pages 8 – 10 of the study guide. Do the exercise before you proceed.

Example 1.2

Study the following example:
Determine the value of the following expression if $x = 2$, $y = 4$ and $z = 5$:

$$\begin{aligned} 3y(z+x) - 2(x+y) &= 3 \times 4 \times (5+2) - 2(2+4) \\ &= 12 \times 7 - 2 \times 6 \\ &= 84 - 12 \\ &= 72. \end{aligned}$$

Activity 1.2

Determine the values of the following expressions and keep $x = 2$, $y = 4$ and $z = 5$:

1. $3(z-x) - 2x(z-y) = 3 \times (5-2) - 2 \times 2 \times (5-4)$

   = ______________________

   = ______________________

   = ______________________

2. $\frac{1}{2}xyz + \frac{1}{4}(2z + 2y + x)$ = ______________________

   = ______________________

   = ______________________

   = ______________________

   = ______________________

   The answer is 25. Did you succeed?

Example 1.3

You are asked to write the following as a mathematical expression:

The sum of a and 4, divided by the difference between 3 and a, is equal to b, can be written as

$$b = \frac{a+4}{3-a}.$$

Activity 1.3

Write the following as mathematical expressions:

1. Three is added to the product of five and x. From this answer the sum of ten and x is subtracted. The final answer is equal to y.
   Start with "*Three is added to the product of five and x.*":

   $$3 + 5 \times x = 3 + 5x.$$

   Then carry on to "*From this answer the sum of ten and x is subtracted.*":

   $$3 + 5x - (\_\_\_\_\_ + \_\_\_\_\_) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

   $$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

   "*The final answer is equal to y*": ______________________________.

2. The variable e is equal to the sum of a and b, multiplied by the difference between c and d.

   ____

3. The monthly salary, P, of a person, is R200 per day plus a monthly bonus of R500. He works d days per month.

   ____

1.3 Worksheet 3a

Worksheet 3a is based on study unit 1.3: *Fractions* (paragraph 1.3.1 – 1.3.2), on pages 11 – 15 of the study guide. Do the activities and exercise on page 19 before you proceed.

Supplementary background

There are different kinds of fractions:

1. **Proper fractions** A fraction is called a proper fraction when the numerator is *smaller* than the denominator. Examples are
$$\frac{1}{2};\quad \frac{1}{4};\quad \frac{3}{4};\quad \frac{1}{3};\quad \frac{5}{6}.$$
2. **Improper fractions** A fraction is called an improper fraction when the numerator is *larger* than the denominator. Examples are
$$\frac{3}{2};\quad \frac{5}{4};\quad \frac{6}{5};\quad \frac{9}{7}.$$
3. **Mixed fractions** A fraction is called a mixed fraction when it consists of an integer (whole number) as well as a fraction. Examples are
$$2\frac{1}{2};\quad 4\frac{1}{3};\quad 5\frac{3}{4}.$$

A *mixed* fraction can be changed to an *improper* fraction and vice versa.

(i) From an *improper* fraction to a *mixed* fraction:

Change $\frac{7}{2}$ to a mixed fraction:

$$7 \div 2 = 3,$$

Divide the numerator by the denominator (to get the integer) and the remainder is then the new numerator.

with a remainder of 1.

Thus,
$$\frac{7}{2} = 3\frac{1}{2}.$$

The integer part is the 3 and the remainder is 1 (the new numerator).

More examples:
$$\frac{5}{3} = 1\frac{2}{3}$$
$$\frac{7}{4} = 1\frac{3}{4}$$
$$\frac{13}{3} = 4\frac{1}{3}.$$

(ii) From a *mixed* fraction to an *improper* fraction:

Change $5\frac{3}{4}$ to an improper fraction:

$$5\frac{3}{4} = \frac{23}{4}.$$

Multiply the denominator by the integer and add the numerator: $5 \times 4 = 20$, then add 3.

More examples:

$$1\frac{1}{2} = \frac{3}{2}$$

$2 \times 1 + 1 = 3$

$$2\frac{1}{4} = \frac{9}{4}$$

$4 \times 2 + 1 = 9$

$$7\frac{5}{6} = \frac{47}{6}.$$

$6 \times 7 + 5 = 47$

Now that you have sharpened your knowledge about these kinds of fractions, the next example will deal with another topic on fractions, namely *equivalent* fractions.

If the numerator *and* denominator of a fraction are multiplied or divided by the same number, the result is called an *equivalent* fraction.

Example 1.4

Determine the value of a in

$$\frac{a}{3} = \frac{16}{24}.$$

The fraction $\frac{16}{24}$ can be rewritten:

$$\frac{16}{24} = \frac{16 \div 8}{24 \div 8} = \frac{2}{3}.$$

To construct an equivalent fraction for $\frac{16}{24}$ we *divide* the numerator (16) and denominator (24) by the same number, in this case 8.

The fraction $\frac{2}{3}$ can be rewritten:

$$\frac{2}{3} = \frac{2 \times 8}{3 \times 8} = \frac{16}{24}.$$

To construct an equivalent fraction for $\frac{2}{3}$ we *multiply* the numerator (2) and denominator (3) by 8.

The number $\frac{8}{8}$ is equal to 1. It is clear that an equivalent fraction has the same value as the original fraction because it was multiplied or divided by a value of 1.

Thus, if

$$\frac{a}{3} = \frac{16}{24},$$

then the value of a is equal to 2.

Activity 1.4

Use the same method as in the previous example to determine the value of b in

$$\frac{15}{b} = \frac{5}{13}.$$

Since $15 = 5 \times 3$, multiply the numerator and denominator of $\frac{5}{13}$ by 3. (Note that $\frac{3}{3} = 1$.) Then

$$\frac{5}{13} = \frac{5\times \square}{13\times \square} = \frac{\square}{\square}.$$

Thus, the value of b is equal to $\square$.

Example 1.5

The following example illustrates the topic of addition and subtraction of fractions:

$$5\frac{3}{4} + 2\frac{5}{6} - 7\frac{1}{3} = \frac{23}{4} + \frac{17}{6} - \frac{22}{3}$$

Convert mixed numbers to improper fractions.

$$= \frac{23 \times 3}{4 \times 3} + \frac{17 \times 2}{6 \times 2} - \frac{22 \times 4}{3 \times 4}$$

Change each fraction into an equivalent fraction so that all the fractions have the same denominator. The number 12 is the smallest number divisible by the denominators 4, 6 and 3.

$$= \frac{69}{12} + \frac{34}{12} - \frac{88}{12}$$

$$= \frac{15}{12}$$

Add or subtract the numerators.

$$= \frac{5}{4}$$

Simplify: $\frac{15 \div 3}{12 \div 3}$.

$$= 1\frac{1}{4}.$$

Write the improper fraction as a mixed fraction.

Activity 1.5

1. Complete the following calculation:

$$\begin{aligned}\left(5\frac{1}{4}+2\frac{1}{5}\right)-\left(2\frac{1}{3}+3\frac{1}{6}\right) &= \left(\frac{21}{4}+\frac{11}{5}\right)-\left(\frac{7}{3}+\frac{19}{6}\right)\\ &= \frac{105+44}{20}-\frac{\square+\square}{6}\\ &= \frac{\square}{20}-\frac{\square}{6}\\ &= \frac{\square-\square}{60}\\ &= \underline{\hspace{4cm}}\\ &= \underline{\hspace{4cm}}\\ &= \underline{\hspace{4cm}}\end{aligned}$$

2. Now try the following calculation on your own:

$$\begin{aligned}3\frac{3}{5}+2\frac{1}{10}-1\frac{3}{10} &= \underline{\hspace{6cm}}\\ &= \underline{\hspace{6cm}}\\ &= \underline{\hspace{6cm}}\\ &= \underline{\hspace{6cm}}\\ &= \underline{\hspace{6cm}}\\ &= \underline{\hspace{6cm}}\\ &= \underline{\hspace{6cm}}\end{aligned}$$

The answer is $4\frac{2}{5}$. *Did you succeed?*

Example 1.6

There are a few steps to follow when multiplying fractions. Let us use the following example to illustrate the steps. Calculate the following:

$$2\frac{3}{4} \times \frac{1}{4} \times \frac{9}{10}.$$

Method 1:

Simplify the fractions by dividing a numerator and a denominator by the same number:

$$\begin{aligned}
2\frac{2}{3} \times \frac{1}{4} \times \frac{9}{10} &= \frac{8}{3} \times \frac{1}{4} \times \frac{9}{10} \\
&= \frac{8}{\cancel{3}_1} \times \frac{1}{4} \times \frac{\cancel{9}^3}{10} \\
&= \frac{8}{1} \times \frac{1}{4} \times \frac{3}{10} \\
&= \frac{\cancel{8}^2}{1} \times \frac{1}{\cancel{4}_1} \times \frac{3}{10} \\
&= \frac{2}{1} \times \frac{1}{1} \times \frac{3}{10} \\
&= \frac{\cancel{2}^1}{1} \times \frac{1}{1} \times \frac{3}{\cancel{10}_5} \\
&= \frac{1}{1} \times \frac{1}{1} \times \frac{3}{5} \\
&= \frac{3}{5}.
\end{aligned}$$

Note: $3 \div 3 = 1$ and $9 \div 3 = 3$.

Note: $4 \div 4 = 1$ and $8 \div 4 = 2$.

Note: $2 \div 2 = 1$ and $10 \div 2 = 5$.

Multiply the numerators and place the answer above the line. Then multiply the denominators and place the answer below the line.

Method 2:

Alternatively you could simply multiply the numerators together and place the answer above the line; then multiply the denominators together and place the answer below the line:

$$\begin{aligned}
2\frac{2}{3} \times \frac{1}{4} \times \frac{9}{10} &= \frac{8}{3} \times \frac{1}{4} \times \frac{9}{10} \\
&= \frac{8 \times 1 \times 9}{3 \times 4 \times 10} \\
&= \frac{72}{120} \\
&= \frac{72 \div 4}{120 \div 4} \\
&= \frac{18}{30} \\
&= \frac{18 \div 6}{30 \div 6} \\
&= \frac{3}{5}.
\end{aligned}$$

Simplify the fraction:
72 and 120 are both divisable by 4.

Simplify the fraction:
18 and 30 are both divisable by 6.

Activity 1.6

Do the following calculation:

Remember, dividing by $\frac{5}{6}$ is equal to multiplying by $\frac{6}{5}$.

$$2\frac{4}{5} \times 1\frac{4}{21} \div \frac{5}{6} = \underline{\hspace{5cm}}$$

$$= \underline{\hspace{5cm}}$$

$$= \underline{\hspace{5cm}}$$

$$= \underline{\hspace{5cm}}$$

$$= \underline{\hspace{5cm}}$$

The answer is 4. Did you succeed?

1.4 Worksheet 3b

Worksheet 3b is based on study unit 1.3: *Fractions* (paragraph 1.3.3), on pages 16 – 18 of the study guide. Do the activities before you proceed.

Example 1.7

Convert the following fractions to decimal notation:

$$\frac{3}{5};\quad 4\frac{3}{5};\quad \frac{16}{5}.$$

The fraction $\frac{3}{5}$ is a *proper* fraction:

$\frac{3}{5} = 3 \div 5 = 0{,}6.$

Divide the numerator by the denominator.

The fraction $4\frac{3}{5}$ is a *mixed* fraction:

$4\frac{3}{5} = 4 + (3 \div 5) = 4{,}6.$

The integer, namely 4, does not change and only the fraction part of the number, namely $\frac{3}{5}$, changes to 0,6.

The fraction $\frac{16}{5}$ is an *improper* fraction:

$\frac{16}{5} = 16 \div 5 = 3{,}2.$

This is the same as with the proper fraction, namely *divide the numerator by the denominator.*

Activity 1.7

1. Convert the following fractions to decimal notation using your calculator:

   (a) $5\frac{3}{8} =$ ____________________

   (b) $\frac{7}{500} =$ ____________________

   (c) $\frac{186}{25} + 9\frac{27}{135} =$ ________ + ________

   $=$ ________________________________

2. Convert the following fraction to decimal notation without using your calculator:

   $50 + 3 + \frac{5}{10} + \frac{5}{1\,000} =$ ____________________

Example 1.8

Convert the decimal number 0,002 to a fraction:

The decimal number can be written as:

$$0{,}002 = \frac{2}{1\,000}$$

Determine the place value of the last digit and use that number as the denominator of the fraction. The place value of the 2 is thousands, therefore the denominator of the fraction is 1 000.

$$= \frac{2 \div 2}{1\,000 \div 2}$$
$$= \frac{1}{500}.$$

Simplify the fraction and always write it in its simplest form.

Activity 1.8

Convert the following decimals to fractions:

1. 7,65 = ____________________
2. 0,085 = ____________________

Example 1.9

- Convert the fraction $\frac{22}{7}$ to decimal notation and round off your answer to three decimal places.

  The fraction $\frac{22}{7}$ converted to decimal notation is 3,1428571... and 3,1428571... $\approx$ 3,143.

  The symbol $\approx$ is read as *approximately eqaul to.*

  The decimal 3,1428571... is an example of a **non-terminating** decimal because the answer goes on and on and is never completed.

- Convert the fraction $\frac{3}{8}$ to decimal notation and round off your answer to one decimal place.

  The fraction $\frac{3}{8}$ converted to decimal notation is 0,375 and
  0,375 ≈ 0,4.

> The decimal 0,375 is an example of a **terminating** decimal because it is exact and complete.

- Convert the fraction $\frac{7}{11}$ to decimal notation and round off your answer to two decimal places.

  The fraction $\frac{7}{11}$ converted to decimal notation is 0,636363... and
  0,636363... ≈ 0,64.

> The decimal 0,636363... is an example of a **recurring** decimal because there are repeating digits. This decimal 0,636363... can also be written as $0,\dot{6}\dot{3}$ and is pronounced as "*naught comma six three recurring*". The dots on the 6 and the 3 tell us that the 6 and the 3 are recurring.

Activity 1.9

1. Round off the following decimal numbers:
   (a) Round 15,6666 off to three decimal places: ____________________

   (b) Round 14,327 off to the nearest tenth (one decimal place): ____________________

   (c) Do the following calculation and round your answer off to two decimal places:

   (1,721 + 3,279) × 5,46 ÷ 1,35 =____________

2. Convert the following fractions to decimal notation:

   (a) $\frac{5}{12}$ = ____________________________
   = ____________________________

   (b) $\frac{65}{99}$ = ____________________________
   = ____________________________

3. Order the following numbers from the largest to the smallest:

   0,6; 0,06; 0,66; 6,6; 6,06.

   ____

1.5 Worksheet 4

Worksheet 4 is based on study unit 1.4: *Powers and roots* on pages 20 – 28 of the study guide. Do the activities and exercise before you proceed.

Example 1.10

Calculate $\sqrt{10^2 - 6^2}$.

Simplifying gives:

$$\begin{aligned} \sqrt{10^2 - 6^2} &= \sqrt{100 - 36} \\ &= \sqrt{64} \\ &= 8. \end{aligned}$$

Activity 1.10

1. Complete:

$$\begin{aligned} (4+1)^3 - (4^3 + 1^3) &= (\underline{\qquad})^3 - (\underline{\quad} + \underline{\quad}) \\ &= \underline{\qquad} - \underline{\qquad} \\ &= \underline{\qquad} \end{aligned}$$

2. Complete:

$$\begin{aligned} \sqrt[3]{4\frac{12}{125}} &= \sqrt[3]{\frac{512}{125}} \\ &= \underline{\qquad\qquad\qquad} \\ &= \underline{\qquad\qquad\qquad} \\ &= \underline{\qquad\qquad\qquad} \end{aligned}$$

3. Try this one on your own:

$$\begin{aligned} 4^2 \times 3^3 \div \sqrt{144} &= \underline{\qquad\qquad\qquad} \\ &= \underline{\qquad\qquad\qquad} \\ &= \underline{\qquad\qquad\qquad} \\ &= \underline{\qquad\qquad\qquad} \end{aligned}$$

The answer is 36. Did you succeed?

4. Complete:

$$\sqrt[3]{10^2+5^2}+4\sqrt{9}+(\sqrt[3]{6})^3 = \sqrt[3]{10\times 10+5\times 5}+4\sqrt{3\times 3}+\sqrt[3]{6}\times\sqrt[3]{6}\times\sqrt[3]{6}$$

= ______________________________

= ______________________________

= ______________________________

= ______________________________

= ______________________________

1.6 Worksheet 5a

Worksheet 5a is based on study unit 1.5: *Ratios, proportions and percentages* (paragraph 1.5.1 – 1.5.2), on pages 29 – 34 of the study guide. Do the activities before you proceed.

Example 1.11

A clothing factory uses 156 buttons for every dozen shirts they make. Determine the ratio of *buttons to shirts*, reduce it and express it as a comparison to one ratio (that is the number of buttons to one shirt).

The ratio of *buttons* to *shirts* is 156 to 12 or $\frac{156}{12}$.
Simplify the fraction to $\frac{13}{1}$.
Hence, the ratio of *butons* to *shirts* is 13 to 1 or 13 : 1.

Activity 1.11

1. Consider the example above and determine the ratio of *shirts to buttons*, reduce it and express it as a comparison to one ratio (ie the number of shirts to one button).

____

____

____

2. John, Jack and Jason sell magazines for a total profit of R572,00. The profit is shared between them in a ratio equal to that of the number of magazines they sell. If John sells 25, Jack sells 38 and Jason sells 41 magazines respectively, how much money should each one receive?

   The ratio of magazines sold is

   John : Jack : Jason

   _____ : _____ : _____

   Add 25, 38 and 41 to obtain a total of ______ magazines that were sold.

   John's part of the profit is $\frac{25}{\quad} \times 572{,}00 =$ __________

   Jack's part of the profit is __________________________

   Jason's part of the profit is __________________________

3. We sometimes encounter a problem given in terms of fractions, **where the total of the fractions is equal to 1**. The fractions can then be converted to a ratio by writing the fractions as equivalent fractions with the same denominator. Consider the following:

A father gives a certain amount of money to his sons. The eldest receives $\frac{2}{5}$, the second son $\frac{3}{8}$ and the youngest son the remainder. If the youngest receives R1 350, what is the total amount of money and how much does each of the other brothers receive?

First calculate the youngest son's fraction of the share. His fraction of the share is

$$\begin{aligned} 1-\left(\frac{2}{5}+\frac{3}{8}\right) &= 1-\left(\frac{2\times 8+3\times 5}{40}\right) \\ &= \frac{40}{40}-\frac{16+15}{40} \\ &= \frac{\square}{40}-\frac{\square}{40} \\ &= \frac{\square}{40}. \end{aligned}$$

The oldest son's fraction of the share is $\frac{2}{5}=\frac{\square}{40}$.

The second son's fraction of the share is $\frac{3}{8}=\frac{\square}{40}$.

The youngest son's fraction of the share is $\frac{\square}{40}$.

The money is divided in the ratio

oldest : second : youngest or _______ : _______ : _______

The youngest son's share of the money is equal to R1 350.

Suppose x is the total amount of money the father gave.

Then $\frac{\square}{40}\times x=\text{R}1\,350$.

Calculate the value of x:

$x =$ ________________________________

____

____

The oldest son: $\frac{}{40} \times$ ____________ = ____________

The oldest son's share of the money is R____________

The second son: $\frac{}{40} \times$ ____________ = ____________

The second son's share of the money is R____________

4. Themba and Joyce receive a sum of money from a rich uncle. For every R50 Themba receives, Joyce receives R10 more. If Joyce receives R5 400, how much does Themba receive?

   The ratio is

Themba	:	Joyce
50	:	______.

   ____

   ____

   ____

5. A manufacturer pays R28 000 for 35 machines. What will the cost of 60 machines be?

   The ratio of cost to the number of machines is ____________ : ____________

   The ratio of cost to one machine is ____________ : ____________

   ____

   ____

   ____

1.7 Worksheet 5b

Worksheet 5b is based on study unit 1.5: *Ratios, proportions and percentages* (paragraph 1.5.3), on pages 34 – 38 of the study guide. Do the activities before you proceed.

Worksheet 5b will cover the following topics on percentages:

- converting a decimal number or a fraction to a percentage
- converting a percentage back to a decimal number or a fraction
- applying percentage to a number
- one number as a percentage of another
- measuring percentage change

Example 1.12

(i) This example illustrates how to convert a *decimal number* and a *fraction* to a *percentage*.

To convert a decimal number or a fraction to a percentage, multiply by 100. The answer is expressed as a %.

Convert 0,093 to a percentage:

$$0{,}093 \times 100 = 9{,}3\%.$$

Convert $2\frac{1}{5}$ to a percentage:

$$\begin{aligned} 2\frac{1}{5} \times 100 &= \frac{11}{5} \times 100 \\ &= \frac{1\,100}{5} \\ &= 220\%. \end{aligned}$$

(ii) This example illustrates how to convert a *percentage* to a *fraction* and a *decimal number*.

To convert a percentage to a fraction or a decimal number, divide by 100 and simplify.

Convert $16\frac{2}{3}\%$ to a fraction:

$$\begin{aligned} 16\frac{2}{3}\% &= \frac{50}{3}\% \\ &= \frac{50}{3} \div 100 \\ &= \frac{50}{3} \times \frac{1}{100} \\ &= \frac{50}{300} \\ &= \frac{1}{6}. \end{aligned}$$

Convert 83% to a decimal:

$$\begin{aligned} 83\% &= \frac{83}{100} \\ &= 0{,}83. \end{aligned}$$

Activity 1.12

1. Complete the following:

   (a) The decimal 0,684 written as a percentage is

   ____

   ____

   (b) Write $37\frac{1}{2}\%$ as a decimal:

   $$\begin{aligned} 37\frac{1}{2}\% &= \frac{75}{2}\% \\ &= \frac{75}{2} \div \underline{\hspace{3cm}} \\ &= \underline{\hspace{5cm}} \\ &= \underline{\hspace{5cm}} \\ &= \underline{\hspace{5cm}} \end{aligned}$$

   (c) The fraction $\frac{13}{80}$ written as a percentage is

   $$\begin{aligned} \frac{13}{80} \times \frac{100}{1} &= \frac{\square}{\square} \\ &= \frac{\square}{\square} \\ &= \underline{\hspace{5cm}} \\ &= \underline{\hspace{5cm}} \end{aligned}$$

   (d) Write $6\frac{1}{4}\%$ as a fraction:

   $$\begin{aligned} 6\frac{1}{4}\% &= \underline{\hspace{5cm}} \\ &= \underline{\hspace{5cm}} \\ &= \underline{\hspace{5cm}} \end{aligned}$$

2. The following exercises cover the topic of applying a percentage to a number. It is explained on page 34 – 35 of the study guide.

   (a) Lucia spends 65% of the R1 485 in her savings account on a holiday and 10% of the money on new shoes. How much money is left in her savings account?

   The percentage money that is left is

   $100 - (\_\_\_\_\_\_ + \_\_\_\_\_\_) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

   $= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

   The amount of money that is left is

   $\_\_\_\_\_\_\% \text{ of R}1\,485 = \frac{}{100} \times 1\,485$

   $= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

   (b) Peter pays R2 500 for furniture after a cash discount of 20%. Calculate the marked price of the furniture.

   A discount of 20% means that Peter pays only ______ % (100% – 20%) of the marked price. Find the marked price.

   ____

   ____

   ____

3. The following exercise covers the topic of expressing one number as a percentage of another. It is explained on page 36 of the study guide.

   Pumi invites 24 people to his birthday party. Only 15 people show up. Calculate the percentage of people that show up.

   ____

   ____

   ____

   ____

Example 1.13

This example covers the topic of calculating percentage change.

Mandile buys a CD player on a sale. It is marked down from R780,00 to R655,20. Calculate the percentage *decrease* on the CD player.

The original price was R780,00 and the new price is R655,20.

The *decrease* in price is

$$\begin{aligned} \textit{decrease} &= \text{original price} - \text{new price} \\ &= 780{,}00 - 655{,}20 \\ &= 124{,}80. \end{aligned}$$

The *decrease* in price is R124,80.

Calculate the percentage *decrease* as

$$\begin{aligned} \text{percentage } \textit{decrease} &= \frac{\text{change}}{\text{original}} \times 100 \\ &= \frac{124{,}80}{780{,}00} \times 100 \\ &= 16\%. \end{aligned}$$

Activity 1.13

If the price of an item *increases* from R215,00 to R247,25 what is the percentage *increase*?

The original price was R__________ and the new price is R__________

Calculate the *increase* in price as

$$\begin{aligned} \textit{increase} &= \text{new price} - \text{original price} \\ &= \_\_\_\_\_\_ - \_\_\_\_\_\_ \\ &= \_\_\_\_\_\_ \end{aligned}$$

The *increase* in price is R__________

Calculate the percentage *increase* as

$$\begin{aligned} \text{percentage } \textit{increase} &= \_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

The answer is 15%. Did you succeed?

1.8 Worksheet 5c

Worksheet 5c is based on extra topics on percentages.

Worksheet 5c will cover the following topics on percentages:

- mark-up percentage on cost and gross margin
- calculation of value-added tax (VAT)

Supplementary background

Mark-up percentage on cost and gross margin

Business enterprises want to make a profit and this profit is the difference between the enterprise's revenue (sales) and expenses (cost). A trading enterprise, for instance, will buy goods at cost price from a manufacturer and resell them at selling price. The difference between the cost price and the selling price is called the **gross profit**, because out of this gross profit the other expenses, such as rent, salaries, electricity and advertising, must be met. Anything left after deducting the expenses is the **net profit**.

Suppose an article was bought for R130 and sold for R180 before administrative costs of R30 were incurred. The gross profit is determined as follows:

$$\begin{aligned}\text{gross profit} &= \text{selling price} - \text{cost price}\\ &= 180 - 130\\ &= 50.\end{aligned}$$

The gross profit is R50.

Operating expenses such as administrative expenses, salaries and finance costs are not taken into account when determining gross profit. *(Gross means "before deductions" and net means "after deductions".)*

Mark-up percentage on cost

If the gross profit is expressed as a percentage of the **cost price**, it is called the **mark-up percentage on cost**, and is calculated as follows:

$$\text{mark-up \% on cost} = \frac{\text{gross profit}}{\text{cost price}} \times 100.$$

In the given example it is calculated as

$$\begin{aligned}\text{mark-up \% on cost} &= \frac{\text{gross profit}}{\text{cost price}} \times 100\\ &= \frac{50}{130} \times 100\\ &= 38{,}46\%.\end{aligned}$$

In this case the cost price is treated as 100% and it is important to remember the following:

selling price = cost price (100%) + mark-up % on cost.

Gross margin

If the gross profit is expressed as a percentage of the **selling price**, it is called the **gross margin**, and is calculated as follows:

$$\text{gross margin} = \frac{\text{gross profit}}{\text{selling price}} \times 100.$$

In the given example it is calculated as

$$\begin{aligned}\text{gross margin} &= \frac{\text{gross profit}}{\text{selling price}} \times 100\\ &= \frac{50}{180} \times 100\\ &= 27{,}78\%.\end{aligned}$$

In this case the selling price is treated as 100% and it is important to remember the following:

$$\boxed{\text{cost price} = \text{selling price (100\%)} - \text{gross margin.}}$$

Activity 1.14

If a trader buys goods for R200 and resells them for R300, determine his mark-up percentage on cost and his gross margin.

The mark-up percentage on cost is calculated as

$$\begin{aligned}\text{mark-up \% on cost} &= \frac{\text{gross profit}}{\text{cost price}} \times 100\\ &= \frac{\text{selling price} - \text{cost price}}{\text{cost price}} \times 100\\ &= \frac{\qquad}{\qquad} \times 100\\ &= \frac{\qquad}{\qquad} \times 100\\ &= \underline{\qquad\qquad}\%.\end{aligned}$$

Calculate the gross margin as

$$\begin{aligned}\text{gross margin} &= \frac{\qquad}{\qquad} \times 100\\ &= \frac{\qquad}{\qquad} \times 100\\ &= \underline{\qquad\qquad}\%.\end{aligned}$$

So far we have used the cost price and selling price to calculate the gross profit percentages. We can also calculate the cost price, selling price or gross profit if the other information is known.

Example 1.14

An item is bought for R350. If the mark-up percentage on cost is 25%, how much will it be sold for?

The following is determined from the question:

- If the mark-up % on cost is given as 25% then the selling price is determined as

$$\begin{aligned} \text{selling price} &= \text{cost price (100\%)} + \text{mark-up \% on cost} \\ &= 100 + 25 \\ &= 125. \end{aligned}$$

  Thus, the selling price is 125% of the cost price.
- Cost price = R350;
- Selling price = x.

The relationship $\dfrac{\text{selling price}}{\text{cost price}}$ can be written as $\dfrac{x}{350} = \dfrac{125}{100}$.

Multiply both sides by 350 and solve for x:

$$\begin{aligned} \frac{x \times 350}{350} &= \frac{125}{100} \times 350 \\ x &= \frac{125}{100} \times 350 \\ &= 437{,}50. \end{aligned}$$

The selling price is R437,50.

Note that the percentage that corresponds to the known amount (R350) is always the denominator (bottom number) in the fraction. The percentage that corresponds to the unknown amount (x) is always the numerator in the fraction.

Activity 1.15

A trader sells his product for R390. If his gross margin is 30%, what is the cost price of the product?

The following is determined from the question:

- If the gross margin is 30% then the cost price is determined as

$$\begin{aligned} \text{cost price} &= \text{selling price} - \text{gross margin} \\ &= 100 - \underline{\hspace{3cm}} \\ &= \underline{\hspace{3cm}} \end{aligned}$$

  The cost price is ________ % of the selling price.

- Selling price = R390;
- Cost price = x.

The relationship $\dfrac{\text{cost price}}{\text{selling price}}$ can be written as $\dfrac{x}{\square} = \dfrac{\square}{\square}$

Multiply both sides by 390 and solve for x:

$$\frac{x \times \square}{\square} = \frac{\square}{\square} \times \square$$

$$x = \frac{\square}{\square} \times \square$$

$$= \_\_\_\_\_\_\_\_$$

The cost price is R237,00. *Did you succeed?*

Supplementary background

Calculation of value-added tax

Value-added tax (VAT) is the tax that is levied whenever a product is sold or a service is rendered. The VAT is added to the selling price that a trader expects for goods and the goods are marked at a price inclusive of VAT. You have probably often seen the following on tax invoices:

Goods as supplied	R150,00
VAT @ 14%	R 21,00
Total	R171,00

The trader collects the R21, which is called **output VAT**, because it has been added to the trader's sales (outputs), on behalf of the South African Revenue Service (SARS). **Input VAT** is the VAT on goods or services purchased. The net amount (output VAT − input VAT) is paid to, or refunded by the SARS at the end of a tax period.

The rate of VAT is decided by the government and is changed from time to time. Currently the rate is 14%.

When an amount is given exclusive of VAT, the amount does not include VAT (net amount). When an amount is inclusive of VAT, the VAT is included in the amount (gross amount). Prices of items are usually given inclusive of VAT. The inclusive price or gross amount is made up of the net amount plus VAT. To find the amount of VAT which has been added to the net amount, the following **VAT formula** can be used:

$$\begin{aligned}\text{VAT} &= \frac{\%\text{ rate of VAT}}{100 + \%\text{ rate of VAT}} \times \text{gross amount} \\ &= \frac{14}{114} \times \text{gross amount.}\end{aligned}$$

Let us suppose that the inclusive price of an item is R1 710 and the rate of VAT is 14%. The amount of VAT is calculated as follows:

$$\begin{aligned}\text{VAT} &= \frac{14}{100 + 14} \times 1\,710 \\ &= \frac{14}{114} \times 1\,710 \\ &= 210.\end{aligned}$$

The VAT is R210.

The net amount is therefore R1 710 − R210 = R1 500.

The net amount can also be calculated as follows:

$$\begin{aligned}\text{net amount} &= \frac{100}{100 + \%\text{ rate of VAT}} \times \text{gross amount}\\ &= \frac{100}{114} \times 1\,710\\ &= 1\,500.\end{aligned}$$

The net amount is R1 500.

Activity 1.16

1. The VAT on an invoice with a gross amount of R5 157,36 is calculated as

$$\begin{aligned}\text{VAT} &= \frac{14}{114} \times \text{gross amount}\\ &= \frac{14}{114} \times \underline{\hspace{3cm}}\\ &= \underline{\hspace{3cm}}\end{aligned}$$

2. If the inclusive price of an article is R969, what is the price exclusive of VAT?
   The price exclusive of VAT is the net amount and is calculated as follows:

$$\begin{aligned}\text{net amount} &= \underline{\hspace{5cm}}\\ &= \underline{\hspace{5cm}}\\ &= \underline{\hspace{5cm}}\end{aligned}$$

3. If the price of an item exclusive of VAT is R500, calculate the inclusive price and the VAT amount.
   If the net amount is calculated as

$$\text{net amount} = \frac{100}{114} \times \text{gross amount},$$

   then the gross amount (inclusive price) is calculated as

$$\begin{aligned}\frac{100}{114} \times \text{gross amount} &= \text{net amount}\\ \text{gross amount} &= \text{net amount} \times \frac{114}{100}.\end{aligned}$$

   The net amount is given as R500, thus the gross amount is calculated as

$$\begin{aligned}\text{gross amount} &= \boxed{} \times \frac{\boxed{}}{\boxed{}}\\ &= \underline{\hspace{4cm}}\end{aligned}$$

   The VAT amount is calculated as

$$\begin{aligned}\text{VAT} &= \text{gross amount} - \text{net amount}\\ &= \underline{\hspace{2cm}} - \underline{\hspace{2cm}}\\ &= \underline{\hspace{2cm}}\end{aligned}$$

1.9 Worksheet 6

Worksheet 6 is based on study unit 1.6: *Signs, notations and counting rules*, on pages 39 – 50 of the study guide. Do the activities and exercise before you proceed.

Example 1.15

Calculate $\sum_{i=1}^{4} x_i$ using the values $x_1 = 5$; $x_2 = 3$; $x_3 = 2$ and $x_4 = 6$.

$$\begin{aligned}\sum_{i=1}^{4} x_i &= x_1 + x_2 + x_3 + x_4 \\ &= 5 + 3 + 2 + 6 \\ &= 16.\end{aligned}$$

Activity 1.17

1. Suppose $x_1 = 5$; $x_2 = 3$; $x_3 = 2$ and $x_4 = 6$. Calculate the following:

   (a) $\sum_{i=1}^{4} x_i^2 =$ __________

   $=$ __________

   $=$ __________

   $=$ __________

   The answer is 74. Did you succeed?

   (b) $\left(\sum_{i=1}^{4} x_i\right)^2 =$ __________

   $=$ __________

   $=$ __________

   $=$ __________

   (c) $\sum_{i=1}^{3} x_i =$ __________

   $=$ __________

   $=$ __________

2. Calculate the value of the following:

   $12 \div (-3) \times 5 + 6 - (5 - 8) =$ __________

   $=$ __________

   $=$ __________

   $=$ __________

   The answer is -11. Did you succeed?

3. How many ways can ten letters be posted in five postboxes, if each of the postboxes can take more than ten letters?

   Each of the ten letters can be posted in any of the five boxes.

   The first letter has five options.
   The second letter has __________ options.
   The tenth letter has __________ options.

   The total number of options is calculated as
   ___ × ___ × ___ × ___ × ___ × ___ × ___ × ___ × ___ × ___ =

   = ________________.

4. In how many ways can the 18 members of a boy scout troop elect a president, a vice-president and a secretary, assuming that no member can hold more than one office?

   Hint: In this case the order of placement is important.

   If the order of placement is important it is a ________________________

   ____

   ____

   ____

   ____

   ____

5. On each trip, a salesman visits four of the twelve cities in his territory. In how many different ways can he schedule his route?

   ____

   ____

   ____

   ____

   ____

6. The value of $\dfrac{6!\,7! - 6!\,5!}{5!\,4!}$ is

   ____

   ____

   ____

   ____

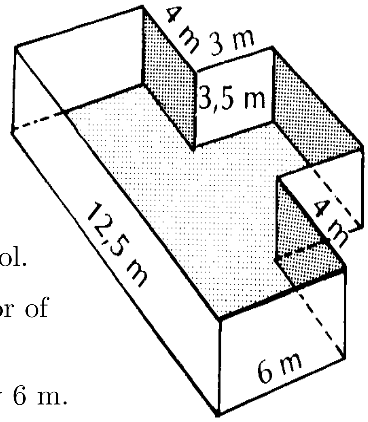
   ____

1.10 Worksheet 7

Worksheet 7 is based on study unit 1.7: *Units and measures*, on pages 51 – 59 of the study guide. Do the activities and exercise before you proceed.

Activity 1.18

1. The following is a diagram of a swimming pool:



(a) Calculate the area of the floor of the swimming pool.

If you study the diagram, you will see that the floor of the swimming pool consists of two rectangles.

The dimensions of the first rectangle are 12,5 m by 6 m.

The dimensions of the second rectangle are $(12{,}5 - (4 + 4))$ m or 4,5 m by 3 m.

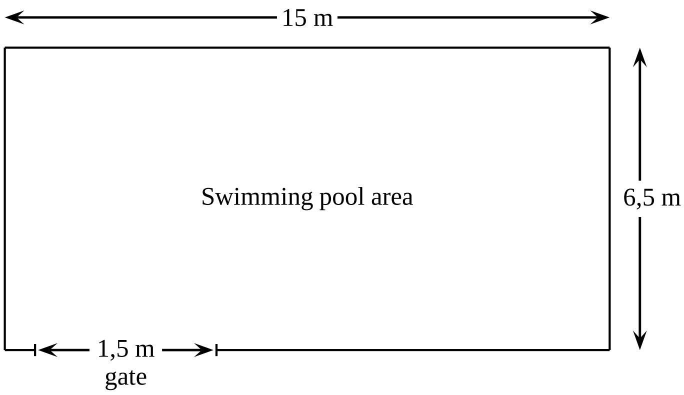
(b) What volume (in cubic metres) of sand was removed to build the swimming pool?

(c) How many kilolitres of water are needed to fill the pool?

2. A fence must be set up around a swimming pool. The dimensions of the area are 15 metres by 6,5 metres and 1,5 metre must be allowed for the gate.

   Find the total cost if the cost of the fence is R125,35 per metre and the cost of the gate is R375,44.

   It is useful to make a sketch to visualise the problem:



Total length of fence = ______________________________

____

____

____

Cost of fence without gate = ______________________________

____

____

Total cost = ______________________________

____

____

1.11 Worksheet 8

Worksheet 8 is a revision of study units 1.1 to 1.7. Complete worksheets 1 to 7 before you proceed.

Activity 1.19

1. Do the following calculations:

   (a) $-12 \div (-4 - (-2)) + 24 \div (-6) \times 5 + (-10 - 4 \times (-2))$

   = ____________________

   = ____________________

   = ____________________

   = ____________________

   = ____________________

   (b) $-10 \times (10 + 2)$ = ____________________

   = ____________________

   (c) $(-10 - 1) \times (10 + 1)$ = ____________________

   = ____________________

   (d) $(10 - 1) \times (3 - 10)$ = ____________________

   = ____________________

   (e) $-2^2 + (-2)^3 + \sqrt{25 - 16} - 7(-3)(-2) + 4 + 3 \times (-10)$

   = ____________________

   = ____________________

   = ____________________

   = ____________________

   = ____________________

(f) $(-7+9)^3 + \sqrt{13-(-2\times 6)} - \frac{-75}{15}$ = ______________________

= ______________________

= ______________________

= ______________________

= ______________________

2. Every day, from Monday to Thursday, it takes mr Seimela one hour and 15 minutes to travel to his office. On Fridays the traffic is less congested, and his travelling time is reduced with 16%. Determine his travelling time on a Friday.

____

____

____

____

3. Frank, Edgar and Trevor buy an old motorcycle for R1 500. Frank contributes R450, Edgar R750 and Trevor the rest. They repair it, sell it and make a profit of R3 500. They share the profit between them according to the ratio of their contributions. What is each person's share of the profit?

____

____

____

____

____

____

____

COMPONENT 2

Collection, presentation and description of data

The following examples and activities will improve your understanding of the basic principles covered in component 6 of the study guide. There is a worksheet for every study unit in component 6. Work through each worksheet after you have studied the relevant study material in the study guide.

2.1 Worksheet 1

Worksheet 1 is based on study unit 6.1: *Data collection*, on pages 161 – 165 of the study guide. Do the activities before you proceed.

Activity 2.1

Activity 2.1:1 is an exercise that covers the topic *Simple random sampling*.

1. A city has 853 medical practitioners. A random sample of 15 medical practitioners must be obtained. Use the following random numbers to select the sample:

253	872	489	230	313
287	875	844	348	134
794	064	437	041	355
898	910	207	194	804

The first step is to number the medical practitioners from 001 to 853. Since the total number of elements in the population is 853, a number larger than 853 is of no use.

The sample will then consist of the medical practitioners whose numbers are drawn.

The first number is 253. The number 872 is larger than 853 and will not be used. The second practitioner is number 489. The random sample is

Sampling unit	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Practitioner no.	253	489													

Activity 2.1:2 is an exercise that covers the topic *Stratified random sampling.*

2. A club has 25 student members:

1.	Knoetze	2.	Keyster	3.	Martin	4.	Gamede	5.	Els
6.	King	7.	Maluleke	8.	Nkosi	9.	Coetzee	10.	Viljoen
11.	Ngubane	12.	De Beer	13.	Moloi	14.	Van Dyk	15.	Hendricks
16.	Haufiku	17.	Moloto	18.	Ndlovu	19.	Erasmus	20.	Singh
21.	De Wet	22.	Bron	23.	Siko	24.	Ndlovu	25.	Williams

and 10 faculty members:

1.	Smit	2.	Pieters	3.	Mlangeni	4.	Brown	5.	Rossouw
6.	Sebola	7.	Sithole	8.	Kekana	9.	Swartz	10.	Davids

The club can send six members to a convention and decides to choose those who will go by random selection. Use random numbers to choose a stratified random sample of the members.

(a) Determine the sample size of each of the strata.

Stratum	Size of stratum	Sample size
Students	25	$\frac{25}{35} \times$ _____ = _____ ≈ _____
Faculty members	10	$\frac{\square}{\square} \times$ _____ = _____ ≈ _____
Total	35	

Therefore, four students and two faculty members can go to the convention.

(b) Draw a simple random sample from each stratum, using the following random numbers:

14 78 85 19 10 34 08 60 25 36 14 06 16 04 83

- Consider the stratum of **Students**.

  The sample consists of the students labeled

  14, _____, _____ and _____.

  Therefore Van Dyk, __________, __________ and __________ will go to the convention.

- Consider the stratum of **Faculty members**.

  The sample consists of the faculty members labeled

  6 and _____.

  Therefore __________ and __________ will go to the convention.

(c) Combine the two samples to form the full sample.

The following members will go to the convention:

__________, __________, __________, __________, __________ and __________.

2.2 Worksheet 2

Worksheet 2 is based on study unit 6.2: *Presentations*, on pages 166 – 174 of the study guide. Do the activities and exercise before you proceed.

Activity 2.2

Activity 2.2 is an exercise that covers the distinction between *qualitative and quantitative data*. State for each of the variables below whether it is qualitative, discrete quantitative or continuous quantitative:

1. length of a person

   This is continuous quantitative data.

2. gender

   This is qualitative data.

3. number of customers in a store

   This is discrete quantitative data.

4. duration of phone calls

   ____

5. mode of travel to work

   ____

6. petrol consumption of a car

   ____

7. size of soccer crowds

   ____

8. students' exam centre

   ____

9. number of cars sold by a dealer in a month

   ____

10. home language

    ____

Example 2.1

1. Consider the following set of data:

   8 13 2 7 17 4 1

   Group the data into the two given intervals:

   [0,5 – 10,5]
   [10,5 – 20,5]

   Start with the first data value, namely 8. Since 0,5 < **8** < 10,5 (which is read as "8 is greater than 0,5 and less than 10,5"), this value fits into the first interval. Indicate it by a tally "|" next to the first interval:

   [0,5 – 10,5] |
   [10,5 – 20,5]

   The next data value is 13. Since 10,5 < **13** < 20,5 this value fits into the second interval. Indicate it by a tally "|" next to the second interval:

   [0,5 – 10,5] |
   [10,5 – 20,5] |

   The next data value is 2. Since 0,5 < **2** < 10,5 this value fits into the first interval. Indicate it by a tally "|" next to the first interval:

   [0,5 – 10,5] ||
   [10,5 – 20,5] |

   The next data value is 7. Since 0,5 < **7** < 10,5 this value fits into the first interval. Indicate it by a tally "|" next to the first interval:

   [0,5 – 10,5] |||
   [10,5 – 20,5] |

   The next data value is 17. Since 10,5 < **17** < 20,5 this value fits into the second interval. Indicate it by a tally "|" next to the second interval:

   [0,5 – 10,5] |||
   [10,5 – 20,5] ||

   The next data value is 4. Since 0,5 < **4** < 10,5 this value fits into the first interval. Indicate it by a tally "|" next to the first interval:

   [0,5 – 10,5] ||||
   [10,5 – 20,5] ||

   The next data value is 1. Since 0,5 < **1** < 10,5 this value fits into the first interval. This will be the fifth element for the first interval. Indicate this by a line drawn across the group of four tallies (~~||||~~ represents a group of five):

   [0,5 – 10,5] ~~||||~~
   [10,5 – 20,5] ||

   This method is called the tally method. The number of tallies in each interval must be counted to find the **frequency** f for each interval:

Interval		**Frequency**
[0,5 - 10,5]	~~||||~~	5
[10,5 - 20,5]	||	2
		7

Activity 2.3

Activity 2.3:1 is an exercise that covers the topic: *Drawing up a frequency table.*

1. Research by the Food and Biomedical Administration shows that acryl amide (a possible cancer-causing substance) forms in high-carbohydrate foods cooked at high temperatures and that acryl amide levels can vary widely even within the same brand of food. The researchers analysed Big Mac's French fries sampled from different franchises and found the following acryl amide levels:

366	155	326	187	245	270	319	223	212	190
193	247	255	235	300	311	180	333	289	245
328	201	260	259	263	313	151	322	270	299

Construct a frequency table for the acryl amide levels.

There are five steps to draw up a frequency table. These steps are explained in detail on page 167 – 169 of the study guide.

(a) The range of the data is

$$\begin{aligned} R &= \text{maximum value} - \text{minimum value} \\ &= \_\_\_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_. \end{aligned}$$

(b) We cannot use the formula $\frac{R}{10}$ to determine the number of intervals because it will give $\frac{215}{10} = 21{,}5$ and 21 or 22 intervals are too many. Another formula that can be used to determine the number of class intervals is given by $K = \sqrt{n}$. This formula is not given in the study guide. The variable n is the number of observations in the data set, namely 30. The number of intervals is calculated as

$$\begin{aligned} K &= \sqrt{30} \\ &= \_\_\_\_\_\_\_\_\_\_. \end{aligned}$$

It is up to you to decide if you are going to use five or six intervals. To get a good idea of the distribution of the data, we decided to use six intervals. Thus, $K = 6$.

(c) The width of the interval is calculated as

$$\begin{aligned} c &= \frac{R}{K} \\ &= \frac{}{} \\ &= \_\_\_\_\_\_\_\_\_\_ \\ &\approx \_\_\_\_\_\_\_\_\_\_. \end{aligned}$$

Use an integer number for the width of an interval.

(d) Determine the interval limits.

The minimum value is 151. Half a unit less is 150,5.

The lower limit of the first interval is therefore __________.

To obtain the upper limit of the first interval we add the interval width to the lower limit:

150,5 + 36 = __________.

Therefore, the first interval is 150,5 − 186,5.
Now, the second interval starts with 186,5 and also has a width of 36.
Therefore, its upper limit is 186,5 + 36 = 222,5.
Thus, the intervals are

150,5	–	186,5
186,5	–	222,5
222,5	–	$222{,}5 + 36$
________	–	________
________	–	________
________	–	________

(e) Group the data into the intervals.
Consider the first value in the data set, 366. Determine into which interval it fits:
Since $330{,}5 < \mathbf{366} < 366{,}5$, it fits into the last interval. Indicate this by a tally "|" next to the last interval.
Now, repeat the process with the next value, 155:
Since $150{,}5 < \mathbf{155} < 186{,}5$, it fits into the first interval. Indicate this by a tally "|" next to the first interval. At this stage it will look as follows:

150,5 – 186,5	|
186,5 – 222,5	
222,5 – 258,5	
258,5 – 294,5	
294,5 – 330,5	
330,5 – 366,5	|

Continue this process for each of the 30 values in the data set. Fit each data value into one of the intervals. Suppose you have four data values that fit into an interval, having ||||.

Now, when a fifth data value also fits into that interval, it is indicated by a line across the group, ~~||||~~, to represent a group of five.

After every value has been fitted into an interval, it will look as follows:

150,5 – 186,5	|||
186,5 – 222,5	~~||||~~
222,5 – 258,5	
258,5 – 294,5	
294,5 – 330,5	
330,5 – 366,5	

Count the lines (or tallies) to find the number of data values (the frequency) in each interval. The frequency table is as follows:

Interval		**Frequency**
150,5 – 186,5	𝍷𝍷𝍷	3
186,5 – 222,5	𝍸	5
222,5 – 258,5	𝍸 𝍷	
258,5 – 294,5	𝍸 𝍷	
294,5 – 330,5	𝍸 𝍷𝍷𝍷	
330,5 – 366,5	𝍷𝍷	
		30

2. Consider the frequency table in activity 2.3:1 and answer the following questions:

   (a) What percentage of the franchises had acryl amide levels higher than 294?

   Acryl amide levels higher than 294 are represented by the intervals

   294,5 – 330,5 and ________ – ________.

   Add ________ and ________ together to obtain the sum of the frequencies of these two intervals as ________. Thus, the percentage of franchises that had acryl amide levels higher than 294 is $\frac{\quad}{30} \times 100 =$ ________%.

   (b) What percentage of the franchises had acryl amide levels lower than 259?

   ____

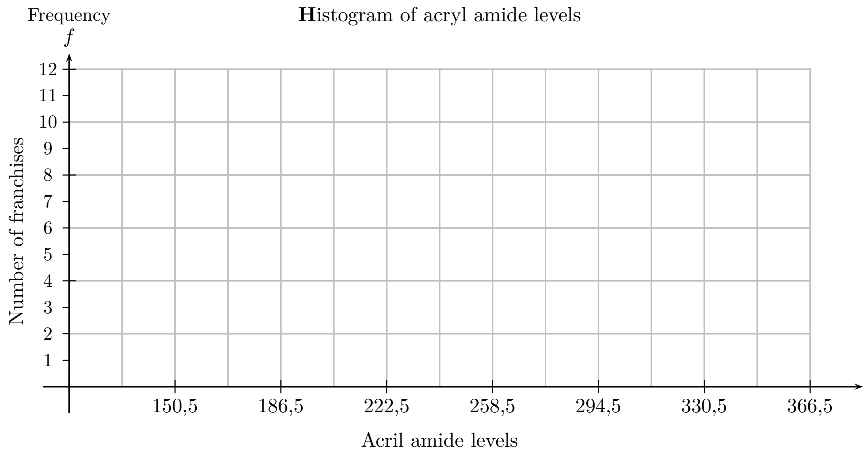
   ____

   ____

   The answer is 46,67%. Did you succeed?

Activity 2.3:3 is an exercise that covers the topic of *drawing a histogram.*

3. Draw a histogram of the data in activity 2.3:1.



Activity 2.3:4 is an exercise that covers the topic of *drawing a pie chart*.

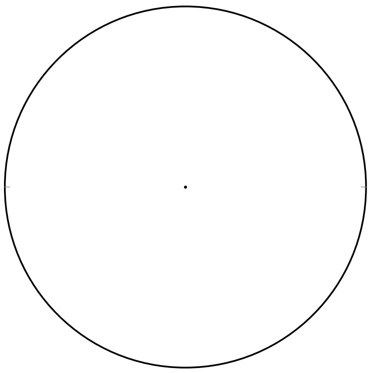
4. Draw a pie chart of the data in activity 2.3:1.

   You first have to calculate the percentage of the area of the circle covered by each interval.

Interval	**Frequency**	**Percentage of area**
150,5 – 186,5	3	$\frac{3}{30} \times 100 = 10\%$
186,5 – 222,5	5	____________
222,5 – 258,5	6	____________
258,5 – 294,5	6	____________
294,5 – 330,5	8	____________
330,5 – 366,5	2	____________
	30	

   Draw the slices of the pie chart. Estimate the sizes according to the percentages that you have calculated.

Pie chart for acryl amide levels

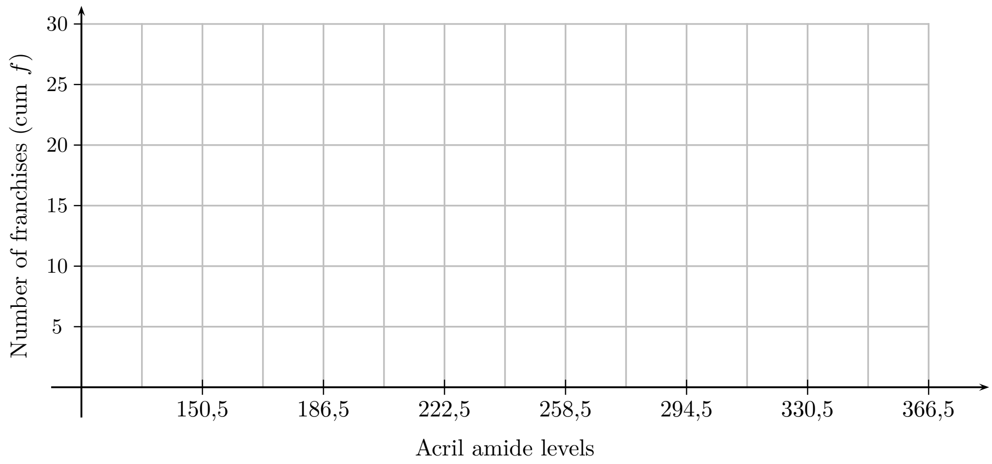


Activity 2.3:5 is an exercise that covers the topic of *drawing a cumulative frequency polygon*.

5. Find the cumulative frequencies for the data in activity 2.3:1 and draw a cumulative frequency polygon.

Upper limit	**Cumulative frequency**
$< 186{,}5$	3
$< 222{,}5$	$3 + 5 = 8$
$< 258{,}5$	$3 + 5 + 6 =$ ____
$< 294{,}5$	$3 + 5 + 6 + 6 =$ ____
$< 330{,}5$	$3 + 5 + 6 + 6 + 8 =$ ____
$< 366{,}5$	$3 + 5 + 6 + 6 + 8 + 2 =$ ____

Cumulative frequency polygon of acryl amide levels



Activity 2.3:6 is an exercise that covers the topic of *drawing a stem-and-leaf diagram.*

6. Construct a stem-and-leaf diagram for the test marks obtained by a sample of 20 students:

78	82	96	74	52	68	82	78	74	76
88	62	66	76	76	84	95	91	58	86

The smallest number is 52 and the largest number is 96. Use the first digit in each number (the tens) as the stem and the last digit (the ones) as the leaf.

Stem	**Leaf**	**Frequency**
5	2 8	2
6	8 2 ___	3
7	8 ___ ___ ___ ___ ___ ___	7
8	2 ___ ___ ___ ___	5
9	6 ___ ___	3

The sorted stem-and-leaf diagram is

Stem	**Leaf**	**Frequency**
5	2 8	2
6	2 6 ___	3
7	___ ___ ___ ___ ___ ___ ___	7
8	___ ___ ___ ___ ___	5
9	___ ___ ___	3

2.3 Worksheet 3

Worksheet 3 is based on study unit 6.3: *Measures of locality*, on pages 175 – 177 of the study guide. Do the activities and exercise before you proceed.

Example 2.2

Find the median of each data set.

1. Over a seven-day period, the number of customers that shop per day at the Hides Leather Shop are as follows:

$$4 \quad 80 \quad 50 \quad 10 \quad 60 \quad 12 \quad 5$$

- Arrange the data in numerical order:

$$4 \quad 5 \quad 10 \quad 12 \quad 50 \quad 60 \quad 80$$

- Determine the position of the median:
There are seven values in the data set, thus $n = 7$. The position of the median is determined as

$$\frac{n+1}{2} = \frac{7+1}{2} = 4.$$

The median is value number 4.

- The fourth value in the numerical list is 12:

$$4 \quad 5 \quad 10 \quad \textcircled{12} \quad 50 \quad 60 \quad 80$$

The median is 12.

- Interpret the result: For 50% of the time fewer than 12 customers visited the shop per day and for 50% of the time more than 12 customers visited the shop per day.

2. A city planner working on bikeways recorded how many minutes it takes bicycle commuters to pedal from home to their destinations. A sample of 12 local bicycle commuters yielded the following times:

$$22 \quad 29 \quad 27 \quad 30 \quad 12 \quad 22 \quad 31 \quad 15 \quad 26 \quad 16 \quad 48 \quad 23$$

- The data arranged in numerical order:

$$12 \quad 15 \quad 16 \quad 22 \quad 22 \quad 23 \quad 26 \quad 27 \quad 29 \quad 30 \quad 31 \quad 48$$

- The position of the median: There are twelve values in the data set, thus $n = 12$. The position of the median is determined as

$$\frac{n+1}{2} = \frac{12+1}{2} = 6{,}5.$$

The median is value number 6,5.

- Count up to value number six in the numerical list. Value number 6,5 falls halfway between 23 and 26.

$$12 \quad 15 \quad 16 \quad 22 \quad 22 \quad \overbrace{23 \quad 26} \quad 27 \quad 29 \quad 30 \quad 31 \quad 48$$

The median is

$$\frac{23+26}{2} = 24{,}5.$$

- Interpret the result: For 50% of the riders it took less than 24,5 minutes to travel to their destinations and 50% took more than 24,5 minutes.

Activity 2.4

The number of reservations made at a hotel over the past fifteen days is

11 20 28 14 19 28 30 23 25 17 15 26 9 22 28.

Calculate the following measures of central tendency for the data and interpret each answer with a short explanation:

1. arithmetic mean
2. median
3. mode

The first step is to write the data in ascending order:

No.	**Data ordered from smallest to largest** (x_i)
1	9
2	11
3	14
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
	$\sum_{i=1}^{15} x_i =$ ________

→ 8th position in data (Data must be ordered from smallest to largest data item.)

The number of observations is $n = 15$.

1. The mean is calculated as

$$\begin{aligned}\overline{x} &= \frac{\sum_{i=1}^{15} x_i}{n} \\ &= \frac{}{15} \\ &= \text{\_\_\_\_\_\_\_\_}.\end{aligned}$$

The mean/average number of reservations over the 15 days is 21. Did you succeed?

2. The position of the median is

$$\begin{aligned}\frac{n+1}{2} &= \frac{15+1}{2} \\ &= 8.\end{aligned}$$

The median is the 8th value in the ordered data set.

The value of the median is ___________.

Half of the reservations were fewer than ________ per day, while the other half of the reservations were more than ________ per day.

3. The mode is ________ because it is the value that occurs most often.

   Interpretation: ____

The data considered in example 2.2 and in activity 2.4 are called **ungrouped** data. When data is classified into a frequency table, it is called **grouped** data. Different methods are used to calculate the measures of location for grouped data.

The following example illustrates how to calculate the mean, median interval and modal interval for data in a frequency table (grouped data).

Example 2.3

The time between breakdowns for equipment in a factory was recorded over a period of several months. During this period 40 breakdowns were observed. The times are shown in the following frequency table:

Time between breakdowns (days)	**Number of breakdowns, frequency** (f)	**Class midpoints** (x)
−0,5 − 4,5	6	2
4,5 − 9,5	10	7
9,5 − 14,5	14	12
14,5 − 19,5	6	17
19,5 − 24,5	4	22
Total	40	

The midpoint of the first interval is

$$\frac{-0{,}5 + 4{,}5}{2} = 2.$$

The midpoint of the second interval is

$$\frac{4{,}5 + 9{,}5}{2} = 7.$$

> The midpoint of an interval divides an interval into two equal parts and is obtained by adding the upper and lower limits of each interval and dividing the result by two. This middle value represents the class interval in calculations.

1. Determine the mean number of days between breakdowns.

   To determine the mean number of days it is necessary to add a column to the table and to calculate the values of $f \times x$. The cumulative frequency column is also added for later use.

Interval	**Frequency** (f)	**Midpoints** (x)	$f \times x$	**Cumulative frequency**
−0,5 − 4,5	6	2	$6 \times 2 = 12$	6
4,5 − 9,5	10	7	$10 \times 7 = 70$	$6 + 10 = 16$
9,5 − 14,5	14	12	$14 \times 12 = 168$	$16 + 14 = 30$
14,5 − 19,5	6	17	$6 \times 17 = 102$	$30 + 6 = 36$
19,5 − 24,5	4	22	$4 \times 22 = 88$	$36 + 4 = 40$
	$\sum f = 40$		$\sum fx = 440$	

The mean is calculated by the formula $\overline{x} = \dfrac{\sum fx}{n}$, where

x is the midpoint of each class,

f is the frequency of each class and

n is the number of observations in the sample ($\sum f$).

The mean is

$$\begin{aligned} \overline{x} &= \frac{\sum fx}{n} \\ &= \frac{440}{40} \\ &= 11. \end{aligned}$$

The mean number of days between breakdowns is 11 days.

2. Determine the median interval.

   Determine the position of the median as $\frac{n}{2}$. (Use the cumulative frequency column in the table.) Compare the position of the median with the cumulative frequency column to determine which one of the intervals contains the median. The median interval is the interval where the cumulative frequency is equal to, or exceeds $\frac{n}{2}$ for the first time. In this case, $\frac{n}{2} = \frac{40}{2} = 20$. The interval where the cumulative frequency is equal to, or exceeds 20 for the first time (the cumulative frequency ≥ 20) is the interval 9,5 – 14,5 with a cumulative frequency of 30. Thus, the median interval is the interval **9,5 – 14,5**.

3. Determine the modal interval.

   The modal interval is the interval with the highest frequency (f). In this case it is the interval **9,5 – 14,5** with a frequency of $f = 14$.

Activity 2.5

The following frequency table shows the time (in minutes) taken to travel to work for a sample of 25 people living in Witbank.

Time in minutes	**Number of people frequency (f)**
15,5 – 21,5	2
21,5 – 27,5	6
27,5 – 33,5	8
33,5 – 39,5	4
39,5 – 45,5	4
45,5 – 51,5	1
	25

1. Calculate the mean time to travel to work.

   The first step is to complete the following table:

Interval	**Frequency** (f)	**Midpoints** (x)	$f \times x$	**Cumulative frequency**
15,5 – 21,5	2	$(15{,}5 + 21{,}5) \div 2 = 18{,}5$	$2 \times 18{,}5 = 37$	2
21,5 – 27,5	6	$(21{,}5 + 27{,}5) \div 2 = 24{,}5$	$6 \times 24{,}5 = 147$	$2 + 6 = 8$
27,5 – 33,5	8	$(27{,}5 + 33{,}5) \div 2 =$ ____	$8 \times$ ____ $=$ ____	$8 + 8 =$ ____
33,5 – 39,5	4	$(33{,}5 + 39{,}5) \div 2 =$ ____	$4 \times$ ____ $=$ ____	____ + ____ = ____
39,5 – 45,5	4	(____ + ____) $\div 2 =$ ____	____ $\times$ ____ $=$ ____	____ + ____ = ____
45,5 – 51,5	1	(____ + ____) $\div 2 =$ ____	____ $\times$ ____ $=$ ____	____ + ____ = ____
	$\sum f = 25$		$\sum fx =$ ________	

   The number of observations is $n = \sum f = 25$.

   The mean is

$$\overline{x} = \frac{\sum fx}{n} = \frac{\quad}{25} = \text{\_\_\_\_\_\_\_\_}.$$

   The mean time to travel to work is 31,7 minutes. Did you succeed?

2. Determine the median interval.

   In this case, $\frac{n}{2} = \frac{25}{2} =$ ________. The interval where the cumulative frequency is equal to, or exceeds 12,5 for the first time is the interval ________________ with a cumulative frequency of ________. Thus the median interval is the interval ________________.

3. Determine the modal interval.

   The modal interval is the interval with the highest frequency (f). In this case it is the interval ________________ with a frequency of________.

   Note that it is not always the case that the median interval is the same interval as the modal interval.

2.4 Worksheet 4

Worksheet 4 is based on study unit 6.4: *Measures of dispersion*, on pages 178 – 184 of the study guide. Do the activities before you proceed.

Activity 2.6

Consider the data from activity 2.4 shown below and calculate the variance and the standard deviation of the data.

The number of observations is $n = 15$ and the mean was calculated as $\overline{x} = 21$.

Add the column $(x - \overline{x})^2$ to the table:

No.	**Data ordered from smallest to largest** (x_i)	$(x - \overline{x})^2 = (x - 21)^2$
1	9	$(9 - 21)^2 = (-12)^2 = 144$
2	11	$(11 - 21)^2 = (-10)^2 = 100$
3	14	$(14 - 21)^2 = (-7)^2 =$
4	15	$(15 - 21)^2 = \qquad =$
5	17	
6	19	
7	20	
8	22	
9	23	
10	25	
11	26	
12	28	
13	28	
14	28	
15	30	
	$\sum_{i=1}^{15} x_i = 315$	$\sum(x - \overline{x})^2 =$ ________

Please note: This data was ordered for the calculation of the median in activity 2.4. It is, however, not necessary to order the data if you want to calculate the mean, variance or standard deviation.

The variance of the data is

$$S^2 = \frac{\sum(x - \overline{x})^2}{n - 1} = \frac{\quad}{14} = \text{\_\_\_\_\_\_\_\_}.$$

The standard deviation is

$$S = \sqrt{\quad} = \text{\_\_\_\_\_\_\_\_}.$$

The answer is 6,68. Did you succeed?

To calculate the variance and standard deviation for grouped data, the following formula is used:

$$S^2 = \frac{\sum(x-\overline{x})^2 f}{n-1}$$

Example 2.4

Consider the data from example 2.3 and determine the variance and standard deviation.

The number of observations is $n = \sum f = 40$ and the mean is $\overline{x} = 11$. Add the column $(x-\overline{x})^2 f$ to the table:

Interval	**Frequency** f	**Midpoint** x	$(x-\overline{x})^2 f = (x-11)^2 f$
−0,5 − 4,5	6	2	$(2-11)^2 \times 6 = 486$
4,5 − 9,5	10	7	$(7-11)^2 \times 10 = 160$
9,5 − 14,5	14	12	$(12-11)^2 \times 14 = 14$
14,5 − 19,5	6	17	$(17-11)^2 \times 6 = 216$
19,5 − 24,5	4	22	$(22-11)^2 \times 4 = 484$
	$\sum f = 40$		$\sum(x-\overline{x})^2 f = 1\,360$

The variance of the data is

$$\begin{aligned} S^2 &= \frac{\sum(x-\overline{x})^2 f}{n-1} \\ &= \frac{1\,360}{39} \\ &= 34{,}87. \end{aligned}$$

The standard deviation is

$$\begin{aligned} S &= \sqrt{34{,}87} \\ &= 5{,}91. \end{aligned}$$

Activity 2.7

1. Consider the data from activity 2.5 and determine the variance and standard deviation.

   We know that the number of observations is $n = \sum f = 25$ and the mean is $\overline{x} = 31{,}7$. Add the column $(x-\overline{x})^2 f$ to the table:

Interval	**Frequency** (f)	**Midpoint** (x)	$\sum(x-\overline{x})^2 f = (x-31{,}7)^2 f$
15,5 − 21,5	2	18,5	$(18{,}5-31{,}7)^2 \times 2 = 348{,}48$
21,5 − 27,5	6	24,5	$(24{,}5-31{,}7)^2 \times$ _____ = _____
27,5 − 33,5	8	30,5	(_____ − _____)^2 × _____ = _____
33,5 − 39,5	4	36,5	(_____ − _____)^2 × _____ = _____
39,5 − 45,5	4	42,5	(_____ − _____)^2 × _____ = _____
45,5 − 51,5	1	48,5	(_____ − _____)^2 × _____ = _____
	$\sum f = 25$		$\sum(x-\overline{x})^2 f =$ ________

The variance of the data is

$$\begin{aligned} S^2 &= \frac{\sum(x-\overline{x})^2 f}{n-1} \\ &= \frac{\quad}{24} \\ &= \text{\_\_\_\_\_\_\_\_\_\_}. \end{aligned}$$

The standard deviation is

$$S = \sqrt{}$$
$$= \underline{\hspace{3cm}}.$$

The answer is 7,94. Did you succeed?

2. Consider the data from example 2.2 and determine the quartile deviation of each data set.

   (a) The number of customers at Hides Leather shop in ascending order:

   4 5 10 12 50 60 80

   It is known that the median is the $\frac{1}{2}(7+1)$th $=$ 4th observation. Thus, $Q_2 = 12$.

   The first quartile, Q_1, is the $\frac{1}{4}(____ + 1)$nd $=$ ____nd observation. Thus, $Q_1 =$ ____.

   The third quartile, Q_3, is the $\frac{3}{4}(____ + 1)$th $=$ ____th observation. Thus, $Q_3 =$ ____.

   The quartile deviation is calculated as

   $$\begin{aligned} Q_D &= \frac{Q_3 - Q_1}{2} \\ &= \frac{ - }{} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

   The answer is 27,5. Did you succeed?

   (b) The commuting time on bicycles in ascending order:

   12 15 16 22 22 23 26 27 29 30 31 48

   It is known that the median is the $\frac{1}{2}(12+1)$th $=$ 6,5th observation. Thus, $Q_2 = 24{,}5$.

   The first quartile, Q_1, is the $\frac{1}{4}(12+1)$th $=$ 3,25th observation.

   Q_1 lies a quarter of the distance between 16 and 22, measured from 16.

   Q_1 is calculated as

   $$\begin{aligned} Q_1 &= 16 + \frac{1}{4} \times (22 - 16) \\ &= 16 + 1{,}5 \\ &= 17{,}5. \end{aligned}$$

   The third quartile, Q_3, is the $\frac{3}{4}(12+1)$th $=$ _____th observation.

   Q_3 lies three quarters of the distance between _____ and _____, measured from _____.

   Q_3 is calculated as

   $$\begin{aligned} Q_3 &= \_\_\_\_ + \frac{3}{4} \times (\_\_\_\_ - \_\_\_\_) \\ &= \_\_\_\_ + \_\_\_\_ \\ &= \_\_\_\_. \end{aligned}$$

The quartile deviation is calculated as

$$\begin{aligned} Q_D &= \frac{Q_3 - Q_1}{2} \\ &= \frac{\square - \square}{\square} \\ &= \_\_\_\_\_\_\_\_ \end{aligned}$$

The answer is 6,13. Did you succeed?

3. A marketing company records the sales of three new types of breakfast cereal in a store for 20 weeks and obtains the following results:

	Type of cereal		
	A	**B**	**C**
Mean number of sales per week ($\overline{x}$)	88	56	100
Standard deviation (s)	16	15	25

The company is not primarily interested in the highest sales, but rather in consistent sales. Which cereal fits this requirement the best?

The coefficient of variation is used to compare two or more sets of data with different means, sample sizes or measurement units. The higher the result, the more variability there is in a set of data.

The coefficient of variation for cereal A is

$$\begin{aligned} CV_A &= \frac{s}{\overline{x}} \times 100 \\ &= \frac{\square}{\square} \times 100 \\ &= \_\_\_\_\_\_\_\_\% . \end{aligned}$$

The answer is 18,18%. Did you succeed?

The coefficient of variation for cereal B is

$$\begin{aligned} CV_B &= \frac{s}{\overline{x}} \times 100 \\ &= \frac{\square}{\square} \times 100 \\ &= \_\_\_\_\_\_\_\_\% . \end{aligned}$$

The coefficient of variation for cereal C is

$$\begin{aligned} CV_C &= \frac{s}{\overline{x}} \times 100 \\ &= \frac{\square}{\square} \times 100 \\ &= \_\_\_\_\_\_\_\_\% . \end{aligned}$$

From the results we see that cereal _____ displays the least variation and is therefore the most consistent in sales.

2.5 Worksheet 5

Worksheet 5 is based on study unit 6.5: *The box-and-whisker diagram*, on pages 185 – 186 of the study guide. Do the exercise before you proceed.

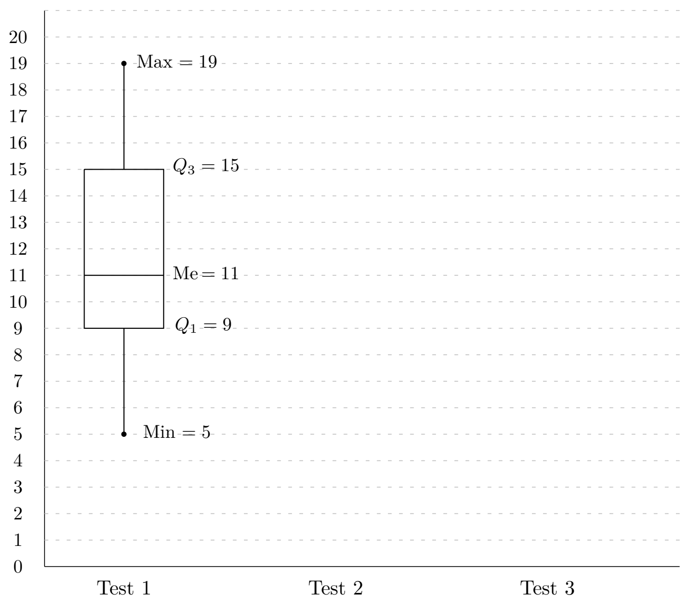
Activity 2.8

A class of students completes three tests. The scores for the three tests are presented in the table below.

Student test scores with marks out of 20, for each test:

	Test		
	1	**2**	**3**
Lowest value	5	7	2
Q_1	9	9	6
Me	11	12	12
Q_3	15	16	17
Highest value	19	20	20

Display this data using a box-and-whisker diagram.



Answer the following questions:

1. Which test(s) shows the widest spread of data?

____

2. In which test(s) did half of the students score a mark of less than 11 out of 20?

____

3. What percentage of scores in test 2 falls between 9 and 16 out of 20?

____

4. In which test(s) did 75% of the students score a mark of less than 15 out of 20?

____

5. What is the range of the test scores for test 1?

____

2.6 Worksheet 6

Worksheet 6 is a revision of some topics in component 6.

Activity 2.9

Answer the following questions:

1. Match each characteristic or description in column B with a measure of central tendency or dispersion in column A.

   For example:

   (a) C, E.

A Measures of central tendency and dispersion	
(a)	Median
(b)	Standard deviation
(c)	Mode
(d)	Mean
(e)	Coefficient of variation
(f)	Range

B Characteristics and description	
A	The difference between the highest and the lowest value in a data set.
B	Measures how much the data differ from the mean.
C	A better measure of central tendency when the data are very skewed.
D	Reliable since it reflects all the values in a data set.
E	The value that occupies the middle position of a group of values in a data set when the data values are arranged in ascending order.
F	Does not exist in many cases, and there may be more than one in others.
G	Compares two or more sets of data with different means, sample sizes or measurement units.
H	It may be affected by extreme values that are not representative of the set as a whole.
I	The value that occurs most often in a data set (data value with the highest frequency).
J	A value of zero means that there is no variation in the data.

2. Last year a small consulting company paid each of its three clerks R152 000, each of its two analysts R190 000 each, the secretary R175 000 and the senior owner R290 000. Calculate the number of people who earned less than the mean salary.

____

____

____

____

____

____

____

____

3. The marks out of 20 for the first assignment of 13 students enrolled in a Business Calculations class were recorded in ascending order as follows:

$$4 \quad 7 \quad 9 \quad 11 \quad 13 \quad 13 \quad 14 \quad 15 \quad 15 \quad 19 \quad 19 \quad 19 \quad 20$$

After calculating the mean, median and mode, an error was discovered. One of the 19s should have been a 15. Without calculating, you know from your knowledge of descriptive measures, that the measure/s of central tendency that will NOT change, is/are the

____

COMPONENT 3

Index numbers and transformations

The following examples and activities will improve your understanding of the basic principles covered in component 5 of the study guide. There is a worksheet for every study unit in component 5. Work through each worksheet after you have studied the relevant study material in the study guide.

3.1 Worksheet 1

Worksheet 1 is based on study unit 5.1: *Index numbers*, on pages 146 – 152 of the study guide. Do the activities and exercise before you proceed.

Example 3.1

The table below shows the prices and quantities of the major raw materials used in a factory in 2006 and 2010.

Material	**Prices R**		**Quantities (kg)**	
	2006	**2010**	**2006**	**2010**
A	40	41	3 000	3 150
B	39	53	2 750	2 000
C	38	30	500	750

Calculate the following indices for 2010 with 2006 as base year:

1. Laspeyres price index
2. Laspeyres quantity index
3. Paasche price index
4. Paasche quantity index
5. value index

Since 2006 is the base year, the 2006 prices and quantities are indicated by p_0 and q_0 respectively, while the 2010 prices and quantities are indicated by p_n and q_n respectively.

The best way of explaining the calculations is by using a table.

Material	p_0	p_n	q_0	q_n	$p_0 \times q_0$ $= p_0q_0$	$p_0 \times q_n$ $= p_0q_n$	$p_n \times q_0$ $= p_nq_0$	$p_n \times q_n$ $= p_nq_n$
A	40	41	3 000	3 150	$40 \times 3\,000$	$40 \times 3\,150$	$41 \times 3\,000$	$41 \times 3\,150$
B	39	53	2 750	2 000	$39 \times 2\,750$	$39 \times 2\,000$	$53 \times 2\,750$	$53 \times 2\,000$
C	38	30	500	750	38×500	38×750	30×500	30×750

Completing the calculations leads to the following:

Material	p_0	p_n	q_0	q_n	p_0q_0 or $p_{2006}q_{2006}$	p_0q_n or $p_{2006}q_{2010}$	p_nq_0 or $p_{2010}q_{2006}$	p_nq_n or $p_{2010}q_{2010}$
A	40	41	3 000	3 150	120 000	126 000	123 000	129 150
B	39	53	2 750	2 000	107 250	78 000	145 750	106 000
C	38	30	500	750	19 000	28 500	15 000	22 500
Total					246 250	232 500	283 750	257 650

1. The Laspeyres price index is

$$\begin{aligned} P_L(n) &= \frac{p_nq_0}{p_0q_0} \times 100 \\ P_L(2010) &= \frac{p_{2010}q_{2006}}{p_{2006}q_{2006}} \times 100 \\ &= \frac{283\,750}{246\,250} \times 100 \\ &= 115{,}23. \end{aligned}$$

For the same quantity of goods bought in 2006 and 2010, the price paid in 2010 is 15,23% higher than in 2006. Note: 115,23% − 100% = 15,23%.

2. The Laspeyres quantity index is

$$\begin{aligned} Q_L(n) &= \frac{p_0q_n}{p_0q_0} \times 100 \\ Q_L(2010) &= \frac{p_{2006}q_{2010}}{p_{2006}q_{2006}} \times 100 \\ &= \frac{232\,500}{246\,250} \times 100 \\ &= 94{,}42. \end{aligned}$$

3. The Paasche price index is

$$\begin{aligned} P_P(n) &= \frac{p_nq_n}{p_0q_n} \times 100 \\ P_P(2010) &= \frac{p_{2010}q_{2010}}{p_{2006}q_{2010}} \times 100 \\ &= \frac{257\,650}{232\,500} \times 100 \\ &= 110{,}82. \end{aligned}$$

The same quantity of goods as bought in 2010, would have cost 10,82% less in 2006.

4. The Paasche quantity index is

$$\begin{aligned} Q_P(n) &= \frac{p_nq_n}{p_nq_0} \times 100 \\ Q_P(2010) &= \frac{p_{2010}q_{2010}}{p_{2010}q_{2006}} \times 100 \\ &= \frac{257\,650}{283\,750} \times 100 \\ &= 90{,}80. \end{aligned}$$

The value index is

$$\begin{aligned} V &= \frac{p_n q_n}{p_0 q_0} \times 100 \\ &= \frac{p_{2010} q_{2010}}{p_{2006} q_{2006}} \times 100 \\ &= \frac{257\,650}{246\,250} \times 100 \\ &= 104{,}63. \end{aligned}$$

The increase in the value of the materials used, is 4,63%.

Activity 3.1

The table gives the average prices and monthly consumption quantities for three items of office equipment in a stationery shop in 2004 and 2009.

Equipment	**2004**		**2009**	
	Price R	**Consumption**	**Price R**	**Consumption**
Ballpoint pens	6,40	400	7,50	500
Pencils	6,50	150	6,50	300
Rulers	6,25	350	6,80	300

Calculate the following indices for 2009 with 2004 as base year:

1. Laspeyres price index
2. Laspeyres quantity index
3. Paasche price index
4. Paasche quantity index
5. value index

The 2004 prices and quantities are indicated by p_0 and q_0 respectively, while the 2009 prices and quantities are indicated by p_n and q_n respectively. Take care that you label the columns correctly. (The order of the columns differs from that of the example.)

Complete the table:

Equipment	**Price (R)**		**Quantities**					
	2004	**2009**	**2004**	**2009**				
	p_0	p_n	q_0	q_n	p_0q_0 or $p_{2004}q_{2004}$	p_0q_n or $p_{2004}q_{2009}$	p_nq_0 or $p_{2009}q_{2004}$	p_nq_n or $p_{2009}q_{2009}$
Ballpoint pens	6,40	7,50	400	500	2 560,0			
Pencils								
Rulers								
Total								

1. The Laspeyres price index is

$$\begin{aligned} P_L(n) &= \frac{p_n q_0}{p_0 q_0} \times 100 \\ P_L(2009) &= \frac{p_{2009} q_{2004}}{p_{2004} q_{2004}} \times 100 \\ &= \frac{\qquad\qquad}{\qquad\qquad} \times 100 \\ &= 111{,}05. \end{aligned}$$

2. The Laspeyres quantity index is

$$\begin{aligned} Q_L(n) &= \frac{p_0 q_n}{p_0 q_0} \times 100 \\ Q_L(2009) &= \frac{\qquad\qquad\qquad}{\qquad\qquad\qquad} \times 100 \\ &= \frac{\qquad\qquad}{\qquad\qquad} \times 100 \\ &= 122{,}76. \end{aligned}$$

3. The Paasche price index is

$$\begin{aligned} P_P(n) &= \frac{\qquad\qquad}{\qquad\qquad} \times 100 \\ P_P(2009) &= \frac{\qquad\qquad\qquad}{\qquad\qquad\qquad} \times 100 \\ &= \frac{\qquad\qquad}{\qquad\qquad} \times 100 \\ &= \underline{\qquad\qquad}. \end{aligned}$$

4. The Paasche quantity index is

$$\begin{aligned} Q_P(n) &= \frac{\qquad\qquad}{\qquad\qquad} \times 100 \\ Q_P(2009) &= \frac{\qquad\qquad\qquad}{\qquad\qquad\qquad} \times 100 \\ &= \frac{\qquad\qquad}{\qquad\qquad} \times 100 \\ &= \underline{\qquad\qquad}. \end{aligned}$$

5. The value index is

$$\begin{aligned} V &= \frac{\qquad\qquad}{\qquad\qquad} \times 100 \\ &= \frac{\qquad\qquad\qquad}{\qquad\qquad\qquad} \times 100 \\ &= \frac{\qquad\qquad}{\qquad\qquad} \times 100 \\ &= \underline{\qquad\qquad}. \end{aligned}$$

3.2 Worksheet 2

Worksheet 2 is based on study unit 5.2: *Transformations and rates*, on pages 153 – 158 of the study guide. Do the activities and exercise before you proceed.

The following example is based on paragraph 5.2.1: *Deflating*.

Example 3.2

The table below contains the yearly earnings of a one-man company, over a five-year period from 2003 to 2007, as well as the consumer price index for the same period.

Year	Actual yearly earnings	Consumer price index (CPI)
2003	280 000	100
2004	315 000	116
2005	325 000	126
2006	380 000	130
2007	410 000	135

Calculate the deflated earnings of the company over the five-year period.

The year 2003 is taken as the base year because it has a CPI of 100.

Year	Actual yearly earnings	Consumer price index (CPI)	$\frac{\text{actual earnings}}{\text{CPI}} \times 100$	Deflated earnings (R)
2003	280 000	100	$\frac{280\,000}{100} \times 100$	280 000,00
2004	315 000	116	$\frac{315\,000}{116} \times 100$	271 551,72
2005	325 000	126	$\frac{325\,000}{126} \times 100$	257 936,51
2006	380 000	130	$\frac{380\,000}{130} \times 100$	292 307,69
2007	410 000	135	$\frac{410\,000}{135} \times 100$	303 703,70

If you could move back the 2004 actual earnings of R315 000 to 2003, it would only have been worth R271 551,71 in 2003. Similarly, if you could move back the 2005 actual earnings of R325 000 to 2003, it would only have been worth R257 936,51 in 2003.

If you could move back the 2006 actual earnings of R380 000 to 2003, it would have been worth R292 307,69 in 2003. Similarly, if you could move back the 2007 actual earnings of R410 000 to 2003, it would have been worth R303 703,70 in 2003.

The actual earnings showed an increase for each of the five years from 2003 to 2007. When these amounts are, however, deflated to 2003 levels, it is clear that the company earned less in real value terms during 2004 and 2005 than was the case in 2003. Earnings again increased in real value terms during 2006 and 2007.

Activity 3.2

The following table shows the yearly living expenses for a typical student at a university over a number of years. Assume 1996 is the base year (with CPI of 100).

Year	**Actual yearly expenses**	**CPI Base year = 1996**
1995	116 000	98,0
2000	125 500	108,5
2005	146 500	121,5

Calculate the deflated expenses of a student over the period. That is, complete the following table:

Year	**Actual expenses**	**CPI Base year = 1996**	$\frac{\textbf{actual expenses}}{\text{CPI}} \times 100$	**Deflated expenses (R)**
1995	116 000	98,0	$\frac{116\,000}{98,0} \times 100$	118 367,35
2000	125 500			
2005				

Consider the deflated expenses and answer the following questions:

1. What would the yearly expenses of a student in 1996 have been if it was R116 000 in 1995?

   ____

2. What would the yearly expenses of a student in 1996 have been if it was R146 500 in 2005?

   ____

The following example is based on paragraph 5.2.2: *Exchange rate.*

Example 3.3

You own a small recording company. You sell CDs through your website for R105, including shipping and handling. You get e-mail from a person who owns a record store in England who would like to sell your CDs in his store. He is willing to pay R75 for each of your CDs and wants to sell them in his store for £11 (eleven pounds). He says his profit from each sale would be £5 and he would split it with you. The exchange rate between the South African rand and the British pound is R14,50 = £1,00.

1. How much profit (in pounds) will you get from the sale of each CD in the British store?

   The profit will be £2,50.

2. How much is that profit in rand?

   In rand it is

$$2{,}50 \times 14{,}50 = 36{,}25.$$

   The profit is R36,25.

3. What is the total amount that you will receive per CD that is sold?

   The total amount is calculated as

$$\begin{aligned} \text{total amount} &= \text{selling price} + \text{profit} \\ &= 75 + 36{,}25 \\ &= 111{,}25. \end{aligned}$$

   The total amount is R111,25.

4. Is that a good business opportunity for you? Why or why not?

   Yes, R111,25 is more than the current selling price of R105.

Activity 3.3

1. A woman travelling to Europe exchanges R24 500 for euros (€).
   The exchange rate is R12,58 = €1.

   (a) Using this exchange rate, how many euros does she receive in return?

   This can be interpreted as a ratio:

$$\begin{aligned} &\text{rand} : \text{euro} \\ &12{,}58 : 1 \\ &\frac{12{,}58}{12{,}58} : \frac{1}{12{,}58} \\ &1 : \_\_\_\_\_ \\ &1 \times 24\,500 : \_\_\_\_\_ \times \_\_\_\_\_ \\ &24\,500 : \_\_\_\_\_\_\_ \end{aligned}$$

   Hence R1 is equal to €______

   Multiply both sides of the ratio by 24 500.

   R24 500 is equal to how many euros?

   The amount of R24 500 is equal to €________

   (b) On her return she still has €56 left. The exchange rate has changed to R11,90 = €1. Using this exchange rate, determine the amount in rand that she receives back.

   This can also be interpreted as a ratio:

euro	:	rand
______	:	______
______	:	______
______	:	______

   The amount of €56 is equal to R_______

2. The following activity is based on paragraph 5.2.3: *Fine ounce.*

   The R/$ (rand / US dollar) exchange rate is R1,00 = $0,135. The price of gold is $934,00 per fine ounce. A trader sells South African gold Kruger rands. Each coin contains one ounce of gold. The price of the coins is determined only by the value of the gold it contains. The following information is supplied:

One fine ounce is equal to 31,10348 grams, one ounce is equal to 28,35 grams.

   Determine the price of one Kruger rand in rand.
   To solve this problem, it is advisable to divide it into smaller parts.

   (a) Determine the gold price per fine ounce in rand value.
   Write the exchange rate as a ratio:

rand : dollar	
$1 : 0{,}135$	R1,00 = $0,135
$\frac{1}{0{,}135} : \frac{\quad}{\quad}$	Divide both sides of the ratio by 0,135.
________ : 1	Obtain the value in rand equal to $1.
________ : ________	Multiply both sides of the ratio by 934.
________ : ________	Obtain the value in rand equal to $934.

   The gold price is $934,00 per fine ounce and $934,00 is equal to R________

   The gold price is thus R________ per fine ounce.

   (b) Determine the price in rand of one gram of gold.

   The price of R________ per fine ounce can be written as a ratio:

rand : gram	
________ : 31,10348	There are 31,10348 grams in one fine ounce.
$\frac{\quad}{31{,}10348} : \frac{31{,}10348}{31{,}10348}$	Divide both sides of the ratio by 31,10348.
________ : 1	Obtain the value in rand of one gram gold.

   One gram of gold costs R________

(c) Determine the price in rand of one **ounce** of gold.

The price of R__________ for one gram of gold can be written as a ratio:

rand : gram

__________ : __________

_______________ : _______________

__________ : __________

Multiply both sides of the ratio by 28,35 to obtain the price in rand of 28,35 g of gold (28,35 g = 1 ounce).

The price of one Kruger rand is R_______________

The following activity is based on paragraph 5.2.4: *Growth rate.*

3. You are given the following information:

Year	Real GDP
2000	517 360
2005	635 820

What was the growth rate from 2000 to 2005?

The period is __________ years.

The growth rate from 2000 to 2005 is

$$\left(\left(\frac{GDP_{2005}}{GDP_{2000}}\right)^{\frac{1}{\square}} - 1\right) \times 100 = \left(\left(\frac{\square}{\square}\right)^{\frac{1}{\square}} - 1\right) \times 100$$

$$= \_\_\_\_\_\_\_\_\_\_$$

COMPONENT 4

Functions and representations of functions

The following examples and activities will improve your understanding of the basic principles covered in component 2 of the study guide. There is a worksheet for every study unit in component 2. Work through each worksheet after you have studied the relevant study material in the study guide.

4.1 Worksheet 1

Worksheet 1 is based on study unit 2.1: *What is a function?*, on pages 62 – 64 of the study guide. Do the activities before you proceed.

Example 4.1

Study the following:

The speed, s, of a car is a function of the distance, d, travelled and the time, t, taken to travel the distance. This relationship can be expressed as

$$s = f(d;\, t) = \frac{d}{t}.$$

Calculate the speed of a car which travelled 354 kilometres in three hours.
From the question, the distance, d, is 354 km and time, t, is three hours.
The speed is

$$\begin{aligned} s &= f(354;\, 3) \\ &= \frac{354}{3} \\ &= 118. \end{aligned} \qquad \frac{\text{kilometre}}{\text{hour}}$$

The speed is therefore 118 kilometre per hour.

Activity 4.1

1. The total cost in rand of renting a sailboat for n days is given by the function

$$C(n) = 6\,500 + 4\,125n.$$

   (a) Calculate the cost in rand to rent the sailboat for one day.

   (b) Nick has R55 000 cash available and wants to take his family on a sailing trip. How many days can he afford to rent the boat?

2. A volleyball court has a rectangular shape. It has a width of x metres and a length of $2x$ metres. Draw a diagram of the court showing the dimensions.

   (a) The area of the court (in square metres) is $A(x) =$ ____________________

   (b) Suppose the court has a width of 12 metres and that the whole area has to be repainted. How much will it cost to repaint the court if the cost of paint and labour is R11,75 per square metre?

4.2 Worksheet 2

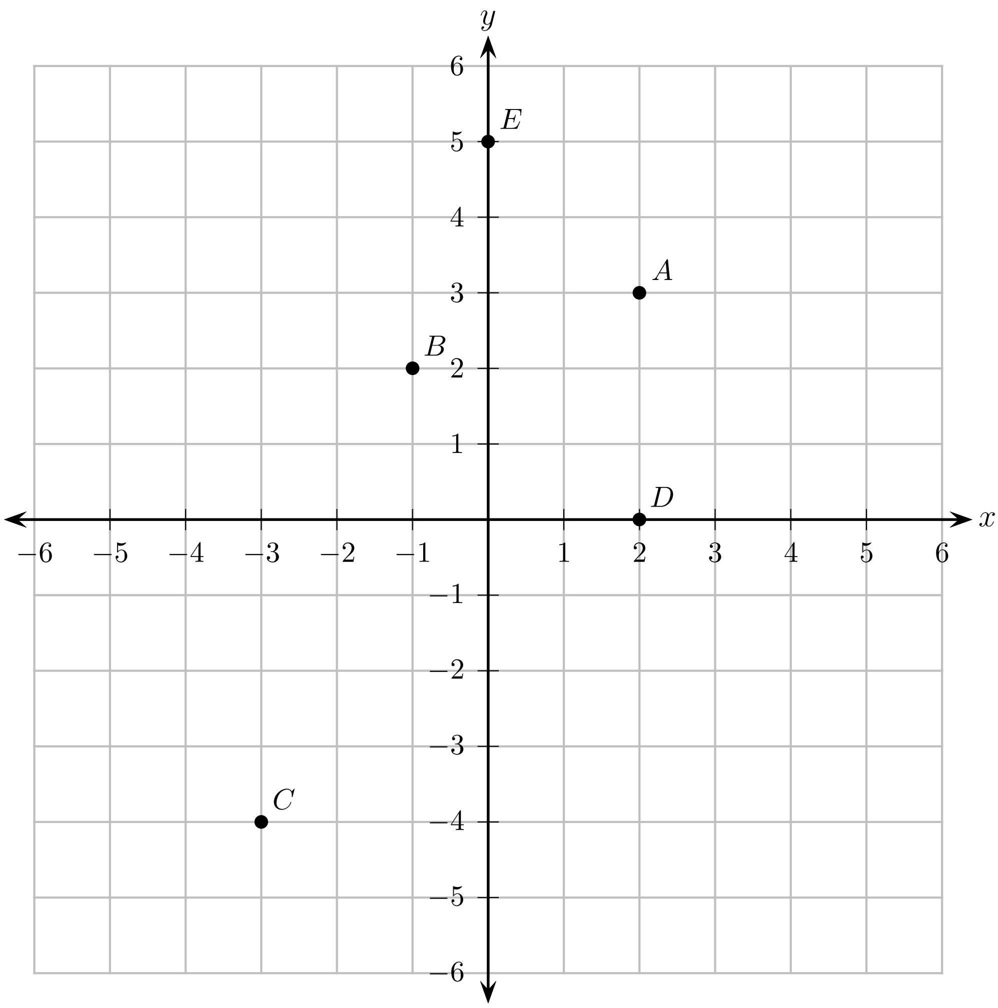
Worksheet 2 is based on study unit 2.2: *Linear functions*, on pages 65 – 83 of the study guide. Do the activities and exercise before you proceed.

Example 4.2

Study the following:

Plot the ordered pairs and name the quadrant or axis in/on which the point lies:

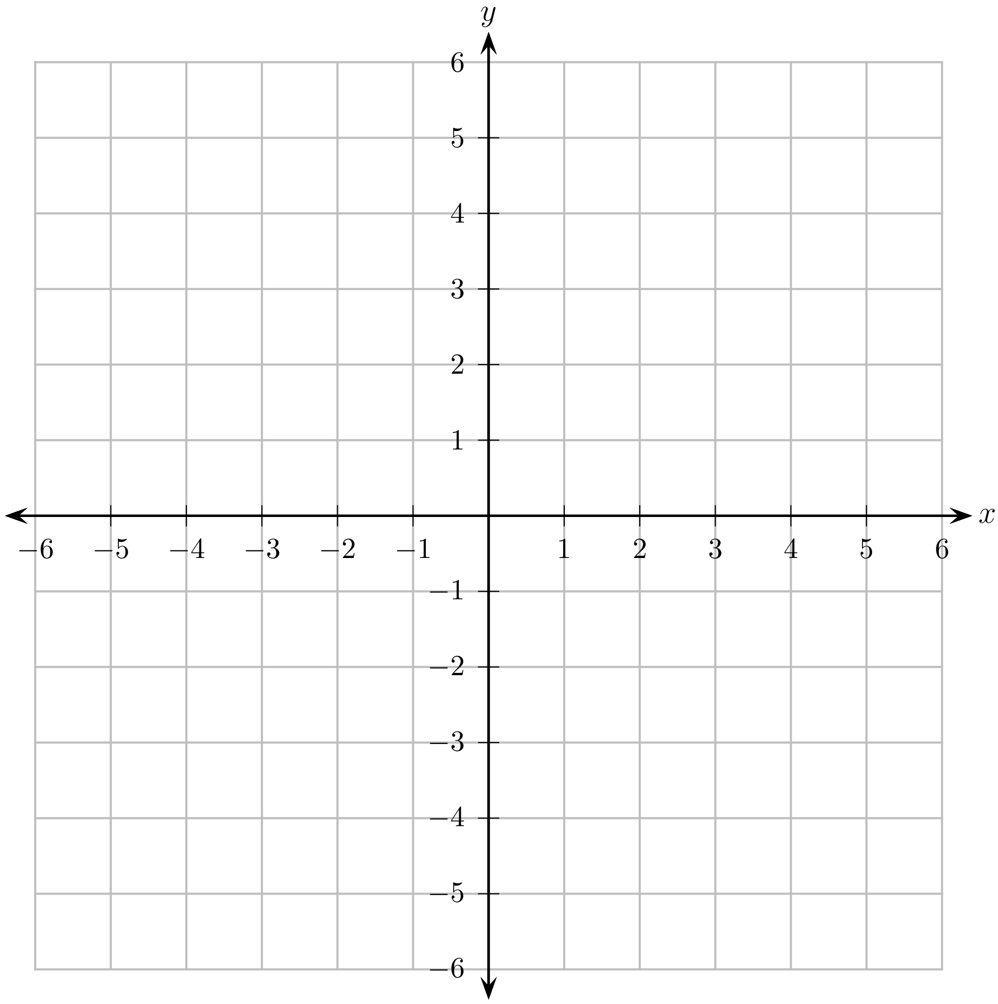
$A = (2; 3)$, $B = (-1; 2)$, $C = (-3; -4)$, $D = (2; 0)$ and $E = (0; 5)$.
Remember that each ordered pair is associated with only one point on the graph.



The point $A = (2; 3)$ lies in quadrant I.
The point $B = (-1; 2)$ lies in quadrant II.
The point $C = (-3; -4)$ lies in quadrant III.
The point $D = (2; 0)$ lies on the x-axis.
The point $E = (0; 5)$ lies on the y-axis.

Activity 4.2

1. Consider the following rectangular coordinate system and answer the questions:



(a) The coordinate system is divided into four areas called ________________________.

(b) The coordinate system is created by the ________ and ________ axes. Coordinates $(x; y)$ are plotted on the coordinate system.

(c) The ________ coordinate, which is the first number, tells us to go _____________ from the origin if it is positive and _____________ from the origin if it is negative.

(d) The ________ coordinate, which is the second number, tells us to go _____________ from the origin if it is positive and _____________ from the origin if it is negative.

(e) Plot the following points on the coordinate system:
$A = (-3;\ 2)$, $B = (4;\ -1)$, $C = (-1;\ 0)$, $D = (0;\ -3)$, $E = (-1{,}5;\ -2)$, $F = (5;\ 3)$.

On page 68 of the study guide, four specific cases are illustrated which can occur when a straight line is drawn. Each one of these cases will be illustrated in the next activity.

2. (a) *The case where $a > 0$ and $b > 0$.*

Graph the linear function

$$y = f(x) = 2x + 4.$$

- For this question, the function is written in the form $y = f(x) = ax + b$ and we see that $a =$ ________ and $b =$ ________.

> The value a is called the slope of the line. This is the steepness with which the straight line ascends or descends. The value b is the intercept on the y-axis.

> Note that $a > 0$ and $b > 0$.

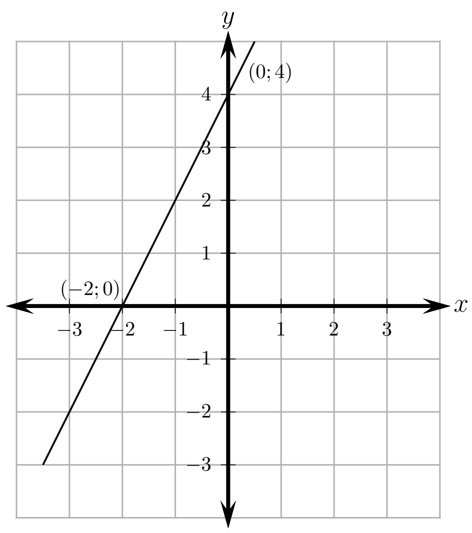
- You need only two points to graph a linear function. These points may be chosen as the x- and y-intercepts of the graph.
- To determine the x-intercept, set $y = f(x) = 0$ and solve for x:

$$\begin{aligned} 2x + 4 &= 0 \\ 2x &= \_\_\_\_\_\_\_\_ \\ x &= \_\_\_\_\_\_\_\_ \end{aligned}$$

- To determine the y-intercept, set $x = 0$ to find $f(0)$:

$$\begin{aligned} y &= f(0) \\ &= 2 \times 0 + 4 \\ &= \_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_ \end{aligned}$$

- The graph of the above function is a line passing through the points (______; ______) and (______; ______) as shown below.



Choose the correct option:

Since the slope, a, is (positive/negative), the function $f(x)$ (increases/decreases) if you move from left to right. The value of the y-intercept, b, is (positive/negative).

(b) *The case where $a > 0$ and $b < 0$.*
Graph the linear function
$$x - 2y = 3.$$

- Rewrite the function to the form $y = f(x) = ax + b$:

$$x - 2y = 3$$
$$x - 2y + 2y - 3 = 3 + 2y - 3$$ Add $2y$ both sides and subtract three both sides.
$$x - 3 = 2y$$
$$2y = x - 3$$
$$y = \frac{\square}{\square}x - \frac{\square}{\square}$$

In this case $a =$ ________ and $b =$ ________ Note that $a > 0$ and $b < 0$.

The answer is $a = \frac{1}{2}$ and $b = -1\frac{1}{2}$. Did you succeed?

- To determine the x-intercept, set $y = f(x) = 0$ and solve for x:

____

____

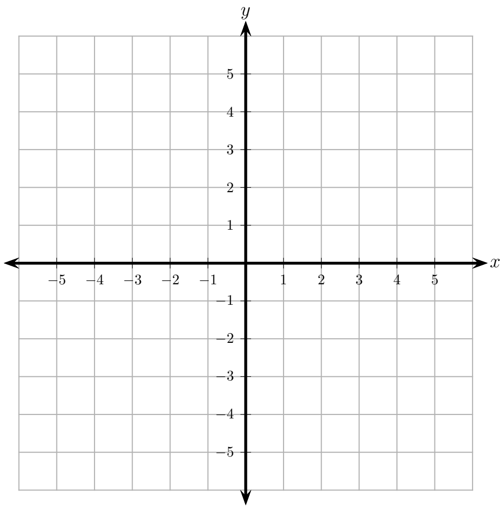
____

____

- To determine the y-intercept, set $x = 0$ to find $f(0)$:

$$y = f(0)$$
$$= \_\_\_\_\_\_\_\_\_\_\_\_$$
$$= \_\_\_\_\_\_\_\_\_\_\_\_$$
$$= \_\_\_\_\_\_\_\_\_\_\_\_$$

- The graph of the above function is a line passing through the points (_____; _____) and (_____; _____). Draw the line below.



Choose the correct option:

Since the slope, a, is (positive/negative), the function $f(x)$ (increases/decreases) if you move from left to right. The value of the y-intercept, b, is (positive/negative).

(c) *The case where $a < 0$ and $b > 0$.*
Graph the linear function

$$2x + 5y = 10.$$

- Rewrite the function to the form $y = f(x) = ax + b$:

$$2x + 5y = 10$$
$$\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_$$
$$\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_$$
$$\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_$$

In this case $a =$ ________ and $b =$ ________ Note that $a < 0$ and $b > 0$.

- To determine the x-intercept, set $y = f(x) = 0$ and solve for x:

____

____

____

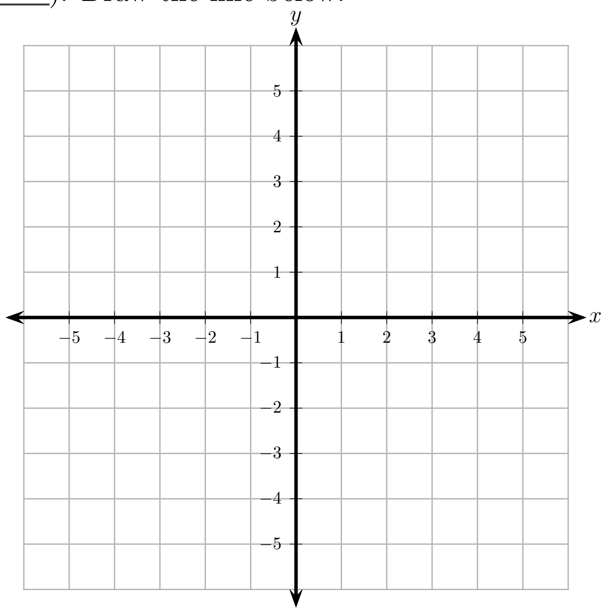
____

The answer is $x = 5$. Did you succeed?

- To determine the y-intercept, set $x = 0$ to find $f(0)$:

$$y = f(0)$$
$$= \_\_\_\_\_\_\_\_\_\_\_\_$$
$$= \_\_\_\_\_\_\_\_\_\_\_\_$$
$$= \_\_\_\_\_\_\_\_\_\_\_\_$$

- The graph of the above function is a line passing through the points (_____; _____) and (_____; _____). Draw the line below.



Choose the correct option:
Since the slope, a, is (positive/negative), the function $f(x)$ (increases/decreases) if you move from left to right. The value of the y-intercept, b, is (positive/negative).

(d) *The case where $a < 0$ and $b < 0$.*
Graph the linear function
$$-3y = 15 + 6x.$$

- Rewrite the function to the form $y = f(x) = ax + b$:

$$-3y = 15 + 6x$$
$$\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_$$
$$\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_$$
$$\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_$$

In this case $a =$ ________ and $b =$ ________ Note that $a < 0$ and $b < 0$.

- To determine the x-intercept, set $y = f(x) = 0$ and solve for x:

____

____

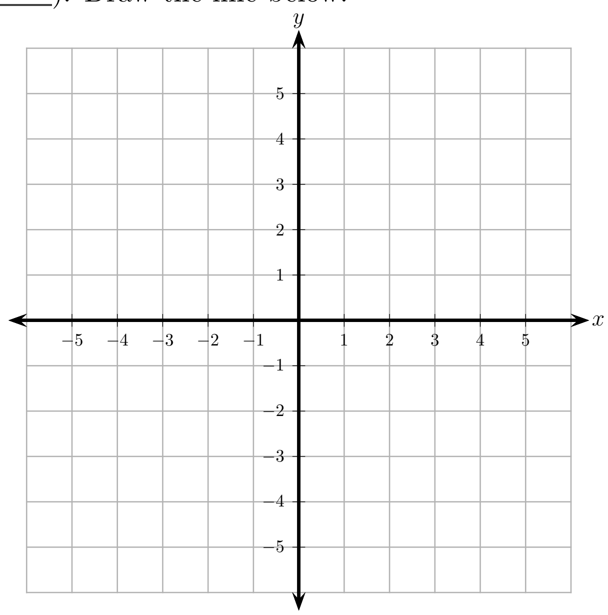
____

____

- To determine the y-intercept, set $x = 0$ to find $f(0)$:

$$\begin{aligned} y &= f(0) \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

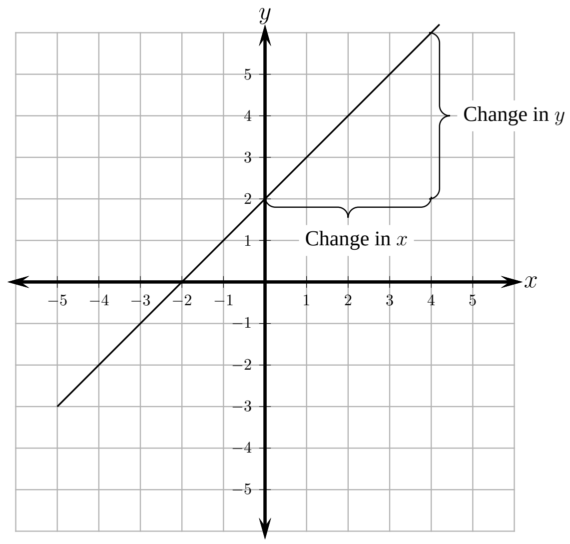
- The graph of the above function is a line passing through the points (_____; _____) and (_____; _____). Draw the line below.



Choose the correct option:
Since the slope, a, is (positive/negative), the function $f(x)$ (increases/decreases) if you move from left to right. The value of the y-intercept, b, is (positive/negative).

3. Find the slopes of the following lines:

(a)



The slope is the ratio of the change in y-values to a given change in x-values.

The change in y-values is 4.

The change in x-values is __________

The size of the slope is calculated as

$$\text{size of slope} = \frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}} = \frac{\square}{\square} = \_\_\_\_\_\_\_\_$$

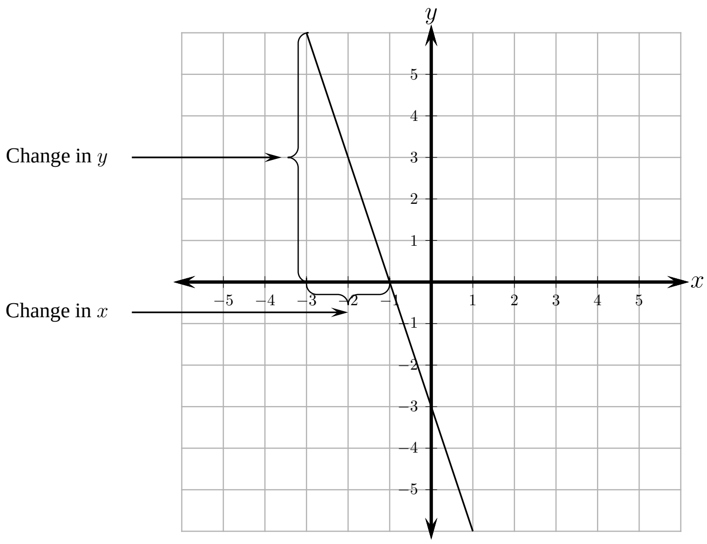
Choose the correct option to determine the sign (+/−) of the slope:

The line is (ascending/descending) from left to right.

The value of a is thus (positive (> 0)/negative (< 0)).

The slope is thus $a =$ __________

(b)



____

____

____

____

Choose the correct option to determine the sign (+/−) of the slope:

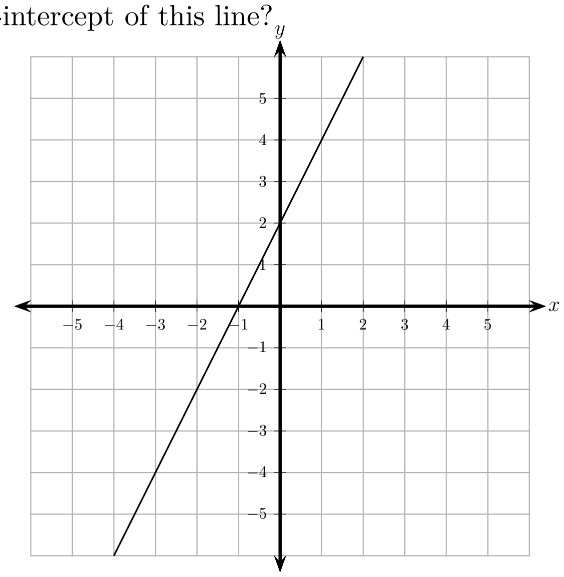
The line is (ascending/descending) from left to right.

The value of a is thus (positive (>0)/negative (<0)).

The slope is thus $a =$ __________

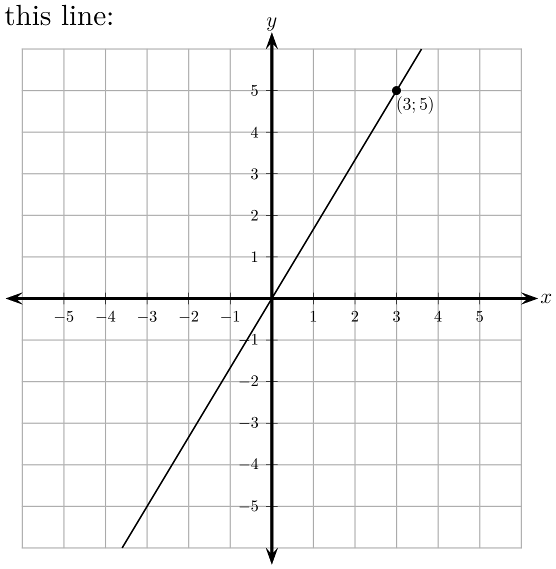
4. Answer the following questions:

(a) What value is the y-intercept of this line?



____

(b) Find the equation of this line:



See paragraph 2.2.6 of the study guide: *Special cases – the case for which the constant term is zero, this is* $b = 0$. This is the case when the line goes through the origin.

Choose a point on the line, for example (3; 5).

The change in y-values is __________.

The change in x-values is __________.

The size of the slope is calculated as

$$\text{size of slope} = \frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}} = \frac{\square}{\square} = \_\_\_\_\_\_\_\_.$$

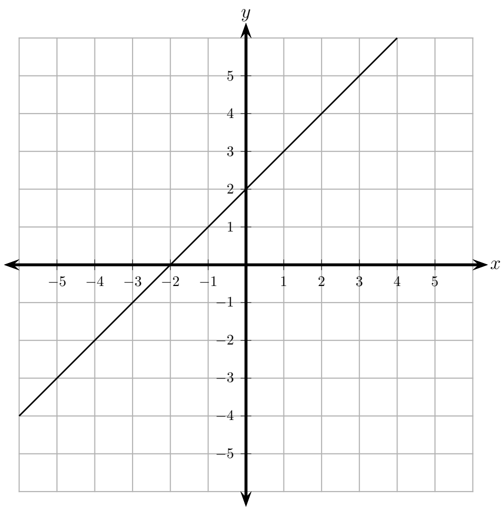
Choose the correct option to determine the signe (+/−) of the slope:

The line is (ascending/descending) from left to right. The value of a is thus (positive (> 0)/negative (< 0)).

The slope is thus $a =$ __________ and the value of $b =$ __________.

The equation for the line is ________________________________.

(c) Find the equation of this line:



Choose any two points through which the line passes. Take the coordinates of the x- and y-intercepts. Here, $(x_1; y_1) = (-2; 0)$ and $(x_2; y_2) = (0; 2)$. Use the formula $a = \frac{y_2 - y_1}{x_2 - x_1}$ to determine the slope of the line:

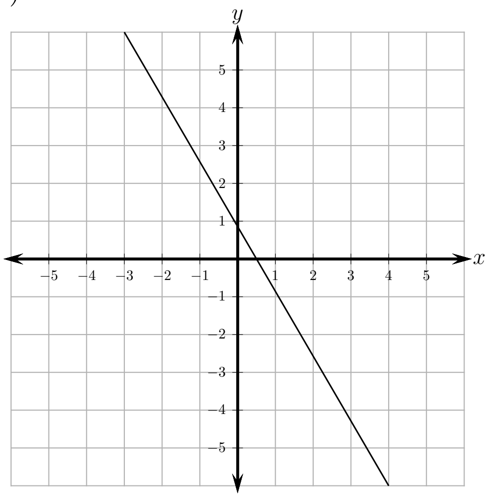
$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\square - \square}{\square - \square} = \_\_\_\_\_\_\_\_.$$

The general expression is $y = ax + b$ which reduces to $y = \square\, x + b$.

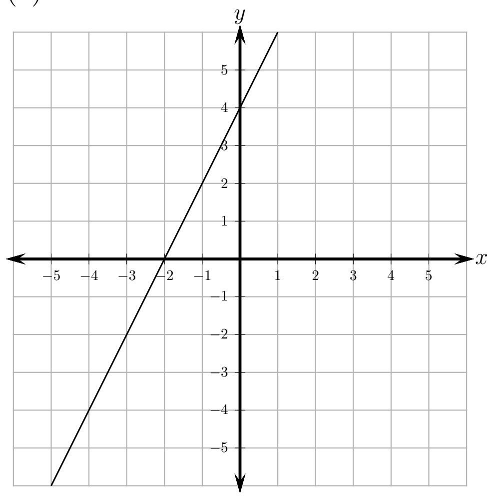
Try to find the value of b on your own and give the equation of the line:

(d) Which graph shows the line $y = 3 - x$?

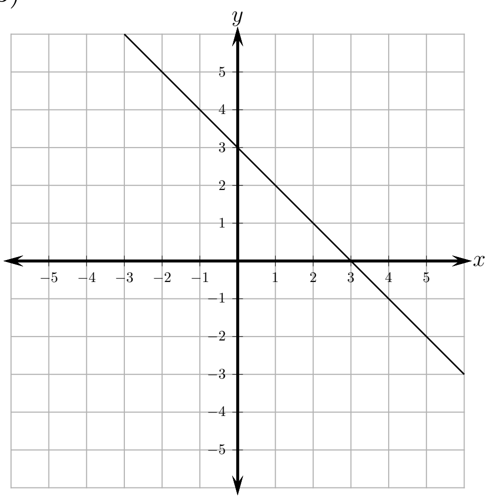
(1)



(2)



(3)



5. See paragraph 2.2.6 of the study guide: *Special cases – two straight lines which have the same slope but different y-intercepts.* This is the case when the lines are parallel.

   Write down the equation of the straight line that is parallel to

   $$2y = 5x + 7,$$

   passing through the point $\left(0; -3\frac{1}{2}\right)$.

   Express the equation $2y = 5x + 7$ in the form $y = ax + b$:

   ____

   ____

   ____

   The value of a is ___________and the value of b is ___________

   From this follows that the line has a slope of ___________

   Any line parallel to this line also has a slope of ___________

   The required straight line parallel to $2y = 5x + 7$ passes through $\left(0;\ -3\frac{1}{2}\right)$.

   Thus, the line has as y-intercept $y = -3\frac{1}{2}$.

   Therefore, the equation of the line is $y =$ ____ $x -$ ____ .

6. Match each sentence or description in column **B** with a specific case of a straight line in column **A**.

A	
Specific cases of a straight line	
1.	Two lines that are parallel.
2.	A straight line that passes through the origin.
3.	A line that is parallel to the x-axis.
4.	Two lines that intersect at the origin.
5.	A line parallel to the y-axis.

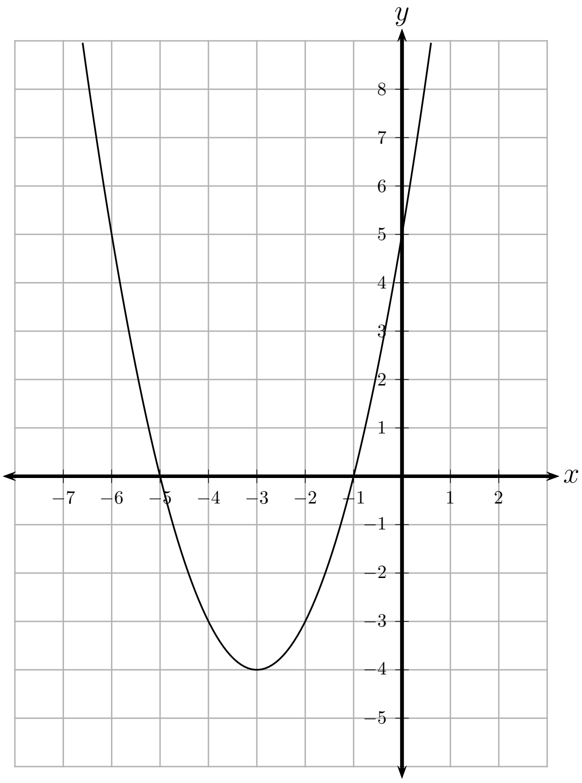
B	
Characteristics and descriptions	
A.	One line has a positive slope while the other has a negative slope – both with $b = 0$.
B.	Expression is $y = b$ where b is the intercept on the y-axis.
C.	There is no constant term (b-value) in the equation.
D.	Expression is $x = c$ where c is the intercept on the x-axis.
E.	Both lines have the same slope.

4.3 Worksheet 3

Worksheet 3 is based on study unit 2.3: *Quadratic functions*, on pages 84 – 93 of the study guide. Do the activities and exercise before you proceed.

Activity 4.3

1. Consider the graph of the quadratic function $f(x) = ax^2 + bx + c$:



Answer the following questions:

(a) What are the intercepts on the x-axis? (Read them off from the graph).

____

(b) What is the y-intercept?

____

(c) What is the value of x at the vertex?

____

(d) What is the minimum value of the function?

____

(e) In the function formula, $f(x) = ax^2 + bx + c$, is the value of a positive or negative? (Don't calculate the value, look at the shape of the graph.)

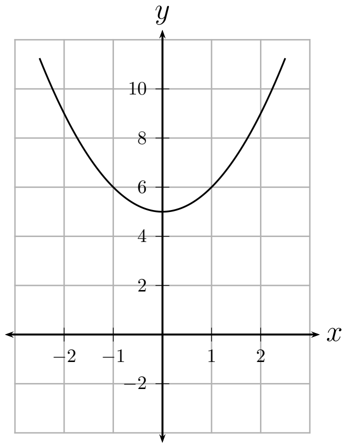
____

(f) Is the value of the discriminant, $b^2 - 4ac$, greater than zero, less than zero or equal to zero? (Don't calculate the value, look at the shape of the graph.)

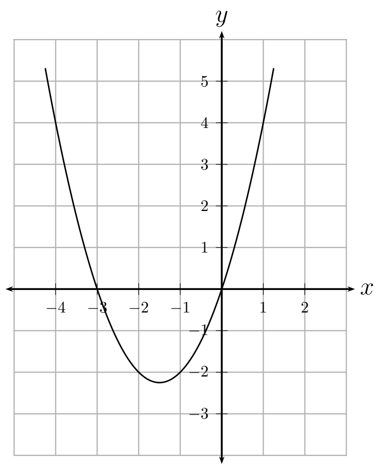
____

2. Consider the following graphs (A - F):

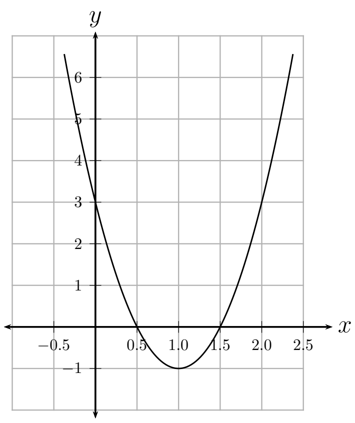
A.



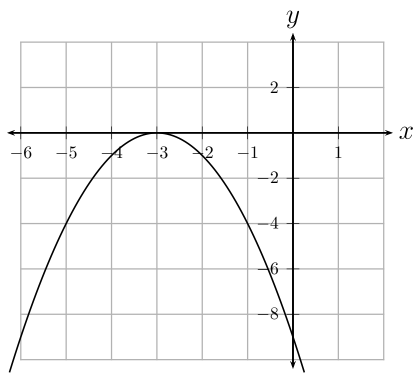
B.



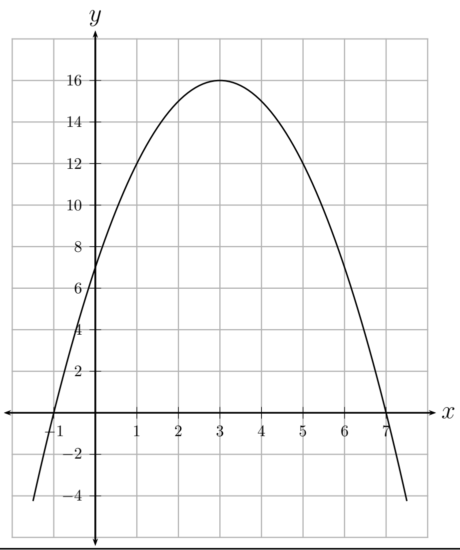
C.



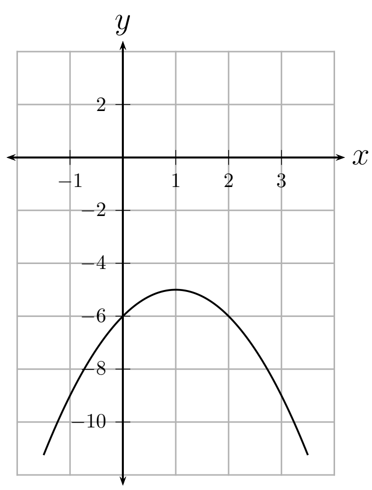
D.



E.



F.



Consider the following quadratic functions:

(a) $y = f(x) = 4x^2 - 8x + 3$
(b) $y = f(x) = -x^2 + 6x + 7$
(c) $y = f(x) = x^2 + 5$
(d) $y = f(x) = -x^2 - 6x - 9$
(e) $y = f(x) = -x^2 + 2x - 6$
(f) $y = f(x) = x^2 + 3x$

Start with the first quadratic function and determine the following for each function:

- coefficients
- minimum or maximum value
- value of x at vertex
- value of y at vertex
- discriminant
- x-intercepts (if any)
- y-intercept: $c = f(0)$
- correct corresponding graph

(a) **Function and coefficients**
From $y = f(x) = 4x^2 - 8x + 3$, we have that $a = 4$, $b = -8$ and $c = 3$.
Minimum or maximum value
The function has a minimum value because $a > 0$.
Value of x at vertex
The value of x at the vertex is

$$\begin{aligned} x_m &= -\frac{b}{2a} \\ &= -\frac{(-8)}{2 \times 4} \\ &= \frac{8}{8} \\ &= 1. \end{aligned}$$

Value of y at vertex $y = f\Big(-\dfrac{b}{2a}\Big)$
The value of y at the vertex, where $x = 1$, is

$$\begin{aligned} f(1) &= 4 \times 1^2 - 8 \times 1 + 3 \\ &= 4 - 8 + 3 \\ &= -1. \end{aligned}$$

Discriminant
The value of the discriminant is

$$\begin{aligned} b^2 - 4ac &= (-8)^2 - 4 \times 4 \times 3 \\ &= 46 - 48 \\ &= 16. \end{aligned}$$

The x-intercepts (if any)
The x-intercepts exists because $b^2 - 4ac > 0$ and are

$$\begin{aligned} x &= \frac{-b - \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-8) - \sqrt{16}}{2 \times 4} \\ &= \frac{8 - 4}{8} \\ &= \frac{4}{8} \\ &= \frac{1}{2} \end{aligned} \qquad \text{and} \qquad \begin{aligned} x &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-8) + \sqrt{16}}{2 \times 4} \\ &= \frac{8 + 4}{8} \\ &= \frac{12}{8} \\ &= 1\frac{1}{2}. \end{aligned}$$

The y-intercept $c = f(0)$
The y-intercept is calculated as

$$\begin{aligned} f(0) &= 4 \times 0^2 - 8 \times 0 + 3 \\ &= 0 + 0 + 3 \\ &= 3. \end{aligned}$$

Choose the corresponding graph
Graph C is the correct graph.

Complete the following table:

Coefficients	Min. or max. value?	At the vertex $x_m = -\frac{b}{2a}$	At the vertex $y = f\left(-\frac{b}{2a}\right)$	$b^2 - 4ac$	x-intercepts (if any)	$c = f(0)$	Corresponding graph
(a) $a = 4$ $b = -8$ $c = 3$	Minimum because $a > 0$	$x_m = 1$	$f(1) = -1$	16	$x = \frac{1}{2}$ and $x = 1\frac{1}{2}$	$f(0) = 3$	C
(b) $a = -1$ $b = 6$ $c = 7$	Maximum because $a < 0$	$x_m = 3$	$f() =$			$f(0) =$	
(c) $a =$ $b =$ $c =$		$x_m =$	$f() =$			$f(0) =$	
(d) $a =$ $b =$ $c =$		$x_m =$	$f() =$			$f(0) =$	
(e) $a =$ $b =$ $c =$		$x_m =$	$f() =$			$f(0) =$	
(f) $a =$ $b =$ $c =$		$x_m =$	$f() =$			$f(0) =$	

(b) **Function and coefficients**
From $y = f(x) = -x^2 + 6x + 7$, we have that $a = -1$, $b = 6$ and $c = 7$.
Minimum or maximum value
The function has a maximum value because $a < 0$.
Value of x at vertex
The value of x at the vertex is

$$\begin{aligned} x_m &= -\frac{b}{2a} \\ &= -\frac{6}{2 \times -1} \\ &= \frac{-6}{-2} \\ &= 3. \end{aligned}$$

Value of y at vertex $y = f\left(-\dfrac{b}{2a}\right)$
The value of y at the vertex, where $x = 3$, is

$$\begin{aligned} f(3) &= -1 \times 3^2 + 6 \times 3 + 7 \\ &= \underline{\hspace{5cm}} \\ &= \underline{\hspace{5cm}} \end{aligned}$$

Discriminant
The value of the discriminant is

$$\begin{aligned} b^2 - 4ac &= 6^2 - 4 \times (-1) \times 7 \\ &= \underline{\hspace{5cm}} \\ &= \underline{\hspace{5cm}} \end{aligned}$$

The x-intercepts (if any)
The x-intercepts exist because $b^2 - 4ac$ __________ and are

$$\begin{aligned} x &= \frac{-b - \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-6 - \sqrt{64}}{2 \times -1} \\ &= \frac{-6 - 8}{-2} \\ &= \frac{\square}{\square} \\ &= \square \end{aligned}$$

and

$$\begin{aligned} x &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-6 + \sqrt{64}}{2 \times -1} \\ &= \frac{-6 + 8}{-2} \\ &= \frac{\square}{\square} \\ &= \square. \end{aligned}$$

The y-intercept $c = f(0)$
The y-intercept is calculated as

$$\begin{aligned} f(0) &= \underline{\hspace{4cm}} \\ &= \underline{\hspace{4cm}} \\ &= \underline{\hspace{4cm}} \end{aligned}$$

Choose the corresponding graph

Graph __________ is the correct graph.

(c) **Function and coefficients**
From $y = f(x) = x^2 + 5$, we have that $a =$ _____, $b =$_____ and $c =$_____.
Minimum or maximum value

____

Value of x at vertex
The value of x at the vertex is

$$\begin{aligned} x_m &= -\frac{b}{2a} \\ &= \frac{}{} \\ &= \_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

Value of y at vertex $y = f\left(-\frac{b}{2a}\right)$
The value of y at the vertex, where $x =$ _____, is

$$\begin{aligned} f(\_\_\_\_\_) &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

Discriminant
The value of the discriminant is

$$\begin{aligned} b^2 - 4ac &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

The x-intercepts (if any)

The x-intercepts ______________________________________

____

____

The y-intercept $c = f(0)$
The y-intercept is calculated as

$$\begin{aligned} f(0) &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

Choose the corresponding graph

Graph __________ is the correct graph.

(d) **Function and coefficients**
From $y = f(x) = -x^2 - 6x - 9$, we have that $a =$ _____, $b =$_____ and $c =$_____.
Minimum or maximum value

____

Value of x at vertex
The value of x at the vertex is

$$\begin{aligned} x_m &= -\frac{b}{2a} \\ &= \frac{\square}{\square} \\ &= \frac{\square}{\square} \\ &= \_\_\_\_\_\_\_\_ \end{aligned}$$

Value of y at vertex $y = f\left(-\frac{b}{2a}\right)$
The value of y at the vertex, where $x =$ _____, is

$$\begin{aligned} f(\_\_\_\_\_) &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

Discriminant
The value of the discriminant is

$$\begin{aligned} b^2 - 4ac &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

The x-intercepts (if any)

The x-intercepts ______________________________________

____

____

The y-intercept $c = f(0)$
The y-intercept is calculated as

$$\begin{aligned} f(0) &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

Choose the corresponding graph

Graph __________ is the correct graph.

(e) **Function and coefficients**
From $y = f(x) = -x^2 + 2x - 6$, we have that $a =$ _____, $b =$_____ and $c =$_____.
Minimum or maximum value

____

Value of x at vertex
The value of x at the vertex is

$$\begin{aligned} x_m &= -\frac{b}{2a} \\ &= \frac{\square}{\square} \\ &= \frac{\square}{\square} \\ &= \_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

Value of y at vertex $y = f\left(-\frac{b}{2a}\right)$
The value of y at the vertex, where $x =$ _____, is

$$\begin{aligned} f(\_\_\_\_\_) &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

Discriminant
The value of the discriminant is

$$\begin{aligned} b^2 - 4ac &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

The x-intercepts (if any)

The x-intercepts ____

____

____

The y-intercept $c = f(0)$
The y-intercept is calculated as

$$\begin{aligned} f(0) &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

Choose the corresponding graph

Graph __________ is the correct graph.

(f) **Function and coefficients**
From $y = f(x) = x^2 + 3x$, we have that $a =$ _____, $b =$_____ and $c =$_____.
Minimum or maximum value

____

Value of x at vertex
The value of x at the vertex is

$$x_m = -\frac{b}{2a}$$
$$= \frac{}{}$$
$$= \frac{}{}$$
$$= \underline{\hspace{3cm}}$$

Value of y at vertex $y = f\left(-\frac{b}{2a}\right)$
The value of y at the vertex, where $x =$ _____, is

$$f(\underline{\hspace{1.5cm}}) = \underline{\hspace{6cm}}$$
$$= \underline{\hspace{6cm}}$$
$$= \underline{\hspace{6cm}}$$
$$= \underline{\hspace{6cm}}$$
$$= \underline{\hspace{6cm}}$$

Discriminant
The value of the discriminant is

$$b^2 - 4ac = \underline{\hspace{6cm}}$$
$$= \underline{\hspace{6cm}}$$
$$= \underline{\hspace{6cm}}$$

The x-intercepts (if any)

The x-intercepts exist because $b^2 - 4ac$ __________ and are

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{}{} \qquad\qquad = \frac{}{}$$
$$= \frac{}{} \qquad\qquad = \frac{}{}$$
$$= \frac{}{} \qquad\qquad = \frac{}{}$$
$$= \underline{\hspace{2cm}} \qquad\qquad = \underline{\hspace{2cm}}.$$

The y-intercept $c = f(0)$
The y-intercept is calculated as

$$\begin{aligned} f(0) &= \underline{\hspace{6cm}} \\ &= \underline{\hspace{6cm}} \end{aligned}$$

Choose the corresponding graph

Graph ___________ is the correct graph.

4.4 Worksheet 4

Worksheet 4 is based on study unit 2.4: *The exponential and logarithmic functions* on pages 94 – 96 of the study guide. Do the activity before you proceed.

Supplementary background

Some background on exponential functions:

Exponential functions look somewhat similar to functions you have seen before, in that they involve exponents, but there is a big difference in that the variable is now the power, rather than the base. Previously, you dealt with such functions as

$$f(x) = x^2,$$

where the variable x was the base and the number 2 was the power. In the case of exponentials, however, you will be dealing with functions such as

$$f(x) = 2^x,$$

where the base is the fixed number, and the power is the variable.

Let's look more closely at the function $f(x) = 2^x$. To evaluate this function, choose a numerical value for x and calculate the function's value at this x.

If x has positive values, for example if $x = 3$, then

$$\begin{aligned} f(3) &= 2^3 \\ &= 2 \times 2 \times 2 \\ &= 8. \end{aligned}$$

If x has negative values, for example if $x = -3$, then

$$\begin{aligned} f(-3) &= 2^{-3} \\ &= \frac{1}{2^3} \\ &= \frac{1}{2 \times 2 \times 2} \\ &= \frac{1}{8}. \end{aligned}$$

The results for positive x-values:

x	$y = 2^x$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
4	$2^4 = 16$
5	$2^5 = 32$

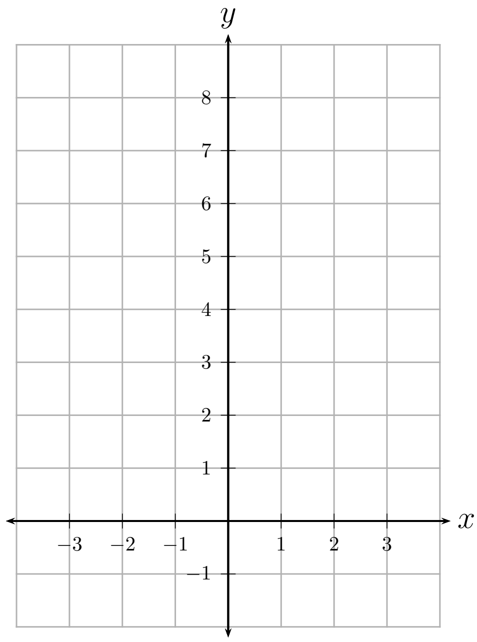
The results for negative x-values:

x	$y = 2^x$
-1	$2^{-1} = \frac{1}{2^1} = \frac{1}{2} = 0{,}5$
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4} = 0{,}25$
-3	$2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0{,}125$
-4	$2^{-4} = \frac{1}{2^4} = \frac{1}{16} \approx 0{,}06$
-5	$2^{-5} = \frac{1}{2^5} = \frac{1}{32} \approx 0{,}03$

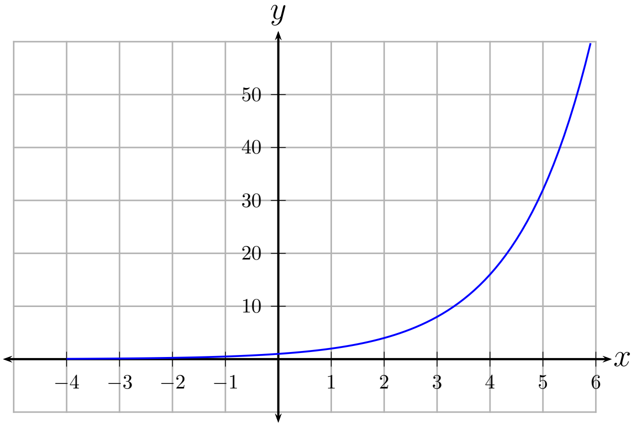
Activity 4.4

Draw a graph of the function $y = 2^x$ for the values of $x = -2, -1, 0, 1, 2$ and 3:

x	$y = 2^x$
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4} = 0{,}25$
-1	$2^{-1} = \frac{1}{2^1} = \frac{1}{2} = 0{,}5$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$



The following is the graph of the function $y = 2^x$ for x-values that range from -4 to 6:



You should expect exponentials to look like this. That is, they start small – very small, so small that they're practically indistinguishable from $y = 0$, which is the x-axis – and then, once they start growing, they grow faster and faster, so fast that they shoot right up through the top of your graph.

You should also expect that your graph will not have many plot points that are easy to plot. For instance, for $x = 6$ and $x = 7$, the y-values are quite large (f(6) = 64 and f(7) = 128), and for just about all the negative x-values (≤ -3), the y-values are too small to see (f(−3) = 0,125 and f(−4) = 0,0625).

You may have heard of the term *exponential growth*. This growth "*starting slow, but then growing faster and faster all the time*" is what they are referring to. Specifically, our function $f(x)$ above doubles each time we increment x. That is, when x increases by one then $y = f(x)$ increases to twice the previous value. This is the definition of exponential growth: there is a consistent fixed period over which the function will double (or triple, or quadruple, etc). The point is that the change is always a fixed proportion. So, if you hear somebody claiming that the world population is doubling every thirty years, you know he/she is claiming exponential growth.

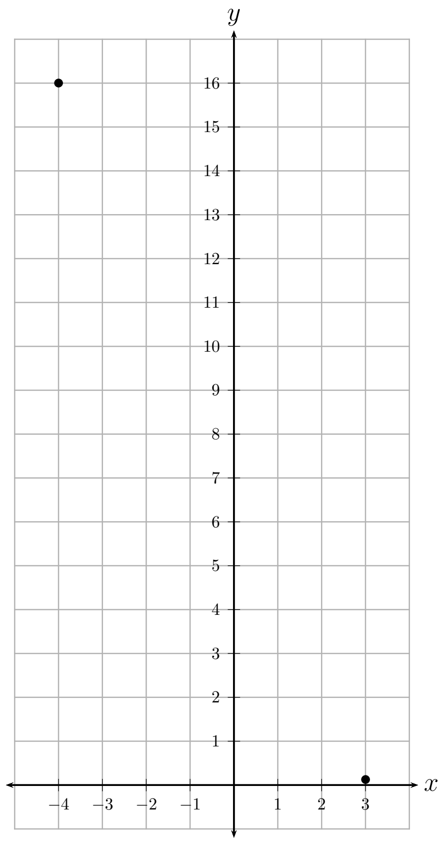
Activity 4.5

1. Consider the exponential function $y = f(x) = \left(\frac{1}{2}\right)^x$.

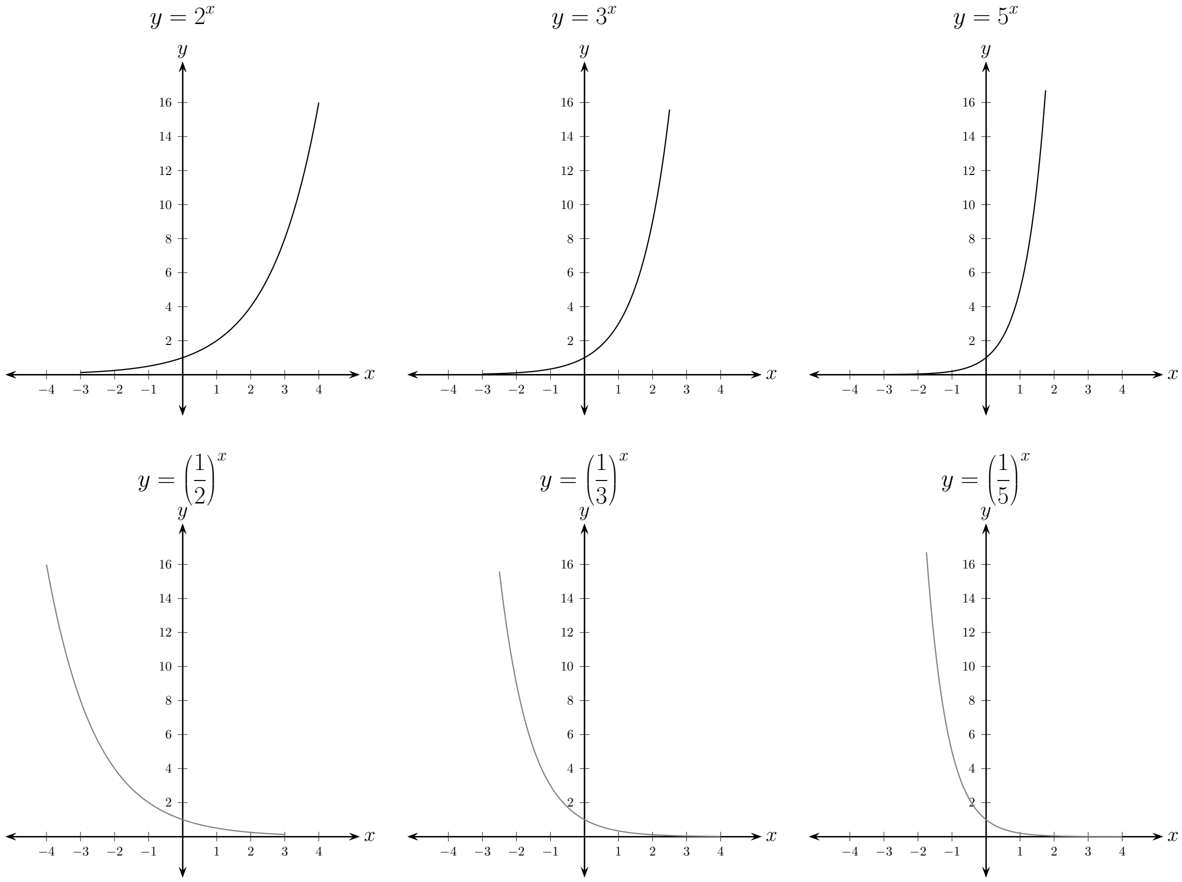
   (a) Complete the following table by calculating the value of $f(x)$ for $x = -4, -3, -2, -1,$ 0, 1, 2 and 3.

x	$y = \left(\frac{1}{2}\right)^x$
-4	$\left(\frac{1}{2}\right)^{-4} = (2^{-1})^{-4} = 2^{-1\times-4} = 2^4 = 16$
-3	$\left(\frac{1}{2}\right)^{-3} = (2^{-1})^{-3} = 2^{-1\times-3} =$
-2	$\left(\frac{1}{2}\right)^{-2} = (2^{-1})^{-2} =$
-1	$\left(\frac{1}{2}\right)^{-1} =$
0	$\left(\frac{1}{2}\right)^{0} =$
1	$\left(\frac{1}{2}\right)^{1} =$
2	$\left(\frac{1}{2}\right)^{2} = \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) =$
3	$\left(\frac{1}{2}\right)^{3} =$

   (b) Draw a graph of the function $y = \left(\frac{1}{2}\right)^x$.



2. Consider the following exponential functions with the form $y = f(x) = b^x$. The variable, b, takes on the values 2, 3, 5, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$.



To test your understanding of the graphs, answer these questions:

(a) What point do all exponent graphs have in common?

____

(b) What do the first three graphs, where $b > 1$, have in common?

____

(c) What do the second three graphs, where $0 < b < 1$, have in common?

____

(d) What is the domain of an exponential function? (The range of the x-values?)

____

(e) What is the range of an exponential function? (The range of the y-values?)

____

Logarithmic functions are the inverse of exponential functions. For example if $(4; 16)$ is a point on the graph of an exponential function, then $(16; 4)$ would be the corresponding point on the graph of the inverse, or logarithmic function.

It is important to understand that A LOG IS ANOTHER WAY TO WRITE AN EXPONENT.

Example 4.3

Evaluate the expression $\log_4 64$ without using a calculator.

Step 1: Set the log equal to x:

$\log_4 64 = x$.

Step 2: Use the definition of log (if $y = a^x$ with $a > 0$ and $a \neq 1$, then $\log_a y = x$) to write the equation in exponential form:

$4^x = 64$.

Step 3: Solve for x:

$4^x = 64$

$4^x = 4 \times 4 \times 4$

$4^x = 4^3$, hence

$x = 3$.

Thus $\log_4 64 = 3$.

Activity 4.6

1. (a) Evaluate the expression $\log_a 1$ without using a calculator.

Step 1: Set the log equal to x:

$\log_a 1 = x$.

Step 2: Use the definition of log to write the equation in exponential form:

$a^x = 1$.

Step 3: Solve for x:

Hint: Any number raised to the power of zero, is equal to one.

$a^x = a^{\square}$, hence

$x =$ ________.

Thus $\log_a 1 =$ ________.

(b) Evaluate the expression $\log_7 7$ without using a calculator.

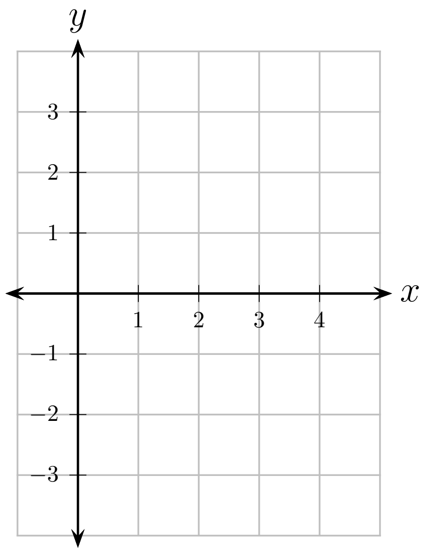
(c) Evaluate the expression $\log_5 \sqrt{5}$ without using a calculator.

(d) Evaluate the expression $\log_6 \frac{1}{36}$ without using a calculator.

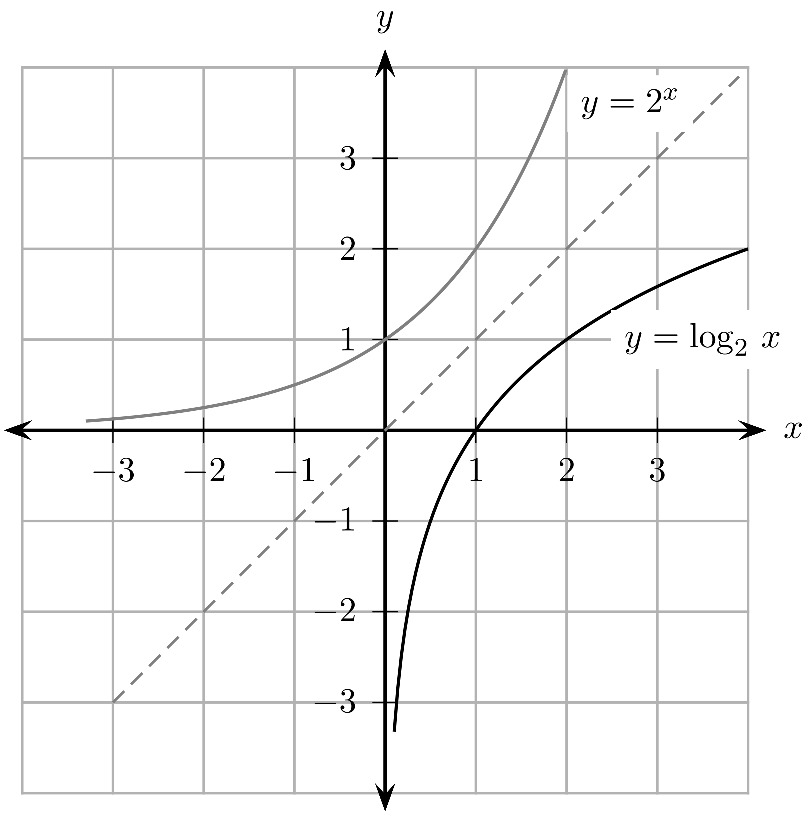
2. (a) Calculate $y = f(x) = \log_2 x$ for $x = \frac{1}{4}, \frac{1}{2}$, 1, 2 and 4:

x	$\log_2 x$	$y = \log_2 x$
$\frac{1}{4}$	$\log_2 \frac{1}{4}$	Because $\frac{1}{4} = 2^{-2}$, it follows from $y = \log_2 \frac{1}{4}$ that $2^y = \frac{1}{4}$ $2^y = 2^{-2}$. Thus, $y = -2$.
$\frac{1}{2}$	$\log_2 \frac{1}{2}$	Because $\frac{1}{2} = 2^{-1}$, it follows from $y = \log_2 \frac{1}{2}$ that $2^y = \frac{1}{2}$ $2^y = 2^{\square}$. Thus, $y = \square$.
1	$\log_2 1$	Because $1 = 2^0$, it follows from $y = \log_2 1$ that $2^y = 1$ $2^y = 2^{\square}$. Thus, $y = \square$.
2	$\log_2 2$	Because $2 = 2^1$, it follows from $y = \log_2 2$ that $2^y = 2$ $2^y = 2^{\square}$. Thus, $y = \square$.
4	$\log_2 4$	Because $4 = 2^2$, it follows from $y = \log_2 4$ that $2^y = 4$ $2^y = 2^{\square}$. Thus, $y = \square$.

(b) Draw the function:



To conclude this component, the two functions $y = f(x) = 2^x$ and $y = \log_2 x$ are represented on the same graph. This illustrates the fact that the exponential function $y = f(x) = 2^x$ and the logarithmic function $y = \log_2 x$ are the inverse of each other:



COMPONENT 5

Linear systems

The following examples and activities will improve your understanding of the basic principles covered in component 3 of the study guide. There is a worksheet for every study unit in component 3. Work through each worksheet after you have studied the relevant study material in the study guide.

5.1 Worksheet 1

Worksheet 1 is based on study unit 3.1: *Linear equations in one variable*, on pages 98 – 102 of the study guide. Do the activities before you proceed.

Example 5.1

Study the following (If you can solve for the unknown variable in fewer steps, it is fine. For the sake of completeness, all the steps are shown):

Solve for the unknown variable:

$$3x + 15 = x + 25$$

$$3x + 15 - x = x + 25 - x$$

Group all the terms containing an x on one side: First, subtract x from both sides.

$$2x + 15 = 25$$

$$2x + 15 - 15 = 25 - 15$$

Second, subtract 15 from both sides.

$$2x = 10$$

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5.$$

Activity 5.1

Solve for the unknown variable:

$$\begin{aligned} -7k + 78 &= 3k - 72 \\ -7k + 78 - 3k &= 3k - 72 - 3k \end{aligned}$$

____________________ = ____________________

____________________ = ____________________

____________________ = ____________________

____________________ = ____________________

Example 5.2

Study the following:

Solve for the unknown variable:

$$\begin{aligned}
6-4(p+3) &= 2(p-1)\\
6-4\times p-4\times 3 &= 2\times p+2\times -1\\
6-4p-12 &= 2p-2\\
-6-4p &= 2p-2\\
-6-4p-2p &= 2p-2-2p\\
-6-6p &= -2\\
-6-6p+6 &= -2+6\\
-6p &= 4\\
\frac{-6p}{-6} &= \frac{4}{-6}\\
p &= -\frac{2}{3}.
\end{aligned}$$

Activity 5.2

Solve for the unknown variable:

$$\begin{aligned}
2x+3 &= 6-(2x-3)\\
2x+3 &= 6-\underline{\qquad\qquad}\\
\underline{\qquad\qquad} &= \underline{\qquad\qquad}\\
\underline{\qquad\qquad} &= \underline{\qquad\qquad}\\
\underline{\qquad\qquad} &= \underline{\qquad\qquad}\\
\underline{\qquad\qquad} &= \underline{\qquad\qquad}\\
\underline{\qquad\qquad} &= \underline{\qquad\qquad}\\
\underline{\qquad\qquad} &= \underline{\qquad\qquad}\\
\underline{\qquad\qquad} &= \underline{\qquad\qquad}\\
\underline{\qquad\qquad} &= \underline{\qquad\qquad}\\
\underline{\qquad\qquad} &= \underline{\qquad\qquad}
\end{aligned}$$

The answer is $x = 1\frac{1}{2}$. *Did you succeed?*

Example 5.3

Study the following:

Solve for the unknown variable:

$$\frac{4(h+2)}{5} = 7 + \frac{5h}{13}$$

$$\frac{65}{1} \times \frac{4(h+2)}{5} = 65 \times 7 + \frac{65}{1} \times \frac{5h}{13}$$

Multiply all the terms by 65 because 65 is the smallest number that is divisible by both 13 and 5 ($5 \times 13 = 65$).

$$\frac{\cancel{65}^{13}}{1} \times \frac{4(h+2)}{\cancel{5}_1} = 65 \times 7 + \frac{\cancel{65}^{5}}{1} \times \frac{5h}{\cancel{13}_1}$$

Simplify the fractions.

$$\begin{aligned}
13 \times 4 \times (h+2) &= 455 + 5 \times 5h \\
52 \times (h+2) &= 455 + 25h \\
52h + 104 &= 455 + 25h \\
52h + 104 - 25h &= 455 + 25h - 25h \\
27h + 104 &= 455 \\
27h + 104 - 104 &= 455 - 104 \\
27h &= 351 \\
\frac{27h}{27} &= \frac{351}{27} \\
h &= 13.
\end{aligned}$$

Activity 5.3

Solve for the unknown variable:

1. $\dfrac{4-5x}{6} - \dfrac{1-2x}{3} = \dfrac{13}{42}$

$$\frac{42}{1} \times \frac{4-5x}{6} - \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} \times \frac{\square}{\square}$$

Multiply all the terms by 42 because 42 is the smallest number that is divisible by 42, 6 and 3 ($42 = 7 \times 6 = 7 \times 3 \times 2$).

Simplify the fractions.

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The answer is $x = \frac{1}{7}$. *Did you succeed?*

2. $\frac{3}{4}d - 2 = \frac{1}{3}d + 3$

The answer is $d = 12$. *Did you succeed?*

Example 5.4

Study the following:

Solve for the unknown variable:

$$\frac{5}{3q} = \frac{25}{27}$$

$$\frac{27q}{1} \times \frac{5}{3q} = \frac{27q}{1} \times \frac{25}{27}$$

Multiply both sides by $27q$ because $27q$ is the smallest number that is divisible by both $3q$ and 27.

$$\frac{{}^{9}\cancel{27q}}{1} \times \frac{5}{{}_{1}\cancel{3q}} = \frac{{}^{1}\cancel{27}q}{1} \times \frac{25}{{}_{1}\cancel{27}}$$

Simplify the fractions.

$$\begin{aligned} 9 \times 5 &= q \times 25 \\ 45 &= 25q \\ 25q &= 45 \\ \frac{25q}{25} &= \frac{45}{25} \\ q &= \frac{9}{5} \\ q &= 1\frac{4}{5}. \end{aligned}$$

Activity 5.4

Solve for the unknown variable:

$\frac{8}{x} = 2$

Multiply both sides by x.

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Example 5.5

On page 101 − 102 of the study guide a different method for solving linear equations is discussed. First you have to manipulate the given equation until it is in the form

$$ax + b = 0.$$

Then

$$ax = -b$$

resulting in

$$x = -\frac{b}{a}$$

which is the solution.

Take example 5.1 of worksheet 1 $(3x + 15 = x + 25)$ and solve for x by rearranging it into the form $ax + b = 0$. This gives

$$\begin{aligned} 3x + 15 &= x + 25 \\ 3x + 15 - x - 25 &= x + 25 - x - 25 \\ 2x - 10 &= 0. \end{aligned}$$

Thus, a is 2 and b is -10 in the equation $ax + b = 0$.
The value of x is calculated as

$$\begin{aligned} x &= -\frac{b}{a} \\ &= \frac{-(-10)}{2} \\ &= 5. \end{aligned}$$

Activity 5.5

Take the equations from activities 5.2 and 5.3.1 and solve for x by rearranging it into the form $ax + b = 0$.

1. Solve for x:

   $2x + 3 = 6 - (2x - 3)$

   ____

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2. Solve for x:

   $\dfrac{4 - 5x}{6} - \dfrac{1 - 2x}{3} = \dfrac{13}{42}$

   ____

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Example 5.6

Linear equations can also be formed by converting given **word statements** to a linear equation. Consider the following example:

The **first** number is three less than two times the **second** number. If their sum is increased by seven, the result is 37. Find the numbers.

Let x be the **second** number.

"*The **first** number is three less than two times the **second** number.*"

Thus, the **first** number is three less than two times x, giving the **first** number as $2x - 3$.

The sum of the **first** and **second** number is

$$\begin{aligned}(2x-3)+x &= 2x-3+x \\ &= 3x-3.\end{aligned}$$

"*If their sum is increased by seven, the result is 37*", giving

$$\begin{aligned}(3x-3)+7 &= 37 \\ 3x-3+7 &= 37 \\ 3x+4 &= 37 \\ 3x+4-4 &= 37-4 \\ 3x &= 33 \\ \frac{3x}{3} &= \frac{33}{3} \\ x &= 11.\end{aligned}$$

Which is the **second** number.

The **first** number is

$$\begin{aligned}2x-3 &= 2\times 11-3 \\ &= 22-3 \\ &= 19.\end{aligned}$$

The numbers are 19 and 11.

This can be checked as follows:

It is true that $19 = 2(11) - 3$,

The first requirement, *"the first number is three less than two times the second"*, is satisfied.

and

it is true that $19 + 11 + 7 = 37$.

The second requirement, *"if their sum is increased by seven, the result is 37"*, is satisfied.

Activity 5.6

Rewrite the following word statements in the form of linear equations and solve for the unknown variable.

1. If one half and one third of a number are added to the number, the result is 44. Find the number.

   Suppose the number is x. Now add one half and one third of x to x and solve for x :

   $\frac{1}{2} \times x + \frac{\quad}{\quad} \times$ ________ + ________ $= 44$

   ____

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2. A person travels $\frac{5}{8}$ of the distance by train, $\frac{1}{4}$ by bus and the remaining 15 kilometers by boat. Find the total distance that he travels.

   Start by setting the total distance the person travels equal to x and solve for x :

   ____

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5.2 Worksheet 2

Worksheet 2 is based on study unit 3.2: *Simultaneous linear equations in two variables*, on pages 103 – 105 of the study guide. Do the activity and exercise before you proceed.

Example 5.7

Solve the following system of equations:

$$\begin{aligned} 5x + 2y &= 13 \\ x + 2y &= 9. \end{aligned}$$

Four methods will be demonstrated on how this system of equations can be solved. Methods 1, 2 and 4 correspond to the methods discussed in the study material.

1. Make y the subject of both equations and set the ys equal to each other.
2. Make x the subject of the first equation and substitute it into the second equation.
3. Use the method of elimination.
4. Use the graphic method.

Method 1: Make y the subject of both equations and set the ys equal to each other.

Equation 1: $5x + 2y = 13$.
Make y the subject of the first equation:

$$\begin{aligned} 5x + 2y - 5x &= 13 - 5x \\ 2y &= 13 - 5x \\ \frac{2y}{2} &= \frac{13}{2} - \frac{5}{2}x \\ y &= \frac{13}{2} - \frac{5}{2}x. \end{aligned}$$

Equation 2: $x + 2y = 9$.
Make y the subject of the second equation:

$$\begin{aligned} x + 2y - x &= 9 - x \\ 2y &= 9 - x \\ \frac{2y}{2} &= \frac{9}{2} - \frac{x}{2} \\ y &= \frac{9}{2} - \frac{x}{2}. \end{aligned}$$

To solve for x, set the two ys equal to each other:

$$\begin{aligned} \frac{13}{2} - \frac{5}{2}x &= \frac{9}{2} - \frac{x}{2} \\ \frac{2}{1} \times \frac{13}{2} - \frac{2}{1} \times \frac{5}{2}x &= \frac{2}{1} \times \frac{9}{2} - \frac{2}{1} \times \frac{x}{2} \\ 13 - 5x &= 9 - x \\ 13 - 5x + x &= 9 - x + x \\ 13 - 4x &= 9 \\ 13 - 4x - 13 &= 9 - 13 \\ -4x &= -4 \\ x &= 1. \end{aligned}$$

Divide both sides by -4.

Substitute the value of $x = 1$ into equation 1:

$$\begin{aligned} y &= \frac{13}{2} - \frac{5}{2}x \\ &= \frac{13}{2} - \frac{5}{2} \times \frac{1}{1} \\ &= \frac{13}{2} - \frac{5}{2} \\ &= \frac{8}{2} \\ &= 4. \end{aligned}$$

Substitute the value of $x = 1$ into equation 2:

$$\begin{aligned} y &= \frac{9}{2} - \frac{x}{2} \\ &= \frac{9}{2} - \frac{1}{2} \\ &= \frac{8}{2} \\ &= 4. \end{aligned}$$

To obtain the value of y, you need only to substitute the value of x in one of the equations. It was done for both the equations to illustrate that y will only have one solution.

The solution is $x = 1$ and $y = 4$. It can also be written as $(1; 4)$.

Method 2: Make x the subject of the first equation and substitute it into the second equation.

Equation 1: $5x + 2y = 13$.
Make x the subject of the **first** equation:

$$\begin{aligned} 5x + 2y &= 13 \\ 5x + 2y - 2y &= 13 - 2y \\ 5x &= 13 - 2y \\ \frac{5x}{5} &= \frac{13}{5} - \frac{2}{5}y \\ x &= \frac{13}{5} - \frac{2}{5}y. \end{aligned}$$

Equation 2: $x + 2y = 9$.
To solve for y, substitute x into the **second** equation:

$$\begin{aligned} x + 2y &= 9 \\ \left(\frac{13}{5} - \frac{2}{5}y\right) + 2y &= 9 \\ \frac{5}{1} \times \left(\frac{13}{5} - \frac{2}{5}y\right) + 5 \times 2y &= 5 \times 9 \\ 13 - 2y + 10y &= 45 \\ 13 + 8y &= 45 \\ 8y &= 45 - 13 \\ 8y &= 32 \\ \frac{8y}{8} &= \frac{32}{8} \\ y &= 4. \end{aligned}$$

To solve for x, substitute $y = 4$ into $x = \frac{13}{5} - \frac{2}{5}y$:

$$\begin{aligned} x &= \frac{13}{5} - \frac{2}{5}y \\ &= \frac{13}{5} - \frac{2}{5} \times \frac{4}{1} \\ &= \frac{13}{5} - \frac{8}{5} \\ &= \frac{5}{5} \\ &= 1. \end{aligned}$$

The solution is $x = 1$ and $y = 4$, or $(1; 4)$.

Alternatively you could have switched the equations:

Make x the subject of the **second** equation:

$$\begin{aligned} x + 2y &= 9 \\ x + 2y - 2y &= 9 - 2y \\ x &= 9 - 2y. \end{aligned}$$

To solve for y, substitute x into the **first** equation:

$$\begin{aligned} 5x + 2y &= 13 \\ 5 \times (9 - 2y) + 2y &= 13 \\ 45 - 10y + 2y &= 13 \\ 45 - 8y &= 13 \\ 45 - 8y - 45 &= 13 - 45 \\ -8y &= -32 \\ \frac{-8y}{-8} &= \frac{-32}{-8} \\ y &= 4. \end{aligned}$$

To solve for x, substitute $y = 4$ into $x = 9 - 2y$:

$$\begin{aligned} x &= 9 - 2y \\ &= 9 - 2 \times 4 \\ &= 9 - 8 \\ &= 1. \end{aligned}$$

The solution is $x = 1$ and $y = 4$, or $(1; 4)$.

Method 3: Two equations are simplified by adding them or subtracting them. This eliminates one of the variables so that the other variable can be found.

The equations are

$$5x + 2y = 13 \qquad (1)$$

and

$$x + 2y = 9. \qquad (2)$$

Subtract (2) from (1):

$$\begin{array}{rcrl} 5x + 2y & = & 13 & \quad (1) \\ -x - 2y & = & -9 & \quad -(2) \\ \hline 4x & = & 4 & \end{array}$$

giving

$$x = 1.$$

Substitute $x = 1$ into (1) to get

$$\begin{aligned} 5x + 2y &= 13 \\ 5 \times 1 + 2y &= 13 \\ 5 + 2y &= 13 \\ 5 + 2y - 5 &= 13 - 5 \\ 2y &= 8 \\ y &= 4. \end{aligned}$$

The solution is $x = 1$ and $y = 4$, or $(1; 4)$.

Method 4: Graphic method

Equation 1:

Write the equation in the form $y = ax + b$.

$$\begin{aligned} 5x + 2y &= 13 \\ 2y &= -5x + 13 \\ y &= -2\frac{1}{2}x + 6\frac{1}{2}. \end{aligned}$$

Determine the ***y*-intercept**. If $x = 0$, then

$$\begin{aligned} y &= -2\frac{1}{2} \times 0 + 6\frac{1}{2} \\ &= 6\frac{1}{2}. \end{aligned}$$

The point is $\left(0; 6\frac{1}{2}\right)$. Call this point A.

Determine the ***x*-intercept**. If $y = 0$, then

$$\begin{aligned} 0 &= -2\frac{1}{2}x + 6\frac{1}{2} \\ 2\frac{1}{2}x &= 6\frac{1}{2} \\ \frac{2}{1} \times \frac{5}{2}x &= \frac{2}{1} \times \frac{13}{2} \\ 5x &= 13 \\ x &= \frac{13}{5} \\ &= 2\frac{3}{5}. \end{aligned}$$

The point is $\left(2\frac{3}{5}; 0\right)$. Call this point B.

Equation 2:

$$\begin{aligned} x + 2y &= 9 \\ 2y &= -x + 9 \\ y &= -\frac{1}{2}x + 4\frac{1}{2}. \end{aligned}$$

Write the equation in the form $y = ax + b$.

Determine the ***y*-intercept**. If $x = 0$, then

$$\begin{aligned} y &= -\frac{1}{2} \times 0 + 4\frac{1}{2} \\ &= 4\frac{1}{2}. \end{aligned}$$

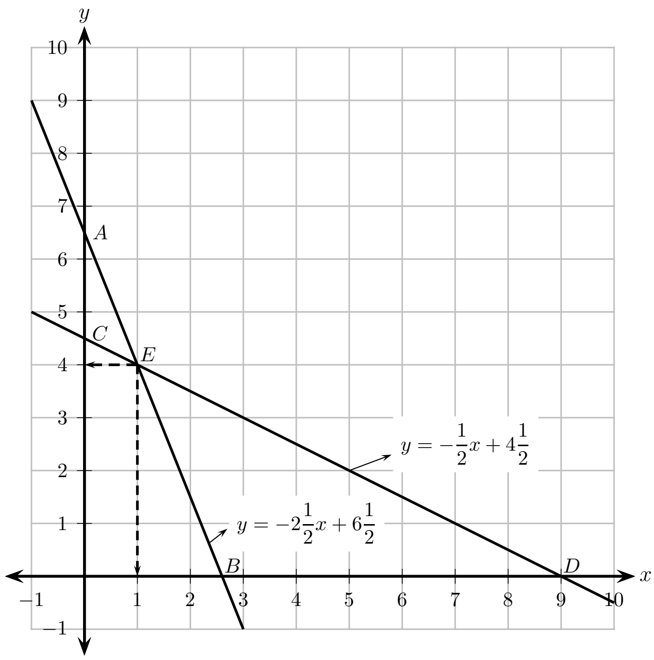
The point is $\left(0;\ 4\frac{1}{2}\right)$. Call this point C.

Determine the ***x*-intercept**. If $y = 0$, then

$$\begin{aligned} 0 &= -\frac{1}{2}x + 4\frac{1}{2} \\ \frac{1}{2}x &= 4\frac{1}{2} \\ \frac{2}{1} \times \frac{1}{2}x &= \frac{2}{1} \times \frac{9}{2} \\ x &= 9. \end{aligned}$$

The point is (9; 0). Call this point D.

The solution is given in the following graph where the two lines intersect. That is at point E with coordinates $(1; 4)$.



Activity 5.7

1. Use method 1 to solve the following system of equations:

$$\begin{aligned} 3x - 2y &= 1 \\ 2x - y &= 1. \end{aligned}$$

(Make y the subject of both equations and set the ys equal to each other.)

Equation 1: $3x - 2y = 1$.
Make y the subject of the first equation:

$$\begin{aligned} 3x - 2y - 3x &= 1 - 3x \\ \_\_\_\_\_\_\_\_ &= \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ &= \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ &= \_\_\_\_\_\_\_\_ \end{aligned}$$

Equation 2: $2x - y = 1$.
Make y the subject of the second equation:

$$\begin{aligned} 2x - y - 2x &= 1 - 2x \\ \_\_\_\_\_\_\_\_ &= \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ &= \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ &= \_\_\_\_\_\_\_\_ \end{aligned}$$

To solve for x, set the two y's equal to each other:

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The answer is $x = 1$. Did you succeed?

Substitute the value of $x = 1$ into equation 1:

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The solution is $x =$ _____ and $y =$ _____. It can also be written as (_____;_____).

2. Use method 2 to solve the following system of equations:

$$\begin{aligned} 2x + 3y &= 7 \\ x - y &= 2. \end{aligned}$$

(Make x the subject of the first equation and substitute it into the second equation.)

Equation 1: $2x + 3y = 7$.
Make x the subject of the **first** equation:

$$\begin{aligned} 2x + 3y &= 7 \\ 2x + 3y - 3y &= 7 - 3y \\ \_\_\_\_\_\_\_\_ &= \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ &= \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ &= \_\_\_\_\_\_\_\_ \end{aligned}$$

Equation 2: $x - y = 2$.
To solve for y, substitute x into the **second** equation:

$x - y = 2$

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To solve for x, substitute $y =$ _____ into $x =$ _______________:

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The solution is $x = 2\frac{3}{5}$ *and* $y = \frac{3}{5}$, *or* $\left(2\frac{3}{5}; \frac{3}{5}\right)$. *Did you succeed?*

3. The elimination method will only work if you can eliminate one of the variables by adding or subtracting the equations as in example 5.7, method 3. But for many simultaneous equations, this is not the case.

   For example, use Method 3 (elimination method) to solve the following system of equations:

$$\begin{aligned} 2x + 3y &= 4 \qquad (1) \\ x - 2y &= -5. \qquad (2) \end{aligned}$$

   Adding or subtracting these equations will not cancel out the x- or the y-terms.

   Before using the elimination method you may have to multiply every term of one, or both of the equations, by some number so that equal terms can be eliminated.

   If equation (2) had a $2x$ instead of an x, we could eliminate it. By multiplying every term in equation (2) by 2, the x will become $2x$:

$$2 \times x - 2 \times 2y = 2 \times -5,$$

   to obtain equation (3):

$$2x - 4y = -10. \qquad (3)$$

   Subtract (3) from (1):

$$\begin{aligned} 2x + 3y &= 4 \qquad (1) \\ \underline{-2x + 4y} &= \underline{10} \qquad -(3) \\ \_\_\_\_\_ &= \_\_\_\_ \end{aligned}$$

   giving

$$\_\_\_\_\_ = \_\_\_\_.$$

   Substitute $y =$ _____ into (1):

   $2x + 3y = 4$

   ____

   ____

   ____

   ____

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   ____

   The solution is $x =$ _____ and $y =$ _____. It can also be written as (_____;_____).

   Sometimes both equations must be modified in order to eliminate a variable. For example, to eliminate the y-terms for this example, we could multiply the first equation by two, and the second equation by three. Then there would be a $6y$ in the first equation and a $-6y$ in the second equation. Adding the equations would then eliminate the y-term.

4. Use method 4 (graphic method) to solve the following system of equations:

$$\begin{aligned} x + 2y &= -5 \\ 3x - 2y &= -3. \end{aligned}$$

Equation 1 rewritten in the form $y = ax + b$ is

$$\begin{aligned} x + 2y &= -5 \\ 2y &= \underline{\hspace{6cm}} \\ y &= \underline{\hspace{6cm}} \end{aligned}$$

Determine the ***y*-intercept**. If $x = 0$, then

$$\begin{aligned} y &= \underline{\hspace{6cm}} \\ &= \underline{\hspace{6cm}} \end{aligned}$$

The point is (0; ______). Call this point A.

Determine the ***x*-intercept**. If $y = 0$, then

$$0 = \underline{\hspace{6cm}}$$

$$\underline{\hspace{7cm}}$$

$$\underline{\hspace{7cm}}$$

$$x = \underline{\hspace{6cm}}$$

The point is (______; 0). Call this point B.

Equation 2 rewritten in the form $y = ax + b$ is

$$\begin{aligned} 3x - 2y &= -3 \\ -2y &= \underline{\hspace{6cm}} \\ y &= \underline{\hspace{6cm}} \end{aligned}$$

Determine the ***y*-intercept**. If $x = 0$, then

$$\begin{aligned} y &= \underline{\hspace{6cm}} \\ &= \underline{\hspace{6cm}} \end{aligned}$$

The point is (0; ______). Call this point C.

Determine the ***x*-intercept**. If $y = 0$, then

$$0 = \underline{\hspace{6cm}}$$

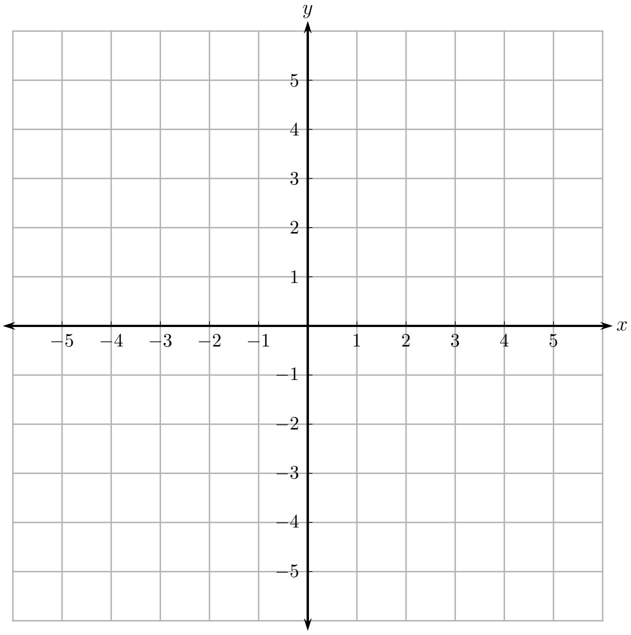
$$\underline{\hspace{7cm}}$$

$$\underline{\hspace{7cm}}$$

$$x = \underline{\hspace{6cm}}$$

The point is (______; 0). Call this point D.

The solution is given in the following graph where the two lines intersect. That is at the point with coordinates (_____;_____).



Example 5.8

There are many situations or problems in the business environment where the values of some unknown quantities have to be found. We can represent the unknown quantities by the names of two variables and form two linear equations involving the variables. Let's look at an example:

A company publishes two magazines, *Fresh Food* and *Big Bite*. It costs R4 to print a copy of *Fresh Food* and R6 to bind it. It costs R5 to print a copy of *Big Bite* and R3 to bind it. The budget of the company only allows R21 000 for binding and R20 000 for printing.

Set up two linear equations that will determine how many copies of each magazine can be published. Note: Set up the two linear equations without solving them.

Let the number of copies of *Fresh Food* magazines published be x.
Let the number of copies of *Big Bite* magazines published be y.

The budget of the company only allows R20 000 for printing and it costs R4 to print a copy of *Fresh Food* and R5 to print a copy of *Big Bite*.

Thus, for printing

$$4x + 5y = 20\,000.$$

Name this equation 1.
The budget of the company only allows R21 000 for binding and it costs R6 to bind a copy of *Fresh Food* and R3 to bind a copy of *Big Bite*.

Thus, for binding

$$6x + 3y = 21\,000.$$

Name this equation 2.

Activity 5.8

1. The price of a pen is R11 more than the price of a pencil. The price of a pen is P and the price of a pencil is R. If you add R3 to the price of the pen and R3 to the price of a pencil, the price of the pen will be twice the price of the pencil. Without solving, write down two linear equations that will determine the values of P and R.

   If the price of a pen is R11 more than the price of a pencil, then

   $$\begin{aligned} \text{price of pencil} + \text{R11} &= \text{price of pen} \\ \_\_\_\_\_\_\_\_ + 11 &= \_\_\_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_\_\_ &= \_\_\_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_\_\_ &= \_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

   Add R3 to the price of a pen and R3 to the price of a pencil, to obtain the price of a pen equal to $P+3$ and the price of a pencil equal to $R+3$. But the price of a pen will be double the price of a pencil, or the price of two pencils will be equal to the price of one pen. Thus the mathematical expression is

   $$\begin{aligned} \_\_\_\_\_\_\_\_\_\_ &= 2 \times (\_\_\_\_\_\_\_\_) \\ \_\_\_\_\_\_\_\_\_\_ &= \_\_\_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_\_\_ &= \_\_\_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_\_\_ &= \_\_\_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_\_\_ &= \_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

2. James buys 6 kg of potatoes and 3 kg of apples for R60. Sam buys 5 kg of potatoes and 2 kg of apples for R45. Without solving, write down two linear equations that will determine the prices of potatoes and apples per kilogram.

   Let p be the price of 1 kg potatoes.
   Let a be the price of 1 kg apples.

   The two equations are

   ____

   and

   ____

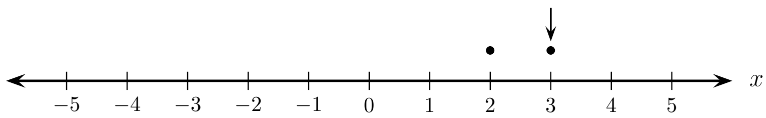
5.3 Worksheet 3

Worksheet 3 is based on study unit 3.3: *Linear inequalities in one variable*, on pages 106 – 109 of the study guide. Do the exercise before you proceed.

Supplementary background

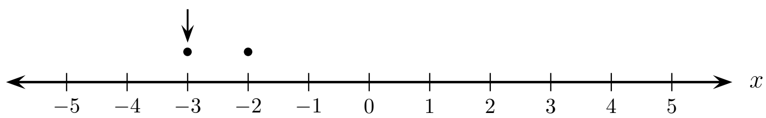
Why does the inequality sign reverse when we divide or multiply both sides of an inequality by the same negative number?

Consider the inequality $x > 2$. Take any number that is greater than 2, for example 3. Then it is true that $3 > 2$. On the number line the positions of 2 and 3 are as follows:



If we multiply both sides of the inequality $x > 2$ by the same negative number, say -1, we obtain $(-x)$ and (-2). We want to know whether $(-x)$ is greater than or smaller than (-2).

Choose $x = +3$.
Then $-x = -3$.
Plot -3 and -2 on the number line:



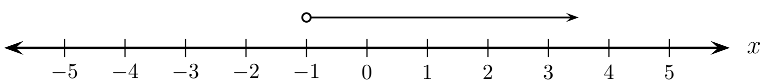
Note that -3 is smaller than -2.
Hence $-3 < -2$.

From this we can conclude that if $x > 2$ then $-x < -2$.

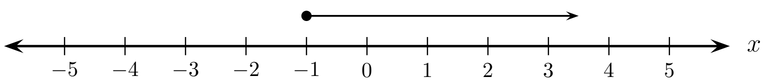
Example 5.9

1. Look at the following graphical representations of inequalities:

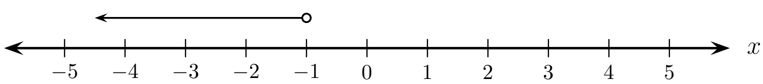
$x > -1$



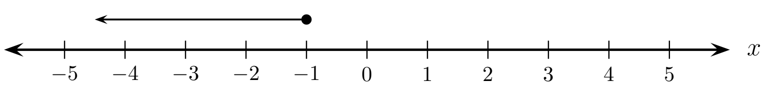
$x \geq -1$



$x < -1$



$x \leq -1$



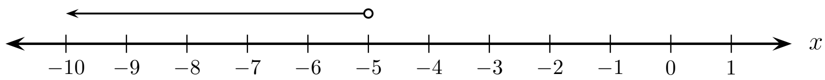
2. Solve the inequality

$$6x + 7 < 4x - 3.$$

To obtain x on the left-hand side, the inequality has to be rewritten as

$$\begin{aligned} 6x + 7 &< 4x - 3 \\ 6x - 4x + 7 &< -3 \\ 2x &< -3 - 7 \\ 2x &< -10 \\ \frac{2x}{2} &< \frac{-10}{2} \\ x &< -5. \end{aligned}$$

It is represented on the number line as follows:



Activity 5.9

1. Solve the inequality $3x \geq 11x + 4$ and represent it on the number line.

To obtain x on the left-hand side, the inequality has to be rewritten as

$3x - 11x \geq 4$

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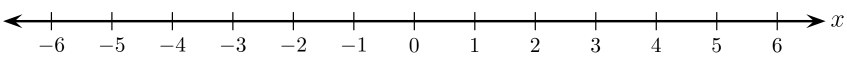
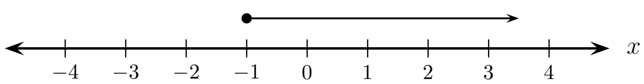
Represent the inequality on the number line:

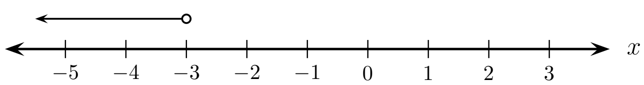


2. Match the inequalities with their graphical representations:

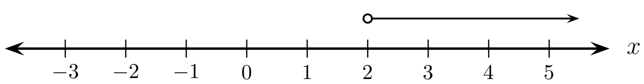
(A)



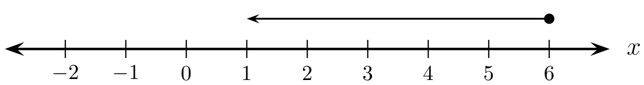
(B)



(C)



(D)



(a) $2x - 6 > 2(1 - x)$

(b) $\dfrac{x}{4} > 16$

(c) $\dfrac{x+3}{2} \leq 2x + 3$

(d) $-4x > 12$

(e) $2(x + 5) - 1 \leq x + 5$

(f) $2(7x - 3) \leq 12x + 6$

Start with the inequalities. Rewrite the inequalities to obtain x on the left-hand side.

(a) The inequality is rewritten as

$$\begin{array}{rcl} 2x-6 & > & 2(1-x) \\ 2x-6 & > & 2-2x \\ \underline{\qquad\qquad} & > & \underline{\qquad\qquad} \\ \underline{\qquad\qquad} & > & \underline{\qquad\qquad} \\ \underline{\qquad\qquad} & > & \underline{\qquad\qquad} \\ \underline{\qquad\qquad} & > & \underline{\qquad\qquad} \\ \dfrac{\boxed{}x}{4} & > & \dfrac{\boxed{}}{4} \\ x & > & \underline{\qquad\qquad} \end{array}$$

This is represented by graph __________.

(b) The inequality is rewritten as

$$\begin{array}{rcl} \dfrac{x}{4} & > & 16 \\ \dfrac{\boxed{}}{1}\times\dfrac{x}{4} & > & \underline{\qquad}\times 16 \\ x & > & \underline{\qquad\qquad} \end{array}$$

This is represented by graph __________.

(c) The inequality is rewritten as

$$\begin{array}{rcl} \dfrac{x+3}{2} & \leq & 2x+3 \\ \dfrac{2}{1}\times\left(\dfrac{\boxed{}}{\boxed{}}\right) & \leq & 2\times\left(\boxed{}\right) \end{array}$$

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This is represented by graph __________.

(d) The inequality is rewritten as

$-4x > 12$

____

____

This is represented by graph __________.

(e) The inequality is rewritten as

$$\begin{aligned} 2(x+5)-1 &\leq x+5 \\ 2x + \_\_\_\_\_ - \_\_\_\_\_ &\leq \_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

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This is represented by graph __________.

(f) The inequality is rewritten as

$2(7x-3) \leq 12x+6$

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This is represented by graph __________.

5.4 Worksheet 4

Worksheet 4 is based on study unit 3.4: *Systems of linear inequalities in two variables* on pages 110 – 115 of the study guide. Do the activities and exercises before you proceed.

Example 5.10

Graph the linear inequality

$$-2x + y \leq 3.$$

Firstly, rewrite the inequality to obtain y on the left-hand side, that is

$$y \leq 2x + 3.$$

Secondly, graph the inequality and find the solution region.

To get the intercepts on the two axes, replace the inequality sign temporarily by =.

The y-intercept is obtained when $x = 0$, thus

$$y = 2 \times 0 + 3 = 3.$$

The point is $(0; 3)$.
The x-intercept is obtained when $y = 0$, thus

$$\begin{aligned} 0 &= 2x + 3 \\ 2x &= -3 \\ x &= -1\frac{1}{2}. \end{aligned}$$

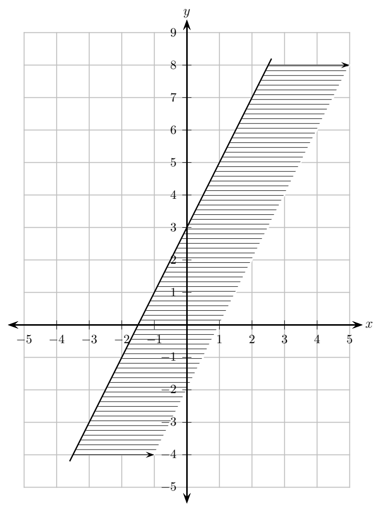
The point is $\left(-1\frac{1}{2}; 0\right)$.
The = is part of the inequality, therefore the line is solid.

Select the point $(0; 0)$ that is not on the line. Substitute it into the inequality to see if it satisfies the inequality, giving

$$0 \leq 2 \times 0 + 3 \quad \text{or} \quad 0 \leq 3,$$

which is true.

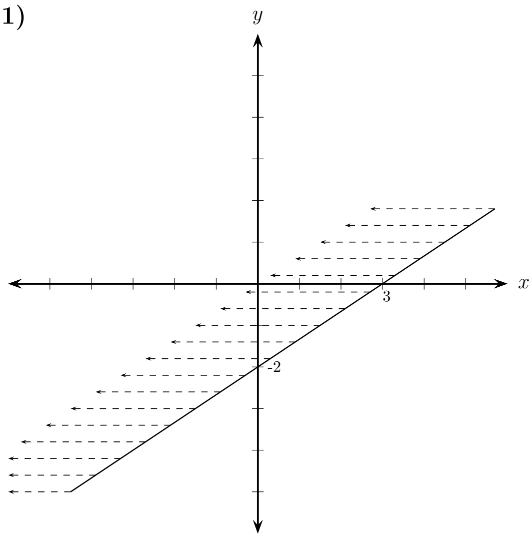
Therefore all the points on the same side of the line as the point $(0; 0)$ satisfy the inequality. Colour or use lines to indicate the region of all $(x; y)$ points that satisfy the inequality.



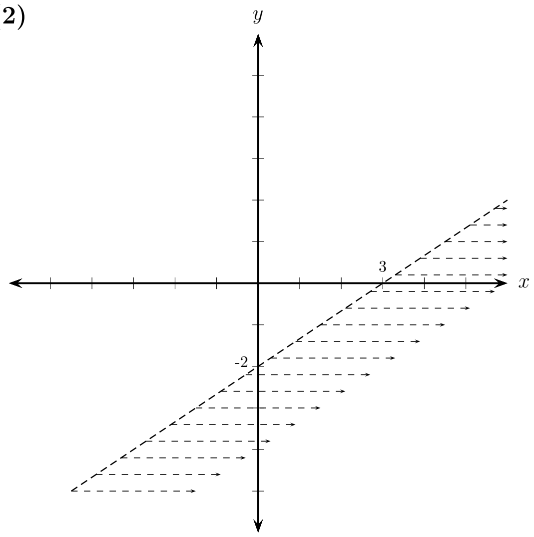
Activity 5.10

Consider the inequality $2x - 3y < 6$. Choose the correct graph that represents this inequality.

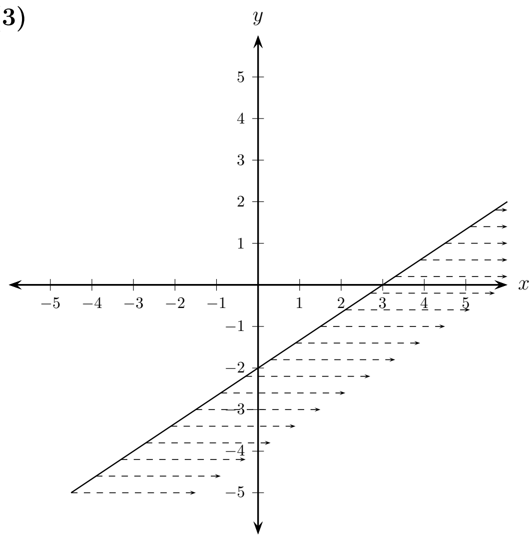
(1)



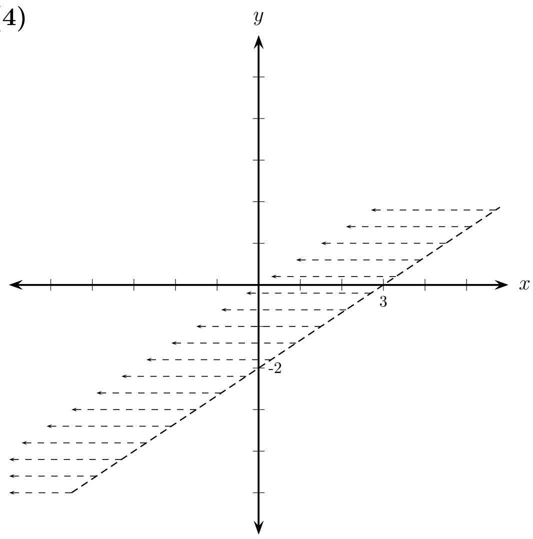
(2)



(3)



(4)



Firstly, rewrite the inequality to obtain y on the left-hand side:
$2x - 3y < 6$

____

____

____

____

Secondly, graph the inequality and find the solution region.
To get the intercepts on the two axes, replace the inequality sign temporarily by =.

The y-intercept is obtained when $x = 0$, thus

$$y = \underline{\hspace{4cm}} = \underline{\hspace{1.5cm}}.$$

The point is (_____;_____).

The x-intercept is obtained when $y = 0$, thus

$$\begin{aligned} 0 &= \underline{\hspace{3cm}} \\ \underline{\hspace{3cm}} &= \underline{\hspace{3cm}} \\ x &= \underline{\hspace{3cm}}. \end{aligned}$$

The point is (_____;_____).

Select the point $(0; 0)$ that is not on the line. Substitute it into the inequality to see if it satisfies the inequality:

____

Thirdly, decide on the solution region.

____

Example 5.11

Solve the following system of inequalities graphically:

$$\begin{aligned} 2x - y &> -3 \quad (1) \\ 4x + y &< 5. \quad (2) \end{aligned}$$

Firstly, rewrite the inequalities to obtain y on the left-hand side.
For inequality (1) it is

$$\begin{aligned} 2x - y &> -3 \\ -y &> -2x - 3 \\ y &< 2x + 3. \end{aligned}$$

The sign changed from > to < because we divide by a negative number.

For inequality (2) it is

$$\begin{aligned} 4x + y &< 5 \\ y &< -4x + 5. \end{aligned}$$

Secondly, graph each inequality and find the overlapping part of the solution regions.

Complete the table below that will help you to draw the two lines.

For the last column in the table, substitute the point $(0; 0)$ into both the inequalities. For example, consider the first inequality

$$y < 2x + 3.$$

The point $(0; 0)$ lies to the right of the line. Substituting gives

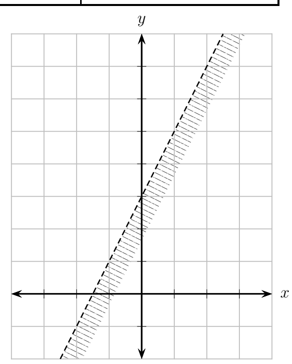
$$\begin{aligned} 0 &< 2 \times 0 + 3 \\ 0 &< 3. \end{aligned}$$

This is true because $0 < 3$. Thus, all the points to the right of the line satisfy the inequality. Write "True" in the last column of the table.

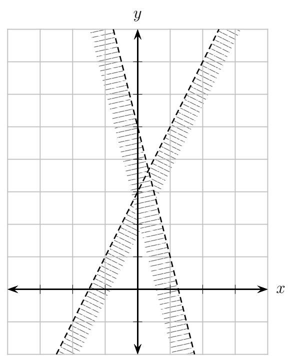
The completed table is given below.

Inequality	**x-intercept**	**y-intercept**	**Line type**	**Point $(0; 0)$**
$y < 2x + 3$	$\left(-1\frac{1}{2}; 0\right)$	$(0; 3)$	dashed	True
$y < -4x + 5$	$\left(1\frac{1}{4}; 0\right)$	$(0; 5)$	dashed	True

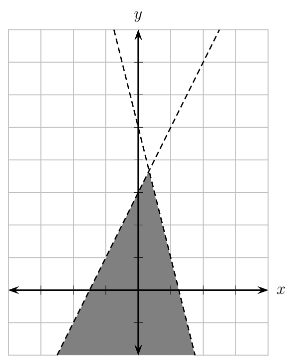
Consider inequality (1): $y < 2x + 3$.
It is a dashed line because of the $<$ sign. The region of all the $(x; y)$ points that satisfy the inequality is indicated by the short lines in the graph.



Consider inequality (2): $y < -4x + 5$.
It is a dashed line because of the $<$ sign. The region of all the $(x; y)$ points that satisfy the inequality is indicated by the short lines in the graph.



Thirdly, find the solution region.
The solution region is the region where the two individual solution regions overlap.
In this case, the solution is the shaded part in the graph.



Note that the solution region is an *unbounded* solution because it continues forever in at least one direction (in this case, forever downward).

Activity 5.11

Solve the following system of inequalities graphically:

$$\begin{aligned} x - y &\leq -2 \quad (1) \\ x - y &\geq 2. \quad (2) \end{aligned}$$

Firstly, rewrite the inequalities to obtain y on the left-hand side.
For inequality (1) it is

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____

____

For inequality (2) it is

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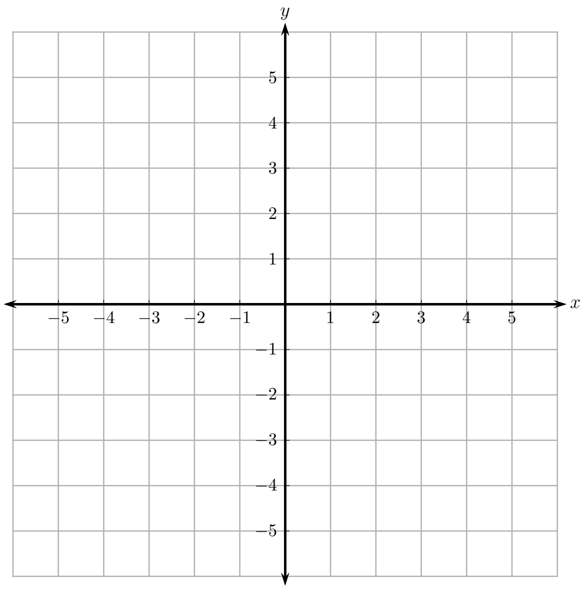
____

____

Secondly, graph each inequality using the table to assist you.

Inequality	**x-intercept**	**y-intercept**	**Line type**	**Point $(0; 0)$**
$x - y \leq -2$				
$x - y \geq 2$				

Draw the inequalities on the graph below.



Thirdly, find the solution region.

____

Example 5.12

Solve the following system of inequalities graphically:

$$\begin{aligned} 2x - 3y &\leq 12 \quad (1) \\ x + 5y &\leq 20 \quad (2) \\ x &> 0. \quad (3) \end{aligned}$$

Firstly, rewrite the inequalities to obtain y on the left-hand side.
For inequality (1) it is

$$\begin{aligned} 2x - 3y &\leq 12 \\ -3y &\leq -2x + 12 \\ y &\geq \frac{2}{3}x - 4. \end{aligned}$$

For inequality (2) it is

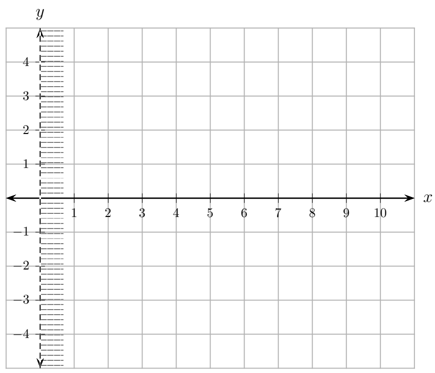
$$\begin{aligned} x + 5y &\leq 20 \\ 5y &\leq -x + 20 \\ y &\leq -\frac{1}{5}x + 4. \end{aligned}$$

Inequality (3) is given as $x > 0$.

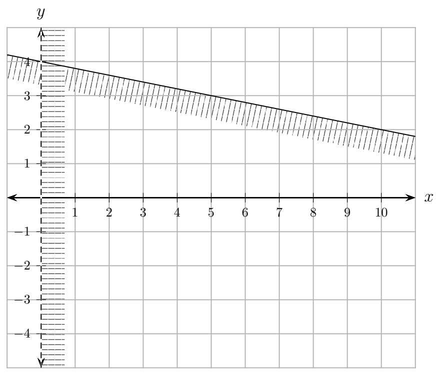
Secondly, graph each inequality using the table to assist you.

Inequality	***x*-intercept**	***y*-intercept**	**Line type**	**Point (0; 0)**
$y \geq \frac{2}{3}x - 4$	(6; 0)	(0; −4)	Solid	True
$y \leq -\frac{1}{5}x + 4$	(20; 0)	(0; 4)	Solid	True

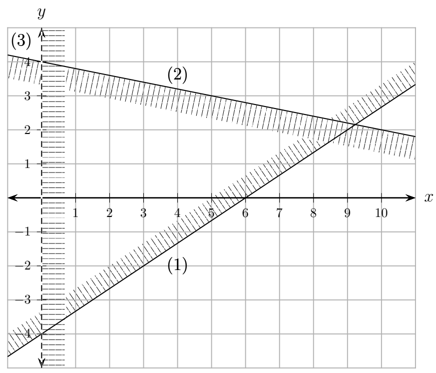
Consider inequality (3): $x > 0$.
We start with the last inequality. The last inequality only allows x to be positive. The line "$x = 0$" is just the y-axis. One needs to remember to dash the line in, because this isn't an "or equal to" inequality, so the boundary (the line) is not included in the solution.



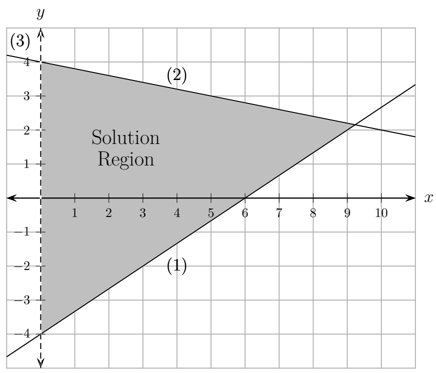
Consider inequality (2): $y \leq -\frac{1}{5}x + 4$.
It is a solid line because of the $\leq$ sign. The region of all the $(x; y)$ points that satisfy the inequality is indicated by the short lines.



Consider inequality (1): $y \geq \frac{2}{3}x - 4$.
It is a solid line because of the $\geq$ sign. The region of all the $(x; y)$ points that satisfy the inequality is indicated by the short lines.



Thirdly, find the solution region.
The solution region is the region where all three individual solution regions overlap.
In this case, the solution is the shaded part in the middle.



Note that the solution region is called a *closed* or *bounded* solution because there are lines on all sides.

Activity 5.12

1. Solve the following system of inequalities graphically:

$$\begin{aligned} 2x + 3y - 6 &\geq 0 \quad (1) \\ x - 3y + 6 &\geq 0 \quad (2) \\ x - 4 &\leq 0 \quad (3) \\ x &\geq 0 \\ y &\geq 0. \end{aligned}$$

Firstly, rewrite the first two inequalities to obtain y on the left-hand side and the third inequality to obtain x on the left-hand side.
For inequality (1) it is

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For inequality (2) it is

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For inequality (3) it is

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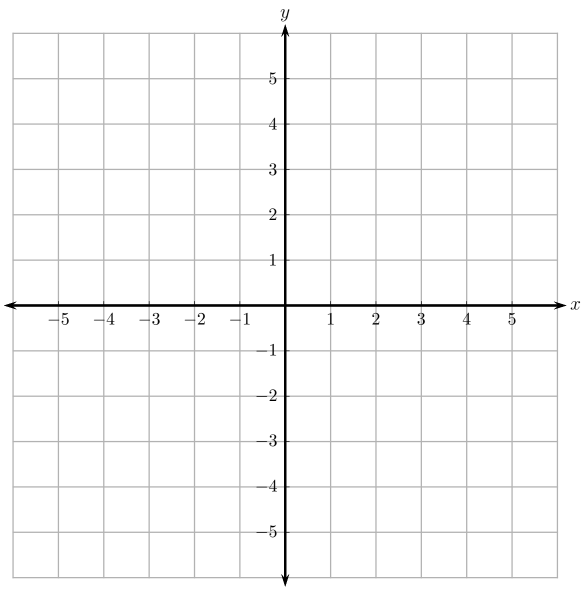
____

Secondly, graph each inequality using the table to assist you.

Inequality	**x-intercept**	**y-intercept**	**Line type**	**Point $(0;0)$**
$2x + 3y - 6 \geq 0$				
$x - 3y + 6 \geq 0$				

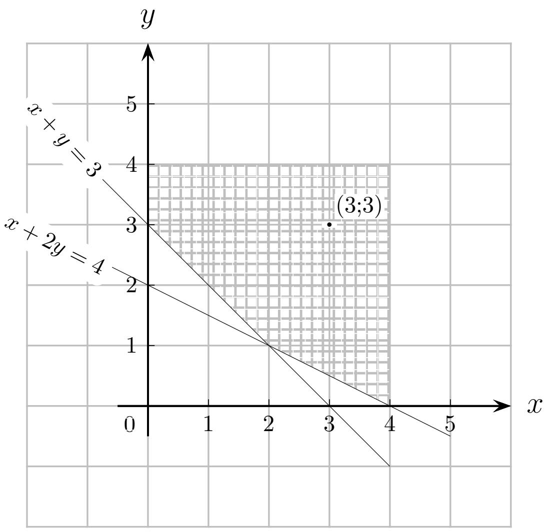
Draw the inequalities on the graph below.



Thirdly, find the solution region.

____

2. Consider the following graph. Which of the following systems of linear inequalities is represented in the graph?



(a) $y > -x + 3$
$y > -\frac{1}{2}x + 2$
$x \geq 0$
$y \geq 0.$

(b) $y < -x + 3$
$y \geq -\frac{1}{2}x + 2$
$x \geq 0$
$y \geq 0.$

(c) $y \geq -x + 3$
$y \geq -\frac{1}{2}x + 2$
$x \geq 0$
$y \geq 0.$

(d) $y \geq -x + 3$
$y < -\frac{1}{2}x + 2$
$x \geq 0$
$y \geq 0.$

To obtain the inequality sign that will make each equation true, take a point that lies in the solution region and substitute it into each equation.

COMPONENT 6

An application of differentiation

> The following examples and activities will improve your understanding of the basic principles covered in component 7 of the study guide. There is a worksheet for every study unit in component 7. Work through each worksheet after you have studied the relevant study material in the study guide.

6.1 Worksheet 1

> Worksheet 1 is based on study unit 7.1: *Marginal profit*, on pages 188 – 193 of the study guide. Do the activities and exercise before you proceed.

Activity 6.1

1. Complete the table by calculating the derivatives of the following functions:

Function $f(x)$	**Derivative** $f'(x)$
$f(x) = x^5$	$f'(x) = 5x^4$
$f(x) = x^{-4}$	$f'(x) = \square \times x^{-4-1} =$
$f(x) = x^{\frac{2}{3}}$	$f'(x) = \frac{2}{3}x^{\frac{2}{3}-\frac{3}{3}} = \frac{2}{3}x^{\square}$
$f(x) = \sqrt{x} = x^{\square}$	$f'(x) =$
$f(x) = 2x^4$	$f'(x) =$
$f(x) = -6x^2$	$f'(x) = -6 \times 2x^1 =$
$f(x) = \frac{3}{2}x^{\frac{2}{3}}$	$f'(x) =$
$f(x) = \frac{-x}{2} = -\frac{\square}{\square}x$	$f'(x) = -\frac{1}{2} \times x^0 =$
$f(x) = 12x^2 - 24x + 8$	$f'(x) =$
$f(x) = \frac{-x^4}{2} + 3x^3 - 2x$ $= -\frac{\square}{\square}x^{\square} + 3x^3 - 2x$	$f'(x) = -\frac{1}{2}\times$ $=$
$f(x) = 12x + 6\sqrt{x} - \frac{4}{x}$ $= 12x + 6x^{\square} - 4x^{\square}$	$f'(x) =$ $=$

2. A factory has found that the total revenue, R, from producing and selling x sound systems is given by the revenue function

$$R(x) = -0{,}02x^2 + 400x.$$

The cost to produce x sound systems is given by the cost function

$$C(x) = 100x + 200\,000.$$

(a) Calculate and interpret the marginal profit when 5 000, 7 500 and 8 000 sound systems are produced.

The profit function is

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= \_\_\_\_\_\_\_\_\_\_ - (\_\_\_\_\_\_\_\_\_\_) \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

The derivative of $P(x)$ is

$$\begin{aligned} P'(x) &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

The answer is $P'(x) = -0{,}04x + 300$. *Did you succeed?*

The **marginal profit when 5 000** sound systems are produced and sold, is

$$\begin{aligned} P'(5\,000) &= -0{,}04 \times 5\,000 + 300 \\ &= \_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

This means that when 5 000 sound systems are produced and sold, the profit will increase by R________ if one additional sound system is produced. Let's test to see if this is indeed the case. Calculate P(5 000) and P(5 001), that is the profit at 5 000 and 5 001 sound systems:

$$\begin{aligned} P(5\,000) &= -0{,}02 \times 5\,000^2 + 300 \times 5\,000 - 200\,000 \\ &= \_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

$$\begin{aligned} P(5\,001) &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_ \\ &\approx \_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

It is indeed the case. When 5 000 sound systems are produced and sold, the profit is R__________ and when 5 001 sound systems are produced and sold, the profit is R__________, an increase of R________.

The **marginal profit when 7 500** sound systems are produced and sold, is

$$\begin{aligned} P'(7\,500) &= -0{,}04 \times \underline{\hspace{2cm}} + 300 \\ &= \underline{\hspace{3cm}} \end{aligned}$$

When 7 500 sound systems are produced and sold, the marginal profit is R__________ which indicates that 7 500 is the number of sound systems that should be produced to maximise the profit.

The **marginal profit when 8 000** sound systems are produced and sold, is

$$\begin{aligned} P'(8\,000) &= \underline{\hspace{5cm}} \\ &= \underline{\hspace{3cm}} \end{aligned}$$

Underline the correct option:

When 8 000 sound systems are produced and sold, the marginal profit is (positive/negative).

This indicates that the profit will (increase/decrease) by R______________ if one additional sound system is produced.

(b) Confirm your findings in (a) that producing and selling 7 500 sound systems will maximise the profit. That is, determine algebraically the number of sound systems that will maximise profit.

To find the number of sound systems that will maximise the profit, determine $P'(x)$. Then solve $P'(x) = 0$. At the x-value where $P'(x) = 0$, profit is maximised:

$$\begin{aligned} P'(x) &= 0 \\ -0{,}04x + 300 &= 0 \\ \underline{\hspace{3cm}} &= \underline{\hspace{3cm}} \\ \underline{\hspace{3cm}} &= \underline{\hspace{3cm}} \\ \underline{\hspace{3cm}} &= \underline{\hspace{3cm}} \end{aligned}$$

The maximum profit is obtained when 7 500 items are produced and sold. Did you succeed?

6.2 Worksheet 2

Worksheet 2 is based on study unit 7.2: *Marginal cost*, on pages 194 – 197 of the study guide. Do the activities and exercise before you proceed.

Activity 6.2

1. A subsidiary of Electra Electronics manufactures programmable pocket calculators. Management determines that the total daily cost (in rand) to produce these calculators, is given by the cost function

$$C(x) = 0{,}0001x^3 - 0{,}08x^2 + 40x + 5\,000,$$

where x is the number of calculators that are manufactured.

(a) Calculate the marginal cost when x calculators are manufactured.

The marginal cost to manufacture x calculators is

$$\begin{aligned} C'(x) &= 0{,}0001 \times 3 \times x^{\square} - 0{,}08 \times \square \times x^{\square} + 40 \times x^{\square} + \square \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

(b) Calculate the marginal cost when 400 calculators are manufactured and interpret the result.

The marginal cost to manufacture 400 calculators is

$$\begin{aligned} C'(400) &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

The marginal cost to manufacture 400 calculators is R24. Did you succeed?

Interpretation: At a production level of __________ calculators, the cost to manufacture one additional calculator, is R__________.

Check that this is true by calculating the cost to manufacture 400 calculators, compared to the cost to calculate 401 calculators. (Note that this step is not compulsory – it is only done for a clearer understanding of the process.)

For 400 calculators:

$$\begin{aligned} C(400) &= 0{,}0001 \times 400^3 - 0{,}08 \times 400^2 + 40 \times 400 + 5\,000 \\ &= 14\,600. \end{aligned}$$

The cost to manufacture 400 calculators is R14 600.

For 401 calculators:

$$\begin{aligned} C(401) &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

The cost to manufacture 401 calculators is R______________.

The difference in cost is

$$\begin{aligned} C(401) - C(400) &= \underline{\hspace{2cm}} - \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

What did you find? ______________________________.

(c) Calculate C'(500) and interpret the result.

$$\begin{aligned} C'(500) &= \underline{\hspace{3cm}} - \underline{\hspace{3cm}} + \underline{\hspace{2cm}} \\ &= \underline{\hspace{4cm}} \end{aligned}$$

Interpretation:

____

____

2. The cost function of a factory that manufactures boxes of nougat bars is given by

$$C(x) = 500 + 5x,$$

where x is the number of boxes of nougat bars produced. The factory cannot produce more than 2 000 boxes of nougat bars per day.

Calculate $C'(100)$, $C'(500)$ and $C'(1\,000)$ and interpret the results.

The marginal cost function is given by $C'(x) =$ ____________________ and

$$\begin{aligned} C'(100) &= \underline{\hspace{2cm}}, \\ C'(500) &= \underline{\hspace{2cm}}, \text{ and} \\ C'(1\,000) &= \underline{\hspace{2cm}}. \end{aligned}$$

Interpretation:

____

____

COMPONENT 7

Mathematics of finance

The following examples and activities will improve your understanding of the basic principles covered in component 4 of the study guide. There is a worksheet for every study unit in component 4. Work through each worksheet after you have studied the relevant study material in the study guide.

7.1 Worksheet 1

Worksheet 1 is based on study unit 4.1: *Simple interest and simple discount*, on pages 118 – 125 of the study guide. Do the activities and exercises before you proceed.

Example 7.1

Calculate the simple interest to be paid on a loan of R10 000 at an interest rate of 12,5% per annum over three and a half years.

The following is given:

$$\begin{aligned} P &= 10\,000 \\ R &= 12{,}5\% = \frac{12{,}5}{100} = 0{,}125 \\ T &= \text{time in years} = 3{,}5 \text{ years} \\ I &= ? \end{aligned}$$

The principal amount, P, interest rate, R, and the time, T, are given.
The interest, I, is required:
? ✓✓✓
$I = PRT$.

The interest is calculated as

$$\begin{aligned} I &= PRT \\ &= 10\,000 \times 0{,}125 \times 3{,}5 \\ &= 4\,375. \end{aligned}$$

The interest is R4 375.

Activity 7.1

If R5 000 is invested for five years at a simple interest rate of 7,5% per annum, what is the amount that will be received at the end of the period?

The following is given:

$$\begin{aligned} P &= 5\,000 \\ R &= 7{,}5\% = \frac{7{,}5}{100} = \underline{\hspace{2cm}} \\ T &= \text{time in years} = \underline{\hspace{2cm}} \\ S &= ? \end{aligned}$$

Two methods can be used.

First calculate I, then $S = P + I$.

The interest is

$$I = PRT$$
$$= \_\_\_\_\_\_ \times \_\_\_\_\_\_ \times \_\_\_\_\_\_$$
$$= \_\_\_\_\_\_.$$

The interest is R________.

The amount at the end of the period is

$$S = P + I$$
$$= \_\_\_\_\_\_ + \_\_\_\_\_\_$$
$$= 6\,875.$$

The amount is R6 875. Did you succeed?

The **second** method is to calculate $S = P(1 + RT)$.

The amount is

$$S = P(1 + RT)$$
$$= \_\_\_\_ \times (1 + \_\_\_\_ \times \_\_\_\_)$$
$$= \_\_\_\_\_\_.$$

> The principal amount, P, the interest rate, R, and the time, T, are given. The interest, I, is required, as well as the amount at the end of the period, S:
> $\overset{?}{I} = \overset{\checkmark\checkmark\checkmark}{PRT}$; $\overset{?}{S} = \overset{\checkmark}{P} + \overset{\checkmark}{I}$.

> The amount at the end of the period, S, can be calculated directly when P, R and T are known, by using the formula
> $\overset{?}{S} = \overset{\checkmark}{P}(1 + \overset{\checkmark\checkmark}{RT})$.

Example 7.2

Calculate the principal that must be invested over four years at 9,5% per annum to earn R779 simple interest.

The following is given:

$$I = 779$$
$$R = 9{,}5\% = \frac{9{,}5}{100} = 0{,}095$$
$$T = \text{time in years} = 4 \text{ years}$$
$$P = ?$$

> The interest, I, the interest rate, R, and the time, T, are given. The principal, P, is required:
> $\overset{\checkmark}{I} = \overset{?}{P}\overset{\checkmark\checkmark}{RT}$.

Simple interest is calculated as

$$I = PRT,$$

therefore, the principal is calculated as

$$P = \frac{I}{R \times T}$$
$$= \frac{779}{0{,}095 \times 4}$$
$$= 2\,050.$$

The principal is R2 050.

Activity 7.2

You want to make an investment at a bank at a simple interest rate of 10% per annum that will yield an amount of R6 000 after four and a half years. Determine the principal to be invested. Also calculate the interest on the investment.

The following is given:

$$\begin{aligned} S &= \_\_\_\_\_\_\_\_\_\_ \\ R &= 10\% = \_\_\_\_\_\_\_\_\_\_ \\ T &= \text{time in years} = \_\_\_\_\_\_\_\_\_\_ \\ P &= ? \end{aligned}$$

The amount at the end of the period, S, the interest rate, R, and the time, T, are given. The principal, P, is required:
✓ ? ✓✓
$S = P(1 + RT)$.

From

$$S = P(1 + RT)$$

the principal is calculated as

$$\begin{aligned} P &= \frac{S}{1 + RT} \\ &= \frac{\boxed{}}{1 + \boxed{} \times \boxed{}} \\ &= \_\_\_\_\_\_\_\_\_\_. \end{aligned}$$

The principal is R__________.

The interest is

$$\begin{aligned} I &= S - P \\ &= \_\_\_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_\_\_ \\ &= 1\,862{,}07. \end{aligned}$$

Did you succeed?

Example 7.3

A client borrows money from a micro-lender who offers short-term loans. The client borrows R2 175 and has to repay R2 349 after six months. What simple interest rate does the micro-lender use to calculate the interest?

The following is given:

$$
\begin{aligned}
I &= S - P = 2\,349 - 2\,175 = 174 \\
P &= 2\,175 \\
T &= \frac{6}{12} = 0{,}5 \text{ year} \\
R &= ?
\end{aligned}
$$

> Manipulate the simple interest formula, $I = PRT$. Interest, I, principal, P, and time, T, are given. The interest rate, R, is required:
> ✓ ✓?✓
> $I = PRT$.

From $I = PRT$ follows that

$$
\begin{aligned}
R &= \frac{I}{P \times T} \\
&= \frac{174}{2\,175 \times 0{,}5} \\
&= 0{,}16.
\end{aligned}
$$

The interest rate is $R = 0{,}16 \times 100 = 16\%$.

Activity 7.3

How long will Lucy have to wait before her R2 500, invested ar 6% simple interest per year, accumulates to R3 100?

The following is given:

$$
\begin{aligned}
I &= \text{interest} = \underline{\qquad\qquad} - \underline{\qquad\qquad} = \underline{\qquad\qquad} \\
R &= \text{interest rate per year} = \underline{\qquad\qquad} \\
P &= \text{principal} = \underline{\qquad\qquad} \\
T &= \text{time in years} = ?
\end{aligned}
$$

> Manipulate the simple interest formula, $I = PRT$. Interest, I, principal, P, and interest rate, R, are given. The time, T, is required:
> ✓ ✓✓?
> $I = PRT$.

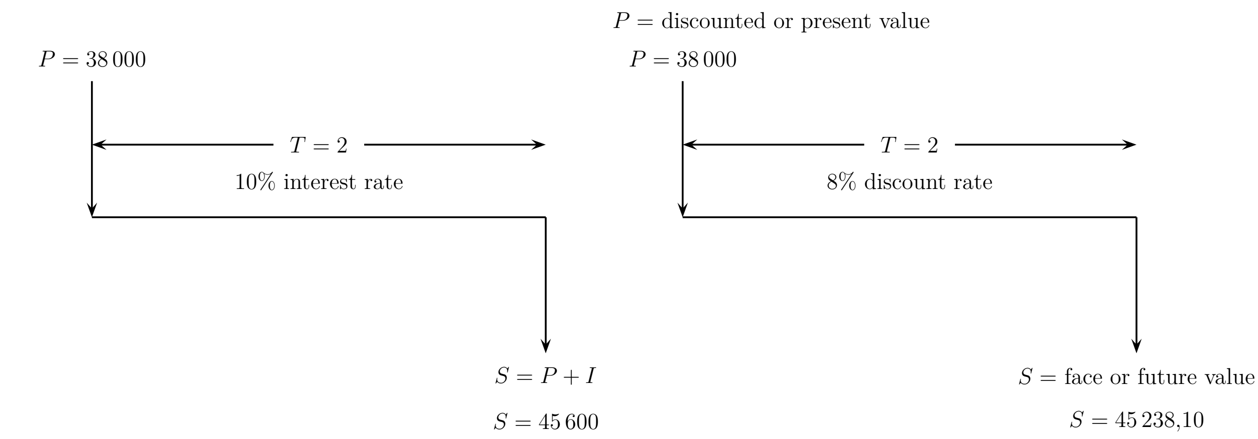
From $I = PRT$ follows that

$$
\begin{aligned}
T &= \frac{I}{P \times R} \\
&= \frac{\boxed{}}{\boxed{} \times \boxed{}} \\
&= \underline{\qquad\qquad}.
\end{aligned}
$$

The time is ______ years.

Compare simple interest and simple discount.

Simple interest	**Simple discount**
The loan is an add-on loan in which the interest is **added** to the **principal**: $S = P + I$.	The interest, called the discount or discounted amount, is **subtracted** from the **face/future** value and only the proceeds are received by the borrower: $P = S - D$.
Example: Happy Homes furniture store wants to borrow R38 000 for two years.	
Community Bank offers them a 10% simple interest loan.	National Bank offers them an 8% simple discount loan.
The following is given: $P = 38\,000$ $R = 0{,}10$ per year $T = 2$ years.	The following is given: $P = 38\,000$ $d = 0{,}08$ per year $T = 2$ years.
The interest that Happy Homes will have to pay on the loan is $I = PRT$ $= 38\,000 \times 0{,}10 \times 2$ $= 7\,600$.	The amount that Happy Homes must request to borrow from the bank in order to actually receive proceeds of R38 000 now, is $P = S(1 - dT)$ $S = \dfrac{P}{1 - dT}$ $= \dfrac{38\,000}{1 - 0{,}08 \times 2}$ $= 45\,238{,}10$.
The amount that Happy Homes will have to repay after two years is $S = P + I$ $= 38\,000 + 7\,600$ $= 45\,600$.	The discount subtracted from the future value of the loan is $D = S - P$ $= 45\,238{,}10 - 38\,000$ $= 7\,238{,}10$.
	The equivalent simple interest rate of the loan is now determined. We have that $P = 38\,000$, $I = 7\,238{,}10$ and $T = 2$. $R = \dfrac{I}{P \times T}$ $S = \dfrac{7\,238{,}10}{38\,000 \times 2}$ $= 0{,}0952$ $= 9{,}5\%$.



Supplementary background

Discount is the interest on a loan computed in advance and deducted at the time the loan is made.

Activity 7.4

1. A loan that has to be repaid in nine months has a discount rate of 12% per year. The present/discounted value is R8 000.

   The following is given:

   $$\begin{aligned} P &= 8\,000 \\ d &= 0{,}12 \text{ per year} \\ T &= \frac{9}{12} = \frac{3}{4} \text{ year.} \end{aligned}$$

   Answer the following questions:

   (a) What is the actual amount of money received by the borrower after the discount has been deducted from the face/future value?

   ____

   (b) What is the face/future value (amount originally borrowed) in order to actually receive R8 000?

   From $P = S(1 - dT)$ follows that

   S = ________________

   = ________________

   = 8 791,21.

   The face/future value is R8 791,21. Did you succeed?

(c) What is the discount that will be subtracted from the face/future value?

The discount is

$$\begin{aligned} D &= SdT \\ &= \underline{\hspace{6cm}} \\ &= \underline{\hspace{6cm}} \end{aligned}$$

or

$$\begin{aligned} D &= S - P \\ &= \underline{\hspace{6cm}} \\ &= \underline{\hspace{6cm}} \end{aligned}$$

(d) Determine the equivalent simple interest rate of the loan.

We have that

$$\begin{aligned} P &= 8\,000 \\ I &= 791{,}21 \\ T &= \frac{3}{4} \text{ year.} \end{aligned}$$

From $I = PRT$ it follows that

$$\begin{aligned} R &= \frac{I}{P \times T} \\ &= \underline{\hspace{6cm}} \\ &= \underline{\hspace{6cm}} \end{aligned}$$

The simple interest rate of the loan is ____________________________%.

2. Mary Ellen takes a loan with a face value of R3 500, a term of six months and a 14% discount rate per year.

The following is given:

$$\begin{aligned} S &= \underline{\hspace{3cm}} \\ d &= \underline{\hspace{3cm}} \\ T &= \underline{\hspace{3cm}}. \end{aligned}$$

Answer the following questions:

(a) What is the discount?

____

____

____

(b) What is the amount that she receives now?

(c) What is the equivalent simple interest rate?

7.2 Worksheet 2

Worksheet 2 is based on study unit 4.2: *Compound interest*, on pages 126 – 128 of the study guide. Do the exercise before you proceed.

Example 7.4

In compound interest calculations, the interest is not necessarily compounded annually (i.e. calculated once a year). Interest can also be compounded half-yearly, quarterly or monthly. In these cases interest is added more frequently than once a year. As a result, there is more opportunity to earn interest on interest.

The following table shows the different compounding periods per year:

Compounding period (interval)	**Descriptive adverb**	**Fraction of one year**
1 month	monthly	$\frac{1}{12}$
3 months	quarterly	$\frac{1}{4}$
6 months	biannually / half-yearly	$\frac{1}{2}$
1 year	annually / yearly	1

Suppose an amount is invested for **two years** at an interest rate of 12% per year, compounded at different periods. The following table shows the different periods, the interest rate per period and the number of periods:

Compounding period (interval)	**Interest rate per period/interval**	**Number of periods/intervals in two years**
monthly	per month: $\frac{0{,}12}{12} = 0{,}01$	months: $2 \times 12 = 24$
quarterly	per quarter: $\frac{0{,}12}{4} = 0{,}03$	quarters: $2 \times 4 = 8$
biannually	per half year: $\frac{0{,}12}{6} = 0{,}02$	half years: $2 \times 2 = 4$
yearly/annually	per year: $\frac{0{,}12}{1} = 0{,}12$	years: $2 \times 1 = 2$

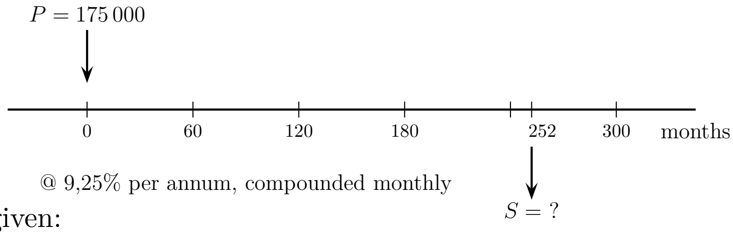
Activity 7.5

Complete the following table that contains different interest rates, compounding periods and terms:

Term	**Interest rate per year**	**Period/ interval**	**Interest rate per period/interval**	**Number of periods/intervals**
5 years	8%	yearly/annually	per year: $\frac{0{,}08}{1} = 0{,}08$	no. of years: $5 \times 1 = 5$
18 months	$7\frac{1}{2}$%	biannually/ half-yearly	per half year: $\frac{0{,}075}{\quad} = \quad$	no. of half years:
6 years	10%	monthly	per month:	no. of months:
2 and a half years	$14\frac{1}{2}$%	yearly/annually	per year:	no. of years:
9 months	15%	quarterly	per quarter:	no. of quarters:
3 and a half years	18%	monthly	per month:	no. of months:
5 and a half years	16%	quarterly	per quarter:	no. of quarters:

Example 7.5

A young man is the beneficiary of a trust fund established 21 years ago at his birth. If the original amount placed in trust was R175 000,00, how much will he receive if the money has earned interest at the rate of 9,25% per annum, compounded monthly?



The following is given:

$$\begin{aligned} P &= 175\,000 \\ R &= \frac{0{,}0925}{12} \\ T &= 21 \times 12 = 252 \text{ months} \\ S &= ? \end{aligned}$$

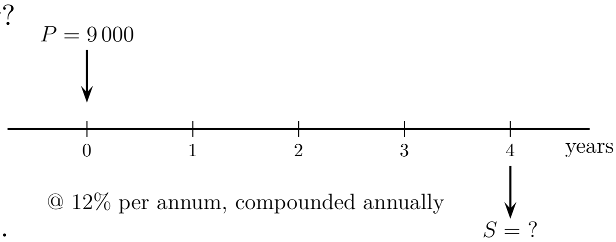
From this we find

$$\begin{aligned} S &= P(1+R)^T \\ &= 175\,000 \times (1 + \frac{0{,}0925}{12})^{252} \\ &= 1\,211\,770{,}15. \end{aligned}$$

The total amount available is R1 211 770,15.

Activity 7.6

How much interest is earned on R9 000 invested for four years at 12% p.a. (per annum), compounded annually?



The following is given:

$$\begin{aligned} P &= 9\,000 \\ R &= \underline{\qquad\qquad} \\ T &= \underline{\qquad\qquad} \\ S &= ? \end{aligned}$$

From this we find

$$\begin{aligned} S &= P(1+R)^T \\ &= \underline{\qquad\qquad\qquad\qquad\qquad} \\ &= \underline{\qquad\qquad\qquad\qquad\qquad} \\ &= 14\,161{,}67. \end{aligned}$$

After four years the total amount is R14 161,67. Did you succeed?

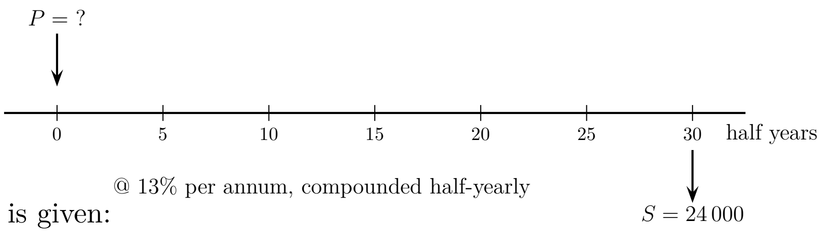
The interest earned is

$$\begin{aligned} \text{interest} &= S - P \\ &= \underline{\hspace{5cm}} \\ &= \underline{\hspace{5cm}} \end{aligned}$$

The interest earned is R_____________.

Example 7.6

A trust fund for a child's education is set up with a single payment so that at the end of 15 years there will be R24 000,00. If the fund earns interest at a rate of 13% per annum, compounded half-yearly, how much money should be paid into the fund initially?



The following is given:

$$\begin{aligned} S &= 24\,000 \\ R &= \frac{0{,}13}{2} = 0{,}065 \\ T &= 15 \times 2 = 30 \text{ half years} \\ P &= ? \end{aligned}$$

From

$$S = P(1+R)^T$$

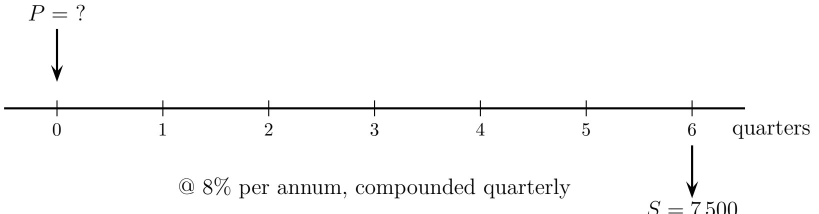
follows that

$$\begin{aligned} P &= \frac{S}{(1+R)^T} \\ &= \frac{24\,000}{(1+0{,}065)^{30}} \\ &= \frac{24\,000}{1{,}065^{30}} \\ &= 3\,628{,}47. \end{aligned}$$

The amount that should originally be invested, is R3 628,47.

Activity 7.7

1. To help her son with his studies, Mrs Maloka borrows an amount of money. After 18 months she has to pay back R7 500. How much did she borrow if the interest rate is 8% per annum, compounded quarterly?



The following is given:

S = ______________

R = ______________

T = ______________

P = ?

From this we find

P = __________________

= __________________

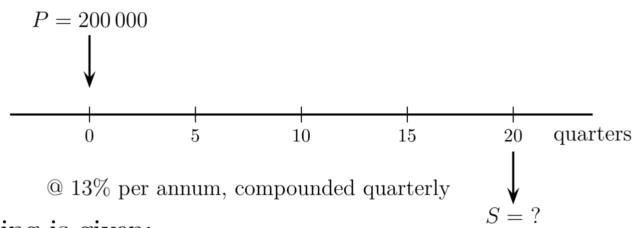
= __________________

= __________________

The amount borrowed, is R_______________.

2. George and Jim are brothers who have each been left an inheritance of R200 000. George invests his money in an account for five years at an annual interest rate of 13%, compounded quarterly. Jim invests his money for five years at a simple annual interest rate of 13,75%. At the end of that time they agree to pool their total amounts available into one account for a further five years with an annual interest rate of 12%, compounded monthly. How much money will they have together after ten years?

   The time line for George is:



   For George the following is given:

   P = 200 000

   R = _____________

   T = _____________ quarters

   S = ?

   The amount available for George after five years is

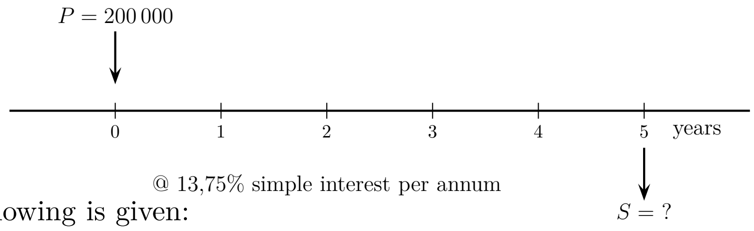
   ____

   ____

   ____

   The amount for George is R_______________.

The time line for Jim is:



For Jim the following is given:

P = 200 000

R = __________

T = __________ years

S = ?

The amount available for Jim after five years is

____

____

____

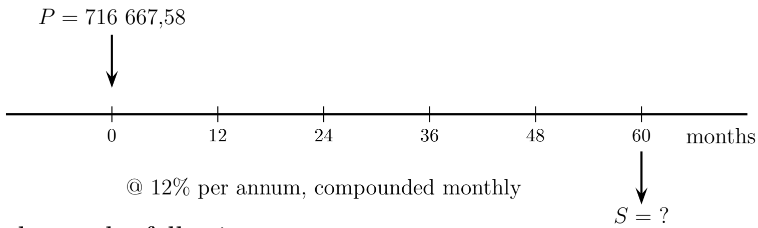
The amount for Jim is R______________.

The total amount of money they have to pool after five years is

______________ + ______________ = ________________.

In total they have R716 667,58. Did you succeed?

The time line for their joint investment is



At this point we know the following:

P = ______________

R = ______________

T = ______________ months

S = ?

The total amount after ten years is

____

____

____

The final amount is R__________________.

7.3 Worksheet 3

> Worksheet 3 is based on study unit 4.3: *The time value of money*, on pages 129 – 134 of the study guide. Do the activity and exercise before you proceed.

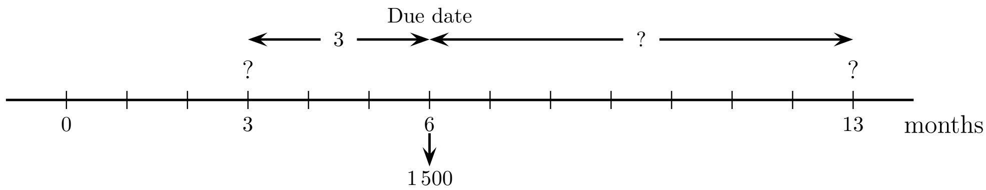
Example 7.7

Study the following example:

Mr Zinta applies for a personal loan and has to pay R1 500 after six months. Interest of 11% per annum, compounded monthly is applicable. Find his obligation

1. three months from now; and
2. 13 months from now.

First, draw a time line. It is important to get a picture of the problem – to understand what is given and what is required:



@ 11% per annum, compounded monthly

This question can be reformulated as follows:

1. If Mr Zinta decides to settle his debt earlier, namely at the end of month three, what will the payment be?

   If he settles his debt earlier than required, he will pay less than the R1 500 because he will save on interest. The debt must be discounted back to month three. From month three to month six is three months. The payment will be

$$\begin{aligned} P &= \frac{S}{(1+R)^T} \\ &= \frac{1\,500}{\left(1+\frac{0{,}11}{12}\right)^3} \\ &= 1\,459{,}49. \end{aligned}$$

   The amount due after three months is R1 459,49.

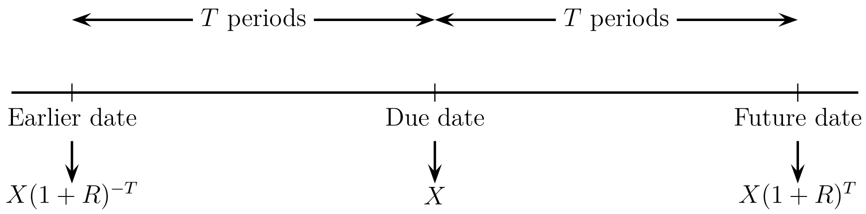
2. If he decides to settle his debt later, namely at the end of month 13, what will the payment be?

   If he settles his debt later than required, he will pay more than the R1 500 because interest will be added. The debt will accumulate. The debt must be moved forward to month 13. From month six to month 13 is seven months. The payment will be

$$\begin{aligned} S &= P \times (1+R)^T \\ &= 1\,500 \times \left(1+\frac{0{,}11}{12}\right)^7 \\ &= 1\,598{,}94. \end{aligned}$$

   The amount due will be R1 598,94.

The following schematic representation illustrates what happens when the due date is shifted: (R = interest per time interval; T = number of time intervals/periods)



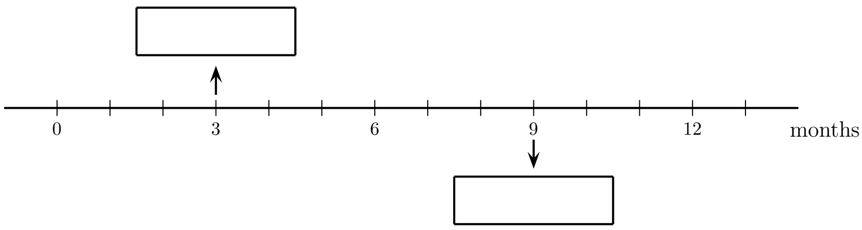
Activity 7.8

Lucas borrows money from his uncle. An amount of R2 700 is due nine months from now. They decide on an interest rate of 15% per annum, compounded quarterly. Consider the following two scenarios:

1. Suppose he wins a competition three months from now and decides to settle his debts immediately. What is the amount that he has to repay his uncle?

   This question is concerned with quarters rather than the months of the example because the interest is compounded quarterly.

   Complete the following time line:



@ _____% per annum, compounded ______________

The payment is moved backwards ___________ months. From month nine to month three.

That is ___________ quarters. The amount that is due, is

$$P = \frac{S}{(1+R)^T}$$

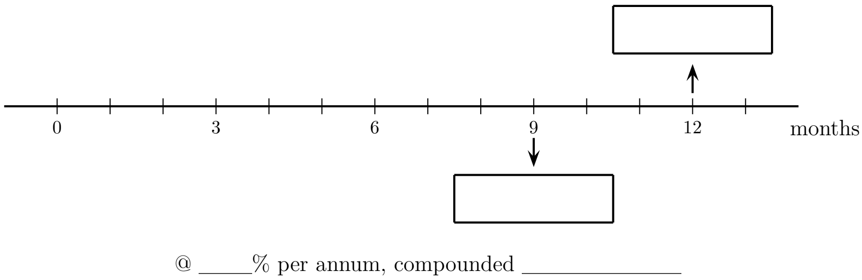
$= $ ________________

$= $ ________________

The amount due is R________________

2. Suppose he has financial difficulties and asks his uncle to postpone the payment that is due in nine months, with three months. What is the amount that he has to repay his uncle after a year?

   Complete the following time line:



   The payment is moved forward __________ months. From month nine to month twelve.

   That is __________ quarter. The amount due is

   $$\begin{aligned} S &= P(1+R)^T \\ &= \underline{\hspace{5cm}} \\ &= \underline{\hspace{5cm}} \end{aligned}$$

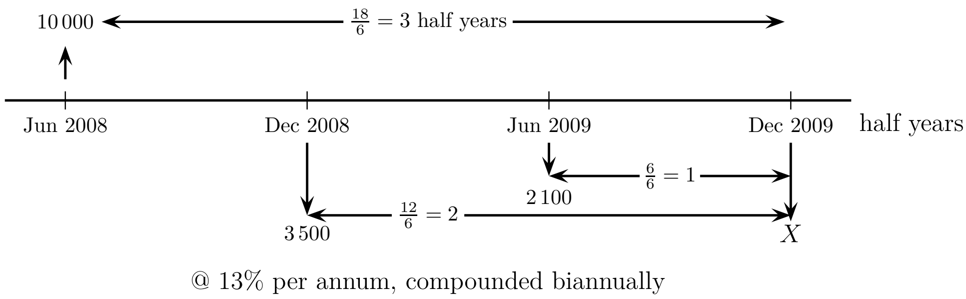
   The amount due is R_______________

Only money (payments or debts) that is brought to a **common date**, may be added together or subtracted from each other. Study the following example:

Example 7.8

Mpho borrows R10 000 at the end of June 2008 at 13% interest per annum, compounded biannually. She pays R3 500 at the end of December 2008 and R2 100 at the end of June 2009. What is the balance due at the end of December 2009? Assume the entire debt and each partial payment earns the same stated rate of interest.

Once again, start off with a timeline. The debts are shown above the line and the payments below.



The **common date** is December 2009. Move all the payments and the debt to this date.

The value of the R3 500 **payment** in December 2008, in December 2009 is

$$\begin{aligned} S &= P \times (1+R)^T \\ &= 3\,500 \times \left(1 + \frac{0{,}13}{2}\right)^2 \\ &= 3\,969{,}79. \end{aligned}$$

One year equals two half years.

The value is R3 969,79.

The value of the R2 100 **payment** in June 2009, is in December 2009

$$\begin{aligned} S &= P \times (1+R)^T \\ &= 2\,100 \times \left(1 + \frac{0{,}13}{2}\right)^1 \\ &= 2\,236{,}50. \end{aligned}$$

Six months equal one half year.

The value is R2 236,50.

The value of the R10 000 **debt** in June 2008, is in December 2009

$$\begin{aligned} S &= P \times (1+R)^T \\ &= 10\,000 \times \left(1 + \frac{0{,}13}{2}\right)^3 \\ &= 12\,079{,}50. \end{aligned}$$

Eighteen months equal three half years.

The value is R12 079,50.

To settle the debt in December 2009, all payments must equal the outstanding debt. Suppose the payment that will settle the remaining debt is X. Then

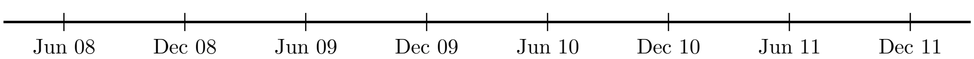
$$\begin{aligned} \text{payments} &= \text{debt} \\ X + 3\,969{,}79 + 2\,236{,}50 &= 12\,079{,}50 \\ X + 6\,206{,}29 &= 12\,079{,}50 \\ X &= 12\,079{,}50 - 6\,206{,}29 \\ &= 5\,873{,}21. \end{aligned}$$

Thus, the balance due at the end of December 2009, is R5 873,21.

Activity 7.9

A student borrows R1 375 from his father in December 2008. He also owes his father R8 250 (interest included) due in December 2011. In June 2009 he pays his father an amount of R5 320 which he earned while doing extra jobs. He does so well with the extra jobs that he plans to settle all his debts in December 2009. All obligations and payments are subject to the same interest rate of 17% per annum, compounded annually. What amount will settle all his debts in December 2009?

Draw the time line with debt above the line and payments below:



@ 17% per annum, compounded annually

The **common date** is December 2009. Move all the payments and the debt to this date.

The value of the R1 375 **debt** incurred in December 2008, in December 2009 is

____

____

____

The value of the R8 250 **debt** payable in December 2011, in December 2009 is

____

____

____

The value of the R5 320 **payment** made in June 2009, in December 2009 is

____

____

____

To find the amount that will settle the debt in December 2009, all payments must be set equal to the outstanding debt. Suppose the payment that will settle the remaining debt is X. Then

payments = debts

____

____

____

____

Thus, the balance due at the end of December 2009, will be R________________.

7.4 Worksheet 4

Worksheet 4 is based on study unit 4.4: *Annuities* on pages 135 – 138 of the study guide. Do the activity and exercise before you proceed.

In everyday life many people are involved in either the payment or receipt of annuities at equal intervals of time. Some examples of annuity payments are as follows:

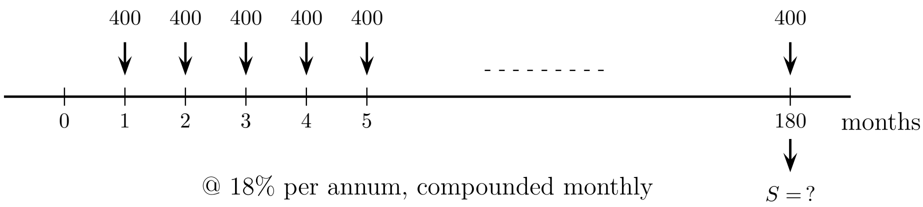
- mortgage payments
- any loan repayment
- insurance premiums
- payment of rent

Firstly we demonstrate how to find the future value of an annuity.

Example 7.9

Suppose that at the end of the month of his 35th birthday Ken decides to deposit R400 into his savings account at the end of each month. The savings account pays interest at a rate of 18% per annum, compounded monthly. What is the amount of money that Ken will have in his account on his 50th birthday?

This can be illustrated with the following time line:



For the future value of the annuity calculation, the following is given:

$$\begin{aligned} R &= 400 \\ i &= \frac{0{,}18}{12} = 0{,}015 \\ n &= 15 \times 12 = 180 \text{ months } (50 - 35 = 15 \text{ years}) \\ S &= ? \end{aligned}$$

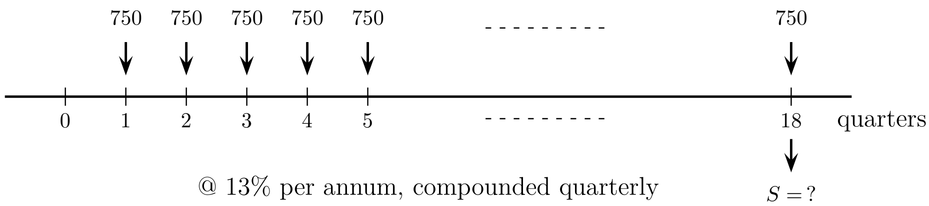
From this follows that

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ &= R \times \left[\frac{(1+i)^n - 1}{i}\right] \\ &= 400 \times \left[\frac{(1+0{,}015)^{180} - 1}{0{,}015}\right] \\ &= 400 \times \left[\frac{1{,}015^{180} - 1}{0{,}015}\right] \\ &= 362\,249{,}81. \end{aligned}$$

The amount in his account will be R362 249,81.

Activity 7.10

What is the accumulated amount of an annuity with a payment of R750 every three months, and an interest rate of 13% per annum, compounded quarterly, at the end of four and a half years?



For the future value of the annuity calculation, the following is given:

$$
\begin{aligned}
R &= 750 \\
i &= \frac{\qquad}{4} = \underline{\qquad\qquad} \\
n &= \underline{\qquad\quad} \times \underline{\qquad\quad} = \underline{\qquad\quad} \text{ quarters} \\
S &= ?
\end{aligned}
$$

From this follows that

$$
\begin{aligned}
S &= Rs_{\overline{n}|i} \\
&= R \times \left[\frac{(1+i)^n - 1}{i}\right] \\
&= \underline{\qquad\qquad\qquad\qquad} \\
&= \underline{\qquad\qquad\qquad\qquad} \\
&= 17\,962{,}29.
\end{aligned}
$$

The accumulated amount is R17 962,29. Did you succeed?

Secondly, we demonstrate how to calculate the payment of an annuity when the future value is known.

The future value is

$$
\begin{aligned}
S &= Rs_{\overline{n}|i} \\
&= R \times \left[\frac{(1+i)^n - 1}{i}\right].
\end{aligned}
$$

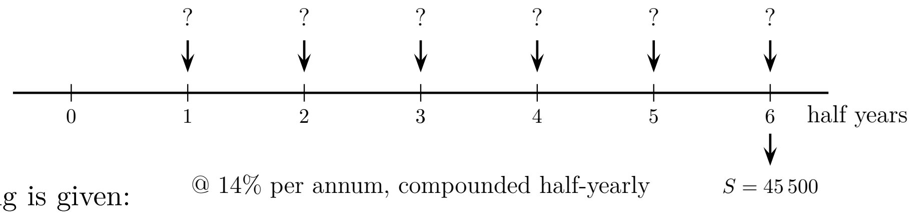
Now, solve for R to get the payment:

$$
\begin{aligned}
R &= S \div \left[\frac{(1+i)^n - 1}{i}\right] \\
&= S \times \left[\frac{i}{(1+i)^n - 1}\right].
\end{aligned}
$$

Example 7.10

The Collins family are planning a trip to Europe and want to have R45 500 in their account in three years' time. The account will pay 14% interest per annum, compounded half-yearly. What amount must they deposit into their account at the end of every six months?

This can be illustrated with the following time line:



The following is given:

$$\begin{aligned} S &= 45\,500 \\ i &= \frac{0{,}14}{2} = 0{,}07 \\ n &= 3 \times 2 = 6 \text{ half years} \\ R &= ? \end{aligned}$$

The size of a half-yearly deposit is

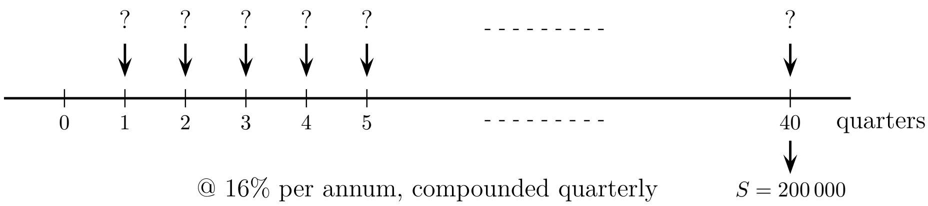
$$\begin{aligned} R &= S \times \left[\frac{i}{(1+i)^n - 1}\right] \\ &= 45\,500 \left[\times \frac{0{,}07}{(1+0{,}07)^6 - 1}\right] \\ &= \frac{45\,500 \times 0{,}07}{1{,}07^6 - 1} \\ &= 6\,360{,}71. \end{aligned}$$

The half-yearly deposits are R6 360,71 each.

Activity 7.11

The Smiths have a small daughter. They want to set up an account to accumulate money for her college education, in which they would like to have R200 000 after ten years. If the account pays 16% interest per year, compounded quarterly, and the Smiths make equal deposits at the end of every quarter, how large must each deposit be for them to reach their goal?

This is illustrated with the following time line:



For the size of the deposit calculation, when the future value of the annuity is given, we have the following information:

S = ____________

i = ____________

n = ____________ quarters

R = ?

The quarterly deposits are

$$\begin{aligned} R &= S \times \left[\frac{i}{(1+i)^n - 1}\right] \\ &= \underline{\hspace{6cm}} \\ &= \underline{\hspace{6cm}} \\ &= 2\,104{,}70. \end{aligned}$$

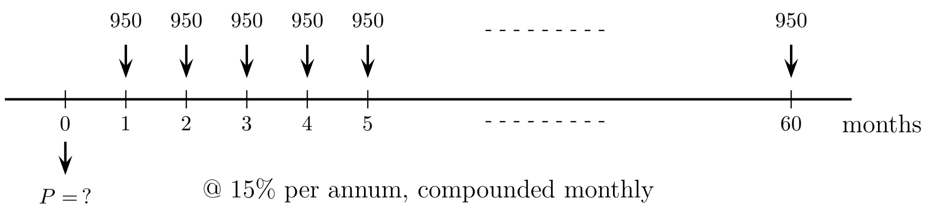
The deposit every quarter will be R2 104,70. Did you succeed?

Thirdly, we demonstrate how to determine the present value of an annuity.

Example 7.11

Sarah borrows money at an interest rate of 15% per annum, compounded monthly, to buy a second-hand car. Every month she has to repay R950,00 for a term of five years. What is the price of the car?

This can be illustrated with the following time line:



For the calculation of the present value of an annuity, we have the following information:

$$\begin{aligned} R &= 950 \\ i &= \frac{0{,}15}{12} = 0{,}0125 \\ n &= 5 \times 12 = 60 \text{ months} \\ P &= ? \end{aligned}$$

The present value is calculated as

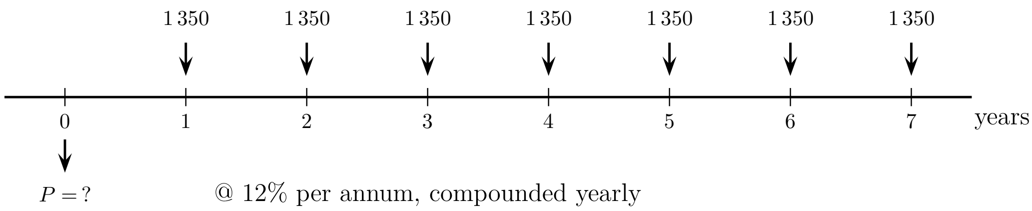
$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ &= R \times \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \\ &= 950 \times \left[\frac{(1+0{,}0125)^{60} - 1}{0{,}0125(1+0{,}0125)^{60}}\right] \\ &= 950 \times \left[\frac{1{,}0125^{60} - 1}{0{,}0125(1{,}0125)^{60}}\right] \\ &= 39\,932{,}86. \end{aligned}$$

The price of the car (present value) is R39 932,86.

Activity 7.12

Determine the present value of an annuity with yearly payments of R1 350 at an interest rate of 12% per annum, compounded annually, with a term of seven years.

This can be illustrated with the following time line:



For the calculation of the present value of an annuity, the following information is given:

R = __________

i = __________

n = __________ years

P = ?

The present value is calculated as

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ &= R \times \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \\ &= \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \\ &= \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \\ &= 6\,161{,}07. \end{aligned}$$

The present value of the annuity is R6 161,07. Did you succeed?

Fourthly, we demonstrate how to calculate the payment when the present value of an annuity is known.

The present value is

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ &= R \times \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]. \end{aligned}$$

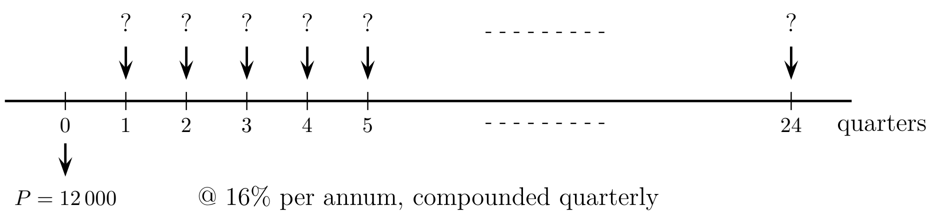
Now, solve for R to get the payment:

$$\begin{aligned} R &= P \div \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \\ &= P \times \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]. \end{aligned}$$

Example 7.12

A loan of R12 000 with an interest rate of 16% per annum, compounded quarterly, is to be repaid by equal quarterly payments over six years. What is the size of the quarterly payments?

This can be illustrated with the following time line:



For the calculation of the size of the payment, when the present value of the annuity is given, we have the following information:

$$\begin{aligned} P &= 12\,000 \\ i &= \frac{0{,}16}{4} = 0{,}04 \\ n &= 6 \times 4 = 24 \text{ quarters} \\ R &= ? \end{aligned}$$

The quarterly payments are calculated as

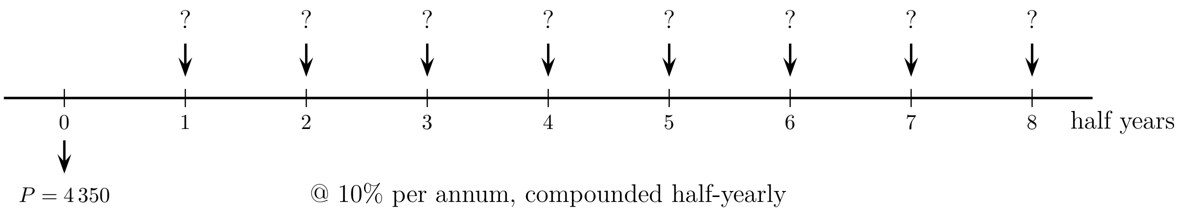
$$\begin{aligned} R &= P \times \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \\ &= 12\,000 \times \left[\frac{0{,}04(1+0{,}04)^{24}}{(1+0{,}04)^{24} - 1}\right] \\ &= \frac{12\,000 \times 0{,}04 \times 1{,}04^{24}}{1{,}04^{24} - 1} \\ &= 787{,}04. \end{aligned}$$

The payment every quarter will be R787,04.

Activity 7.13

Max pays R1 250 as a deposit on a new TV. He borrows the rest of the money and has to repay it in equal payments at the end of every six months. Interest is charged at 10% per annum, compounded half-yearly. How much will he pay every six months if the cost of the TV is R5 600 and he has four years to settle his debt?

This can be illustrated with the following time line:



For the calculation of the size of the payment, when the present value of the annuity is given, we have the following information:

$P = 5\,600 - 1\,250 =$ ____________

$i =$ ________________

$n =$ ________________ half years

$R = ?$

The half-yearly payments are

$$R = P \times \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

= ________________________________

= ________________________________

= 673,04.

The payment every six months is R673,04. Did you succeed?

7.5 Worksheet 5

Worksheet 5 is based on study unit 4.5: *Amortisation* on pages 139 – 143 of the study guide. Do the exercise before you proceed.

Example 7.13

Sibongile purchases a computer for R9 500 and obtains a three year loan at 12% interest per annum, compounded biannually. Calculate the six-monthly payments and draw up an amortisation table.

The following is given:

$$\begin{aligned} P &= 9\,500 \\ i &= \frac{0{,}12}{2} = 0{,}06 \\ n &= 3 \times 2 = 6 \text{ half years} \\ R &= ? \end{aligned}$$

The six-monthly payments are

$$\begin{aligned} R &= P \times \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \\ &= 9\,500 \times \left[\frac{0{,}06(1+0{,}06)^6}{(1+0{,}06)^6 - 1}\right] \\ &= 1\,931{,}94. \end{aligned}$$

The six-monthly payments are R1 931,94 each.

The amortisation table is shown below:

Half year	**Outstanding principal at half-year beginning**	**Interest due at half-year end (simple) $I = PRT$ ***	**Payment**	**Principal repaid**
1	9 500,00	9 500,00 × 0,12 × 0,5 = 570,00	1 931,94	1 931,94 − 570,00 = 1 361,94
2	9 500,00 − 1 361,94 = 8 138,06	8 138,06 × 0,12 × 0,5 = 488,28	1 931,94	1 931,94 − 488,28 = 1 443,66
3	8 138,06 − 1 443,66 = 6 694,40	6 694,40 × 0,12 × 0,5 = 401,66	1 931,94	1 931,94 − 401,66 = 1 530,28
4	6 694,40 − 1 530,28 = 5 164,13	5 164,12 × 0,12 × 0,5 = 309,85	1 931,94	1 931,94 − 309,85 = 1 622,09
5	5 164,13 − 1 622,09 = 3 542,04	3 542,04 × 0,12 × 0,5 = 212,52	1 931,94	1 931,94 − 212,52 = 1 719,42
6	3 542,04 − 1 719,42 = 1 822,62	1 822,62 × 0,12 × 0,5 = 109,36	1 931,94	1 931,94 − 109,36 = 1 822,58
	Total	2 091,67	11 591,64	9 499,97

* The interest due at half-year end is calculated as $I = PRT$, where P is the outstanding principal at the beginning of the half year, R is the interest rate per year, and T is the time in years, and in this case it is half years, therefore $T = 0{,}5$.

The following is a very important concept; make sure that you understand it 100%. Sometimes it happens that the interest rate changes during the given period, say after one year the interest rate drops to 11%. The first thing that you have to calculate is the present value of the loan at that stage. What is the present value of the loan after one year?

After one year, **two** payments have been made. The number of payments that remains, is the initial number of payments minus the number of payments already made, thus

$$\begin{aligned} &= 6 - 2 \\ &= 4. \end{aligned}$$

The present value of the loan after one year (when **four** payments remain) is

$$\begin{aligned} P &= R \times \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] \\ &= 1\,931{,}94 \left[\times \frac{(1+0{,}06)^4 - 1}{0{,}06(1+0{,}06)^4} \right] \\ &= 6\,694{,}38. \end{aligned}$$

The present value of the loan after one year is R6 694,38.

This number can also be seen in the amortisation table column for **outstanding principal** at the beginning of the third half year. This outstanding principal of R6 694,38 must be amortised over the remaining four half years at an interest rate of $(11 \div 2)\% = 5{,}5\%$ interest per half year.

The new payments are now

$$\begin{aligned} R &= P \times \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right] \\ &= 6\,694{,}38 \times \left[\frac{0{,}055(1+0{,}055)^4}{(1+0{,}055)^4 - 1} \right] \\ &= 1\,909{,}87. \end{aligned}$$

The payments are R1 909,87.

Activity 7.14

A teacher borrows R25 000 from a bank to remodel the dining room and kitchen of her home. The bank charges interest at 8% per annum, compounded quarterly over two years. What is her payment every three months? The following is given:

P = ____________

i = ____________

n = ____________ quarters

R = ?

The quarterly payments are

R = ________________________

= ________________________

= 3 412,74.

The quarterly payments are R3 412,74 each. Did you succeed?

Draw up an amortisation schedule for the loan.

The interest due at the end of the quarter is calculated as $I = PRT$, where P is the outstanding principal at the beginning of the quarter, R is the interest rate per year and T is the time in years. In this case it is quarters of years, therefore $T = 0{,}25$. Complete the table:

Quar- -ter	**Outstanding principal at beginning of quarter**	**Interest due at end of quarter (simple) $I = PRT^*$**	**Payment**	**Principal repaid**
1	25 000,00	$25\,000{,}00 \times 0{,}08 \times 0{,}25$ $= 500{,}00$	3 412,74	$3\,412{,}74 - 500{,}00$ $= 2\,912{,}74$
2	$25\,000{,}00 - 2\,912{,}74$ $= 22\,087{,}26$	$22\,087{,}26 \times 0{,}08 \times 0{,}25$ $=$	3 412,74	$3\,412{,}74 -$ $=$
3			3 412,74	$3\,412{,}74 -$ $=$
4			3 412,74	$3\,412{,}74 -$ $=$
5			3 412,74	$3\,412{,}74 -$ $=$
6			3 412,74	$3\,412{,}74 -$ $=$
7			3 412,74	$3\,412{,}74 -$ $=$
8			3 412,74	$3\,412{,}74 -$ $=$
	Total	2 301,96	27 301,92	24 999,96

After 15 months the interest rate rises to 9%. What is the present value of the loan after 15 months?

After 15 months, __________ payments have been made. Determine the number of payments that remains as

$$\begin{aligned} \text{no. of payments that remains} &= \text{initial no. of payments} - \text{no. of payments already made} \\ &= \_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_. \end{aligned}$$

The present value of the loan after 15 months (when ________ payments remain) is

$$\begin{aligned} P &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ &= 9\,841{,}94. \end{aligned}$$

The present value of the loan after 15 months is R9 841,94. Did you succeed?

What are the new quarterly payments?

The new interest rate is ____________________% per quarter.

The new payments are now

$$\begin{aligned} R &= \rule{5cm}{0.4pt} \\ &= \rule{5cm}{0.4pt} \\ &= 3\,429{,}37. \end{aligned}$$

The new payments are R3 429,37. Did you succeed?

APPENDIX A

Answers to activities

Component 1. Numbers and working with numbers

Activity 1.1:1
Simplifying gives

$$\begin{aligned}(50-(3+4\times 6)+15\div 5\times 3)-(36\div 4+2) &= (50-(3+24)+3\times 3)-(9+2)\\ &= (50-27+9)-11\\ &= (23+9)-11\\ &= 32-11\\ &= 21.\end{aligned}$$

Activity 1.1:2
Simplifying gives

$$\begin{aligned}(13+12)\times(20+(8\times 5-10)\div 5) &= 25\times(20+(40-10)\div 5)\\ &= 25\times(20+30\div 5)\\ &= 25\times(20+6)\\ &= 25\times 26\\ &= 650.\end{aligned}$$

Activity 1.2:1
Simplifying gives

$$\begin{aligned}3(z-x)-2x(z-y) &= 3\times(5-2)-2\times 2\times(5-4)\\ &= 3\times 3-4\times 1\\ &= 9-4\\ &= 5.\end{aligned}$$

Activity 1.2:2
Simplifying gives

$$\begin{aligned}\frac{1}{2}xyz+\frac{1}{4}(2z+2y+x) &= \frac{1}{2}\times 2\times 4\times 5+\frac{1}{4}(2\times 5+2\times 4+2)\\ &= 1\times 4\times 5+\frac{1}{4}\times(10+8+2)\\ &= 4\times 5+\frac{1}{4}\times 20\\ &= 20+5\\ &= 25.\end{aligned}$$

Activity 1.3:1
Three is added to the product of five and x:

$$3 + 5 \times x = 3 + 5x.$$

From this answer the sum of ten and x is subtracted:

$$\begin{aligned} 3 + 5x - (10 + x) &= 3 + 5x - 10 - x \\ &= 4x - 7. \end{aligned}$$

The final answer is equal to

$$y = 4x - 7.$$

Activity 1.3:2
The statement "*The variable e is equal to the sum of a and b, multiplied by the difference between c and d*," can be written as

$$e = (a + b) \times (c - d).$$

Activity 1.3:3
The monthly salary can be written as

$$P = 200d + 500.$$

Activity 1.4
Multiplying the numerator and denominator of $\frac{5}{13}$ by 3 gives

$$\frac{5}{13} = \frac{5 \times 3}{13 \times 3} = \frac{15}{39}.$$

Dividing the numerator and denominator of $\frac{15}{39}$ by 3 gives

$$\frac{15}{39} = \frac{15 \div 3}{39 \div 3} = \frac{5}{13}.$$

Thus, the value of b is equal to 39.

Activity 1.5:1
Simplifying gives

$$\begin{aligned} \left(5\frac{1}{4} + 2\frac{1}{5}\right) - \left(2\frac{1}{3} + 3\frac{1}{6}\right) &= \left(\frac{21}{4} + \frac{11}{5}\right) - \left(\frac{7}{3} + \frac{19}{6}\right) \\ &= \frac{105 + 44}{20} - \frac{14 + 19}{6} \\ &= \frac{149}{20} - \frac{33}{6} \\ &= \frac{447 - 330}{60} \\ &= \frac{117}{60} \\ &= \frac{39}{20} \\ &= 1\frac{19}{20}. \end{aligned}$$

Note: $\frac{21}{4} = \frac{21 \times 5}{4 \times 5} = \frac{105}{20}$; $\frac{11}{5} = \frac{11 \times 4}{5 \times 4} = \frac{44}{20}$; $\frac{7}{3} = \frac{7 \times 2}{3 \times 2} = \frac{14}{6}$.

Note: $\frac{149}{20} = \frac{149 \times 3}{20 \times 3} = \frac{447}{60}$; $\frac{33}{6} = \frac{33 \times 10}{6 \times 10} = \frac{330}{60}$.

Note: $\frac{117 \div 3}{60 \div 3}$.

Write the improper fraction as a mixed fraction.

Activity 1.5:2
The solution is

$$
\begin{aligned}
3\frac{3}{5} + 2\frac{1}{10} - 1\frac{3}{10} &= \frac{18}{5} + \frac{21}{10} - \frac{13}{10} \\
&= \frac{36}{10} + \frac{21}{10} - \frac{13}{10} \\
&= \frac{36 + 21 - 13}{10} \\
&= \frac{44}{10} \\
&= \frac{22}{5} \\
&= 4\frac{2}{5},
\end{aligned}
$$

Note: $\frac{18\times 2}{5\times 2} = \frac{36}{10}$.

Note: $\frac{44\div 2}{10\div 2} = \frac{22}{5}$.

or

$$
\begin{aligned}
3\frac{3}{5} + 2\frac{1}{10} - 1\frac{3}{10} &= \frac{18}{5} + \frac{21}{10} - \frac{13}{10} \\
&= \frac{18}{5} + \frac{21 - 13}{10} \\
&= \frac{18}{5} + \frac{8}{10} \\
&= \frac{18}{5} + \frac{4}{5} \\
&= \frac{22}{5} \\
&= 4\frac{2}{5}.
\end{aligned}
$$

Note: $\frac{8\div 2}{10\div 2} = \frac{4}{5}$.

Activity 1.6
Following method 1 gives

$$
\begin{aligned}
2\frac{4}{5} \times 1\frac{4}{21} \div \frac{5}{6} &= \frac{\cancel{14}^{\,2}}{5} \times \frac{25}{\cancel{21}_{\,3}} \times \frac{6}{5} \\
&= \frac{2}{5} \times \frac{25}{3} \times \frac{6}{5} \\
&= \frac{2}{\cancel{5}_{\,1}} \times \frac{\cancel{25}^{\,5}}{3} \times \frac{6}{5} \\
&= \frac{2}{1} \times \frac{5}{3} \times \frac{6}{5} \\
&= \frac{2}{1} \times \frac{\cancel{5}^{\,1}}{\cancel{3}_{\,1}} \times \frac{\cancel{6}^{\,2}}{\cancel{5}_{\,1}} \\
&= \frac{2}{1} \times \frac{1}{1} \times \frac{2}{1} \\
&= 4.
\end{aligned}
$$

Note: $14 \div 7 = 2$ and $21 \div 7 = 3$.

Note: $25 \div 5 = 5$ and $5 \div 5 = 1$.

Note: $5 \div 5 = 1$;
$6 \div 3 = 2$ and $3 \div 3 = 1$.

Following method 2 gives

$$\begin{aligned} 2\frac{4}{5} \times 1\frac{4}{21} \div \frac{5}{6} &= \frac{14}{5} \times \frac{25}{21} \times \frac{6}{5} \\ &= \frac{14 \times 25 \times 6}{5 \times 21 \times 5} \\ &= \frac{2\,100}{525} \\ &= 4. \end{aligned}$$

Activity 1.7:1(a)

The fraction $5\frac{3}{8}$ converted to a decimal is $= 5{,}375$.

Activity 1.7:1(b)

The fraction $\frac{7}{500}$ converted to a decimal is $= 0{,}014$.

Activity 1.7:1(c)

The sum of the fractions converted to a decimal is calculated as

$$\begin{aligned} \frac{186}{25} + 9\frac{27}{135} &= 7{,}44 + 9{,}2 \\ &= 16{,}64. \end{aligned}$$

Activity 1.7:2

The total is calculated as

$$\begin{aligned} 50 + 3 + \frac{5}{10} + \frac{5}{1\,000} &= 50 + 3 + 0{,}5 + 0{,}005 \\ &= 53{,}505. \end{aligned}$$

Activity 1.8:1

The decimal converted to a fraction is

$$\begin{aligned} 7{,}65 &= 7\frac{65}{100} \\ &= 7\frac{65 \div 5}{100 \div 5} \\ &= 7\frac{13}{20}. \end{aligned}$$

Activity 1.8:2

The decimal converted to a fraction is

$$\begin{aligned} 0{,}085 &= \frac{85}{1\,000} \\ &= \frac{17}{200}. \end{aligned}$$

Activity 1.9:1(a)

The number 15,6666 rounded off to three decimal places is 15,667.

Activity 1.9:1(b)
The number 14,327 rounded off to the nearest tenth is 14,3.

Activity 1.9:1(c)
The calculation rounded off to two decimal places is

$$\begin{aligned}(1{,}721 + 3{,}279) \times 5{,}46 \div 1{,}35 &= 5 \times 5{,}46 \div 1{,}35 \\ &= 20{,}22.\end{aligned}$$

Activity 1.9:2(a)
The fraction converted to decimal notation is

$$\begin{aligned}\frac{5}{12} &= 0{,}4166666... \\ &= 0{,}41\dot{6}.\end{aligned}$$

Activity 1.9:2(b)
The fraction converted to decimal notation is

$$\begin{aligned}\frac{65}{99} &= 0{,}656565... \\ &= 0{,}\dot{6}\dot{5}.\end{aligned}$$

Activity 1.9:3
The numbers ordered form the largest to the smallest is

$$6{,}6; \quad 6{,}06; \quad 0{,}66; \quad 0{,}6; \quad 0{,}06.$$

Activity 1.10:1
Simplifying gives

$$\begin{aligned}(4+1)^3 - (4^3 + 1^3) &= 5^3 - (64+1) \\ &= 125 - 65 \\ &= 60.\end{aligned}$$

Activity 1.10:2
Simplifying gives

$$\begin{aligned}\sqrt[3]{4\frac{12}{125}} &= \sqrt[3]{\frac{512}{125}} \\ &= \sqrt[3]{\frac{8\times 8\times 8}{5\times 5\times 5}} \\ &= \frac{8}{5} \\ &= 1\frac{3}{5},\end{aligned}$$

or

$$\begin{aligned}\sqrt[3]{4\frac{12}{125}} &= \sqrt[3]{\frac{512}{125}} \\ &= \frac{512^{\frac{1}{3}}}{125^{\frac{1}{3}}} \\ &= \frac{8}{5} \\ &= 1\frac{3}{5}.\end{aligned}$$

Activity 1.10:3
Simplifying gives

$$\begin{aligned}4^2 \times 3^3 \div \sqrt{144} &= 4 \times 4 \times 3 \times 3 \times 3 \div 12 \\ &= 16 \times 27 \div 12 \\ &= 432 \div 12 \\ &= 36.\end{aligned}$$

Activity 1.10:4
Simplifying gives

$$\begin{aligned}\sqrt[3]{10^2 + 5^2} + 4\sqrt{9} + (\sqrt[3]{6})^3 &= \sqrt[3]{10 \times 10 + 5 \times 5} + 4\sqrt{3 \times 3} + \sqrt[3]{6} \times \sqrt[3]{6} \times \sqrt[3]{6} \\ &= \sqrt[3]{100 + 25} + 4 \times 3 + \sqrt[3]{6 \times 6 \times 6} \\ &= \sqrt[3]{125} + 12 + 6 \\ &= 5 + 12 + 6 \\ &= 23.\end{aligned}$$

Activity 1.11:1
The ratio of *shirts* to *buttons* is 12 to 156 or $\frac{12}{156}$. Simplify this fraction to $\frac{1}{13}$.
Hence, the ratio of *shirts* to *buttons* is 1 to 13 which can be written as

$$1 : 13$$

or

$$0{,}0769 : 1.$$

Note: $\frac{1}{13} = 0{,}0769$.

Activity 1.11:2

The ratio John : Jack : Jason of magazines sold can be written as 25 : 38 : 41.

Add 25, 38 and 41 to obtain a total of 104 magazines that were sold.

For John: $\frac{25}{104} \times 572{,}00 = 137{,}50$. John's part of the profit is R137,50.

For Jack: $\frac{38}{104} \times 572{,}00 = 209{,}00$. Jack's part of the profit is R209,00.

For Jason: $\frac{41}{104} \times 572{,}00 = 225{,}50$. Jason's part of the profit is R225,50.

Activity 1.11:3
First calculate the youngest son's fraction of the share. His fraction of the share is

$$\begin{aligned} 1-\left(\frac{2}{5}+\frac{3}{8}\right) &= 1-\left(\frac{2\times 8+3\times 5}{40}\right) \\ &= \frac{40}{40}-\frac{16+15}{40} \\ &= \frac{40}{40}-\frac{31}{40} \\ &= \frac{9}{40}. \end{aligned}$$

The oldest son's fraction of the share is $\frac{2}{5}=\frac{16}{40}$.

The second son's fraction of the share is $\frac{3}{8}=\frac{15}{40}$.

The youngest son's fraction of the share is $\frac{9}{40}$.

The money is divided in the ratio oldest : second : youngest or 16 : 15 : 9.
The value of x is calculated as

$$\begin{aligned} \frac{9}{40}\times x &= 1\,350 \\ \frac{9}{40}\times\frac{40}{9}\times x &= \frac{1\,350}{1}\times\frac{40}{9} \\ x &= \frac{54\,000}{9} \\ &= 6\,000. \end{aligned}$$

The total amount of money the father gives to his sons, is R6 000.

The oldest son: $\frac{16}{40}\times 6\,000 = 2\,400$. His share of the money is R2 400.

The second son: $\frac{15}{40}\times 6\,000 = 2\,250$. His share of the money is R2 250.

(Check: R2 400 + R2 250 + R1 350 = R6 000.)

Activity 1.11:4
The ratio is

	Themba	:	Joyce
	50	:	60
or	$\frac{50}{60}$	:	$\frac{60}{60}$
or	$\frac{5}{6}$	:	1.

Joyce receives R5 400. Thus, for Themba

$$\frac{5}{6}\times 5\,400 = 4\,500.$$

Themba will receive R4 500.

$\left(\text{Check: } \frac{4\,500}{5\,400}=\frac{5}{6}.\right)$

Activity 1.11:5
The ratio required is

$$28\,000 \quad \text{to} \quad 35$$

or

$$\frac{28\,000}{35} \quad \text{to} \quad \frac{35}{35}$$

which is

$$800 : 1.$$

He will pay R800 per machine, and for 60 machines

$$800 \times 60 = 48\,000.$$

He will pay R48 000 for 60 machines.

Activity 1.12:1(a)
The decimal 0,684 written as a percentage is

$$0{,}684 \times 100 = 68{,}4\%.$$

Activity 1.12:1(b)
The percentage $37\frac{1}{2}\%$ written as a decimal is

$$\begin{aligned} 37\frac{1}{2}\% &= \frac{75}{2}\% \\ &= \frac{75}{2} \div 100 \\ &= \frac{75}{2} \times \frac{1}{100} \\ &= \frac{75}{200} \\ &= 0{,}375. \end{aligned}$$

Activity 1.12:1(c)

The fraction $\frac{13}{80}$ written as a percentage is

$$\begin{aligned} \frac{13}{80} \times \frac{100}{1} &= \frac{1\,300}{80} \\ &= \frac{65}{4} \\ &= 16\frac{1}{4} \\ &= 16{,}25\%. \end{aligned}$$

Note: $\frac{1\,300 \div 20}{80 \div 20} = \frac{65}{4}$.

Activity 1.12:1(d)

The percentage $6\frac{1}{4}\%$ written as a fraction is

$$
\begin{aligned}
6\frac{1}{4}\% &= \frac{25}{4}\% \\
&= \frac{25}{400} \\
&= \frac{1}{16}.
\end{aligned}
$$

Note: $\frac{25}{4} \div 100 = \frac{25}{4} \times \frac{1}{100}$.

Note: $\frac{25 \div 25}{400 \div 25} = \frac{1}{16}$.

Activity 1.12:2(a)
The percentage money that is left is

$$
\begin{aligned}
100 - (65 + 10) &= 100 - 75 \\
&= 25\%.
\end{aligned}
$$

The amount of money that is left is

$$
\begin{aligned}
25\% \text{ of } 1\,485 &= \frac{25}{100} \times 1\,485 \\
&= 0{,}25 \times 1\,485 \\
&= 371{,}25.
\end{aligned}
$$

There is R371,25 left.

Activity 1.12:2(b)
A discount of 20% means that Peter pays only 80% (100% − 20%) of the marked price, thus

$$
\begin{aligned}
80\% \text{ of the } \textit{price} &= 2\,500 \\
0{,}8 \times \textit{price} &= 2\,500 \\
\textit{price} &= \frac{2\,500}{0{,}8} \\
&= 3\,125.
\end{aligned}
$$

The marked price is R3 125.

Activity 1.12:3
The percentage of people that show up at Pumi's birthday party is

$$
\begin{aligned}
\frac{15}{24} \times \frac{100}{1} &= 0{,}625 \times 100 \\
&= 62{,}5\%.
\end{aligned}
$$

Alternatively you can say that the ratio of people that show up is 15 : 24 which reduces to

$$\frac{15}{24} : \frac{24}{24} \text{ or } 0{,}625 : 1.$$

Multiplying by a hundred gives 62,5%.

Activity 1.13
The original price was R215,00 and the new price is R247,25.

Calculate the *increase* in price as

$$\begin{aligned} \textit{increase} &= \text{new price} - \text{original price} \\ &= 247{,}25 - 215{,}00 \\ &= 32{,}25. \end{aligned}$$

The *increase* in price is R32,25.

Calculate the percentage *increase* as

$$\begin{aligned} \text{percentage } \textit{increase} &= \frac{\text{change}}{\text{original}} \times 100 \\ &= \frac{32{,}25}{215{,}00} \times 100 \\ &= 15\%. \end{aligned}$$

Activity 1.14
The mark-up % on cost is calculated as

$$\begin{aligned} \text{mark-up \% on cost} &= \frac{\text{gross profit}}{\text{cost price}} \times 100 \\ &= \frac{\text{selling price} - \text{cost price}}{\text{cost price}} \times 100 \\ &= \frac{300 - 200}{200} \times 100 \\ &= \frac{100}{200} \times 100 \\ &= 50\%. \end{aligned}$$

The gross profit is 50% of the cost price. (The trader has marked up the cost price by 50%.) Calculate the gross margin as

$$\begin{aligned} \text{gross margin} &= \frac{\text{gross profit}}{\text{selling price}} \times 100 \\ &= \frac{300 - 200}{300} \times 100 \\ &= \frac{100}{300} \times 100 \\ &= 33\frac{1}{3}\%. \end{aligned}$$

The gross profit is $33\frac{1}{3}\%$ of the selling price.

Activity 1.15
The following is determined from the question:

- If the gross margin is given as 30% then the cost price is determined as

$$\begin{aligned} \text{cost price} &= \text{selling price} - \text{gross margin} \\ &= 100 - 30 \\ &= 70. \end{aligned}$$

  The cost price is 70% of the selling price.

- Selling price = R390;
- Cost price = x.

The relationship $\dfrac{\text{cost price}}{\text{selling price}}$ can be written as $\dfrac{x}{390} = \dfrac{70}{100}$.

Multiply both sides by 390 and solve for x, thus

$$\begin{aligned}\frac{x \times 390}{390} &= \frac{70}{100} \times 390 \\ x &= \frac{70}{100} \times 390 \\ &= 273{,}00.\end{aligned}$$

The cost price is R273,00.

Activity 1.16:1
The VAT on an invoice with a gross amount of R5 157,36 is calculated as

$$\begin{aligned}\text{VAT} &= \frac{14}{114} \times \text{gross amount} \\ &= \frac{14}{114} \times 5\,157{,}36 \\ &= 633{,}36.\end{aligned}$$

The amount of VAT is R633,36.

Activity 1.16:2
The price exclusive of VAT is the net amount and is calculated as

$$\begin{aligned}\text{net amount} &= \frac{100}{114} \times \text{gross amount} \\ &= \frac{100}{114} \times 969 \\ &= 850{,}00.\end{aligned}$$

The net amount is R850,00.

Activity 1.16:3
The net amount is given as R500, thus the gross amount (inclusive price) is calculated as

$$\begin{aligned}\text{gross amount} &= 500 \times \frac{114}{100} \\ &= 570.\end{aligned}$$

The inclusive price is R570.
The VAT amount is calculated as

$$\begin{aligned}\text{VAT} &= \text{gross amount} - \text{net amount} \\ &= 570 - 500 \\ &= 70.\end{aligned}$$

The amount of VAT is R70.

Activity 1.17:1(a)
The values are $x_1 = 5$; $x_2 = 3$; $x_3 = 2$ and $x_4 = 6$. Simplifying gives

$$\begin{aligned}\sum_{i=1}^{4} x_i^2 &= x_1^2 + x_2^2 + x_3^2 + x_4^2 \\ &= 5^2 + 3^2 + 2^2 + 6^2 \\ &= 25 + 9 + 4 + 36 \\ &= 74.\end{aligned}$$

Activity 1.17:1(b)
The values are $x_1 = 5$; $x_2 = 3$; $x_3 = 2$ and $x_4 = 6$. Simplifying gives

$$\begin{aligned}\left(\sum_{i=1}^{4} x_i\right)^2 &= (x_1 + x_2 + x_3 + x_4)^2 \\ &= (5 + 3 + 2 + 6)^2 \\ &= 16^2 \\ &= 256.\end{aligned}$$

Activity 1.17:1(c)
The values are $x_1 = 5$; $x_2 = 3$ and $x_3 = 2$. Simplifying gives

$$\begin{aligned}\sum_{i=1}^{3} x_i &= x_1 + x_2 + x_3 \\ &= 5 + 3 + 2 \\ &= 10.\end{aligned}$$

Activity 1.17:2
Simplifying gives

$$\begin{aligned}12 \div (-3) \times 5 + 6 - (5 - 8) &= -4 \times 5 + 6 - (-3) \\ &= -20 + 6 + 3 \\ &= -14 + 3 \\ &= -11.\end{aligned}$$

Note: $12 \div (-3) = -4; \quad (5 - 8) = (-3).$

Note: $-4 \times 5 = -20; \quad -(-3) = 3.$

Activity 1.17:3
Each of the ten letters can be posted in any of the five boxes.

The first letter has five options.
The second letter has five options.
The tenth letter has five options.

The total number of options is calculated as

$$\begin{aligned}\text{number of options} &= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \\ &= 5^{10} \\ &= 9\,765\,625.\end{aligned}$$

Activity 1.17:4
This is a permutation. The number of ways the members can be elected is

$$\begin{aligned} {}_{18}\mathrm{P}_3 &= \frac{18!}{(18-3)!} \\ &= \frac{18!}{15!} \\ &= \frac{18 \times 17 \times 16 \times 15!}{15!} \\ &= 18 \times 17 \times 16 \\ &= 4\,896. \end{aligned}$$

Activity 1.17:5
This is a combination because nothing is mentioned about the order in which he visits the cities. The number of ways in which he can schedule his route is

$$\begin{aligned} {}_{12}\mathrm{C}_4 &= \frac{12!}{(12-4)! \times 4!} \\ &= \frac{12!}{8! \times 4!} \\ &= \frac{12 \times 11 \times 10 \times 9 \times 8!}{8! \times 4!} \\ &= \frac{12 \times 11 \times 10 \times 9 \times \cancel{8!}}{\cancel{8!} \times 4!} \\ &= \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} \\ &= \frac{11\,880}{24} \\ &= 495. \end{aligned}$$

Activity 1.17:6
Simplifying gives

$$\begin{aligned} \frac{6!\,7! - 6!\,5!}{5!\,4!} &= \frac{6!\,7!}{5!\,4!} - \frac{6!\,5!}{5!\,4!} \\ &= \frac{6 \times 5! \times 7 \times 6 \times 5 \times 4!}{5!\,4!} - \frac{6 \times 5! \times 5 \times 4!}{5!\,4!} \\ &= \frac{6 \times \cancel{5!} \times 7 \times 6 \times 5 \times \cancel{4!}}{\cancel{5!}\cancel{4!}} - \frac{6 \times \cancel{5!} \times 5 \times \cancel{4!}}{\cancel{5!}\cancel{4!}} \\ &= 6 \times 7 \times 6 \times 5 - 6 \times 5 \\ &= 1\,260 - 30 \\ &= 1\,230. \end{aligned}$$

If you find this explanation difficult to follow, you can study the following detailed explanation of the same question:

Simplifying gives

$$\frac{6!\,7! - 6!\,5!}{5!\,4!}$$
$$= \frac{6!\,7!}{5!\,4!} - \frac{6!\,5!}{5!\,4!}$$
$$= \frac{6\times5\times4\times3\times2\times1\times7\times6\times5\times4\times3\times2\times1}{5\times4\times3\times2\times1\times4\times3\times2\times1} - \frac{6\times5\times4\times3\times2\times1\times5\times4\times3\times2\times1}{5\times4\times3\times2\times1\times4\times3\times2\times1}$$
$$= \frac{6\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}\times7\times6\times5\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}{\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}} - \frac{6\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}\times5\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}{\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}$$
$$= 6\times7\times6\times5 - 6\times5$$
$$= 1\,260 - 30$$
$$= 1\,230.$$

Activity 1.18:1(a)
Add the areas of the two rectangles together to obtain the area of the floor of the swimming pool as

$$\begin{aligned}\text{area} &= \text{length}\times\text{width} + \text{length}\times\text{width}\\ &= 12{,}5\times6 + (12{,}5-(4+4))\times3\\ &= 12{,}5\times6 + (12{,}5-(8))\times3\\ &= 12{,}5\times6 + 4{,}5\times3 && \text{Note: m} \times \text{m} + \text{m} \times \text{m.}\\ &= 75 + 13{,}5 && \text{Note: m}^2 + \text{m}^2.\\ &= 88{,}5. && \text{Note: m}^2.\end{aligned}$$

The area of the floor of the swimming pool is 88,5 m^2.

Activity 1.18:1(b)
The volume is calculated as

$$\begin{aligned}\text{volume} &= \text{length}\times\text{width}\times\text{height} + \text{length}\times\text{width}\times\text{height}\\ &= 12{,}5\times6\times3{,}5 + 4{,}5\times3\times3{,}5 && \text{Note: m} \times \text{m} \times \text{m} + \text{m} \times \text{m} \times \text{m.}\\ &= 262{,}5 + 47{,}25 && \text{Note: m}^3 + \text{m}^3.\\ &= 309{,}75. && \text{Note: m}^3.\end{aligned}$$

The volume is 309,75 m^3.

Alternatively, we know that the area of the floor of the swimming pool is 88,5 m^2. The volume is calculated as

$$\begin{aligned}\text{volume} &= \text{area}\times\text{height}\\ &= 88{,}5\times3{,}5 && \text{Note: m}^2 \times \text{m.}\\ &= 309{,}75. && \text{Note: m}^3.\end{aligned}$$

The volume is 309,75 m^3.

Activity 1.18:1(c)
We know that $1\,l = 10^3\text{ cm}^3$, thus

$$\begin{aligned} 1\,l &= 1\,000\text{ cm}^3 \\ &= 10\text{ cm} \times 10\text{ cm} \times 10\text{ cm} \\ &= 0{,}1\text{ m} \times 0{,}1\text{ m} \times 0{,}1\text{ m} && \text{Note: } 10\text{ cm} = 0{,}1\text{ m.} \\ &= 0{,}001\text{ m}^3. \end{aligned}$$

Therefore,

$$\begin{aligned} 1\,000\,l &= 1\text{ m}^3 && \text{Multiply both sides by } 1\,000. \\ 1\text{ k}l &= 1\text{ m}^3 && \text{Note: } 1\,000\ l = 1\text{ k}l. \\ 309{,}75\text{ k}l &= 309{,}75\text{ m}^3. && \text{Multiply both sides by } 309{,}75. \end{aligned}$$

Thus, 309,75 kl of water is necessary to fill the pool.

Activity 1.18:2
The total length of the fence is calculated as

$$\begin{aligned} \text{length} &= \text{length of fence around area} - \text{length of gate} \\ &= 2 \times 15 + 2 \times 6{,}5 - 1{,}5 \\ &= 30 + 13 - 1{,}5 && \text{The unit is metre (m).} \\ &= 41{,}5. \end{aligned}$$

The total length of the fence is 41,5 m.
The cost of the fence without the gate is calculated as

$$\begin{aligned} \text{cost} &= \text{length of fence} \times \text{cost of fence per metre} \\ &= 41{,}5 \times 125{,}35 && \text{Note: m} \times \text{R/m} = \text{m} \times \text{R} \times \text{m}^{-1} = \text{R.} \\ &= 5\,202{,}03. \end{aligned}$$

The cost of the fence without the gate is R5 202,03.
The total cost of the fence is calculated as

$$\begin{aligned} \text{total cost} &= \text{cost of fence} + \text{cost of gate} \\ &= 5\,202{,}03 + 375{,}44 \\ &= 5\,577{,}47. \end{aligned}$$

The total cost of the fence is R5 577,47.

Activity 1.19:1(a)
Simplifying gives

$$\begin{aligned} -12 \div (-4 - (-2)) + 24 \div (-6) \times 5 + (-10 - 4 \times (-2)) &= -12 \div (-4 + 2) - 4 \times 5 + (-10 + 8) \\ &= -12 \div (-2) - 20 + (-2) \\ &= 6 - 20 - 2 \\ &= -14 - 2 \\ &= -16. \end{aligned}$$

Activity 1.19:1(b)
Simplifying gives

$$\begin{aligned} -10 \times (10+2) &= -10 \times 12 \\ &= -120. \end{aligned}$$

Activity 1.19:1(c)
Simplifying gives

$$\begin{aligned} (-10-1) \times (10+1) &= -11 \times 11 \\ &= -121. \end{aligned}$$

Activity 1.19:1(d)
Simplifying gives

$$\begin{aligned} (10-1) \times (3-10) &= 9 \times -7 \\ &= -63. \end{aligned}$$

Activity 1.19:1(e)
Simplifying gives

$$\begin{aligned} -2^2 + (-2)^3 + \sqrt{25-16} - 7(-3)(-2) + 4 + 3 \times (-10) &= -(2 \times 2) + (-2)(-2)(-2) \\ &\quad + \sqrt{9} + 21(-2) + 4 - 30 \\ &= -4 + 4(-2) + 3 - 42 + 4 - 30 \\ &= -4 - 8 + 3 - 42 + 4 - 30 \\ &= -77. \end{aligned}$$

Activity 1.19:1(f)
Simplifying gives

$$\begin{aligned} (-7+9)^3 + \sqrt{13 - (-2 \times 6)} - \frac{-75}{15} &= 2^3 + \sqrt{13 - (-12)} - (-5) \\ &= 8 + \sqrt{13+12} + 5 \\ &= 8 + \sqrt{25} + 5 \\ &= 8 + 5 + 5 \\ &= 18. \end{aligned}$$

Activity 1.19:2
Every weekday, except on a Friday, his travelling time is 1 hour and 15 minutes. When converted to minutes it gives

$$60 + 15 = 75.$$

He takes 75 minutes on Mondays to Thursdays to travel to work.
On a Friday his travelling time is reduced by 16%. Converted to minutes gives

$$\begin{aligned} 16\% \text{ of } 75 &= \frac{16}{100} \times 75 \\ &= 12. \end{aligned}$$

Thus, 16% of 75 minutes is 12 minutes. On a Friday is travelling time is calculated as

$$75 - 12 = 63.$$

His travelling time on a Friday is 63 minutes.

Activity 1.19:3
The ratio

$$\text{Frank} : \text{Edgar} : \text{Trevor}$$

can be written as

$$450 : 750 : (1\,500 - (450 + 750))$$

or

$$450 : 750 : 300.$$

Divide each term by 50, then

$$9 : 15 : 6.$$

Further divide each term by three to get

$$3 : 5 : 2.$$

Add three, five and two together to obtain ten parts in total.

For Frank: $\dfrac{3}{10} \times 3\,500 = 1\,050$. His share is R1 050.

For Edgar: $\dfrac{5}{10} \times 3\,500 = 1\,750$. His share is R1 750.

For Trevor: $\dfrac{2}{10} \times 3\,500 = 700$. His share is R700.

Component 2. Collection, presentation and description of data

Activity 2.1:1
The random sample:

Sampling unit	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Practitioner no.	253	489	230	313	287	844	348	134	794	64	437	41	355	207	194

Activity 2.1:2(a)
The size of each of the strata:

Stratum	Size of stratum	Sample size
Students	25	$\frac{25}{35} \times 6 = 4{,}29 \approx 4$
Faculty members	10	$\frac{10}{35} \times 6 = 1{,}71 \approx 2$
Total	35	6

Therefore, four students and two faculty members can go to the convention.

Activity 2.1:2(b)
The stratum of **Students**.

The sample consists of the students labeled

14, 19, 10 and 8.

Therefore Van Dyk, Erasmus, Viljoen and Nkosi will go to the convention.

Activity 2.1:2(b)
The stratum of **Faculty members**.

The sample consists of the faculty members labeled

6 and 4.

Therefore Sebola and Brown will go to the convention.

Activity 2.1:2(c)
The following members will go to the convention:

Van Dyk, Erasmus, Viljoen, Nkosi, Sebola and Brown.

Activity 2.2:1
This is continuous quantitative data.

Activity 2.2:2
This is qualitative data.

Activity 2.2:3
This is discrete quantitative data.

Activity 2.2:4
This is continuous quantitative data.

Activity 2.2:5
This is qualitative data.

Activity 2.2:6
This is continuous quantitative data.

Activity 2.2:7
This is discrete quantitative data.

Activity 2.2:8
This is qualitative data.

Activity 2.2:9
This is discrete quantitative data.

Activity 2.2:10
This is qualitative data.

Activity 2.3:1(a)
The data is

366	155	326	187	245	270	319	223	212	190
193	247	255	235	300	311	180	333	289	245
328	201	260	259	263	313	151	322	270	299

The range of the data is

$$\begin{aligned} R &= \text{maximum value} - \text{minimum value} \\ &= 366 - 151 \\ &= 215. \end{aligned}$$

Activity 2.3:1(b)
The formula to use is $K = \sqrt{n}$. The variable, n, is equal to 30. The number of intervals is

$$\begin{aligned} K &= \sqrt{30} \\ &= 5{,}48. \end{aligned}$$

We decided to use six intervals. Thus, $K = 6$.

Activity 2.3:1(c)
The width of the intervals is

$$
\begin{aligned}
c &= \frac{R}{K} \\
&= \frac{215}{6} \\
&= 35{,}83 \\
&\approx 36.
\end{aligned}
$$

Activity 2.3:1(d)
Determine the interval limits.
The minimum value is 151. Half a unit less is 150,5.
The lower limit of the first interval is therefore 150,5.
To obtain the upper limit of the first interval we add the interval width to the lower limit: 150,5 + 36 = 186,5.
Therefore, the first interval is 150,5 – 186,5.
Now, the second interval starts with 186,5 and also has a width of 36.
Therefore, its upper limit is 186,5 + 36 = 222,5.
Thus, the intervals are

$$
\begin{array}{ccc}
150{,}5 & - & 186{,}5 \\
186{,}5 & - & 222{,}5 \\
222{,}5 & - & 258{,}5 \\
258{,}5 & - & 294{,}5 \\
294{,}5 & - & 330{,}5 \\
330{,}5 & - & 366{,}5.
\end{array}
$$

Activity 2.3:1(e)
Group the data into the intervals. After every value has been fitted into an interval, it will look as follows:

150,5 – 186,5	|||
186,5 – 222,5	卌
222,5 – 258,5	卌 |
258,5 – 294,5	卌 |
294,5 – 330,5	卌 |||
330,5 – 366,5	||

The frequency table:

Interval		Frequency
150,5 – 186,5	|||	3
186,5 – 222,5	~~||||~~	5
222,5 – 258,5	~~||||~~ |	6
258,5 – 294,5	~~||||~~ |	6
294,5 – 330,5	~~||||~~ |||	8
330,5 – 366,5	||	2
		30

Activity 2.3:2(a)
Acryl amide levels higher than 294 are represented by the intervals

294,5 – 330,5 and 330,5 – 366,5.

Add eight and two together to obtain the sum of the frequencies of these two intervals as ten. Thus, the percentage of franchises that had acryl amide levels higher than 294 is

$$\frac{10}{30} \times 100 = 33{,}33\%.$$

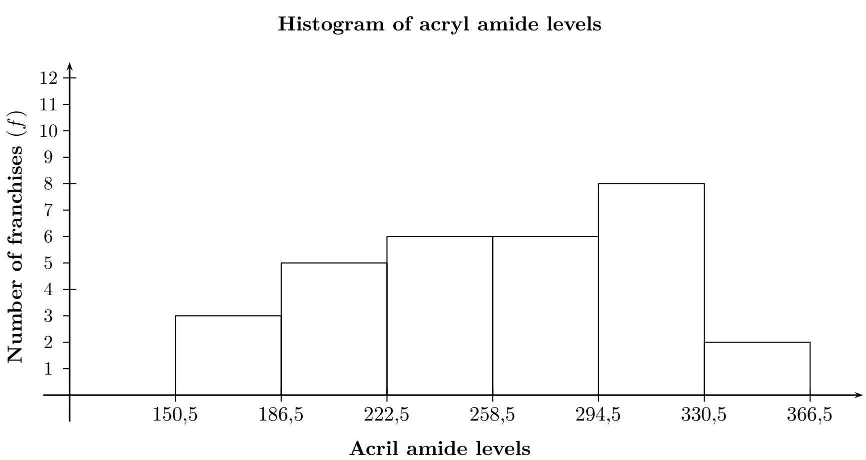
Activity 2.3:2(b)
Acryl amide levels lower than 259 are represented by the intervals

150,5 – 186,5, 186,5 – 222,5 and 222,5 – 258,5.

Add three, five and six together to obtain the sum of the frequencies of these three intervals as fourteen. Thus, the percentage of franchises that had acryl amide levels lower than 259 is

$$\frac{14}{30} \times 100 = 46{,}67\%.$$

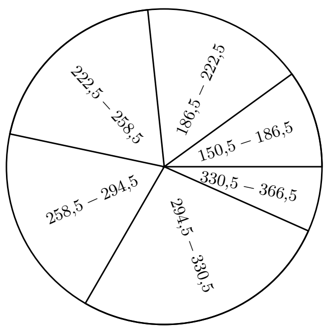
Activity 2.3:3



Activity 2.3:4

Interval	Frequency	Percentage of area
150,5 – 186,5	3	$\frac{3}{30} \times 100 = 10\%$
186,5 – 222,5	5	$\frac{5}{30} \times 100 = 16{,}67\%$
222,5 – 258,5	6	$\frac{6}{30} \times 100 = 20\%$
258,5 – 294,5	6	$\frac{6}{30} \times 100 = 20\%$
294,5 – 330,5	8	$\frac{8}{30} \times 100 = 26{,}67\%$
330,5 – 366,5	2	$\frac{2}{30} \times 100 = 6{,}67\%$
	30	100,01

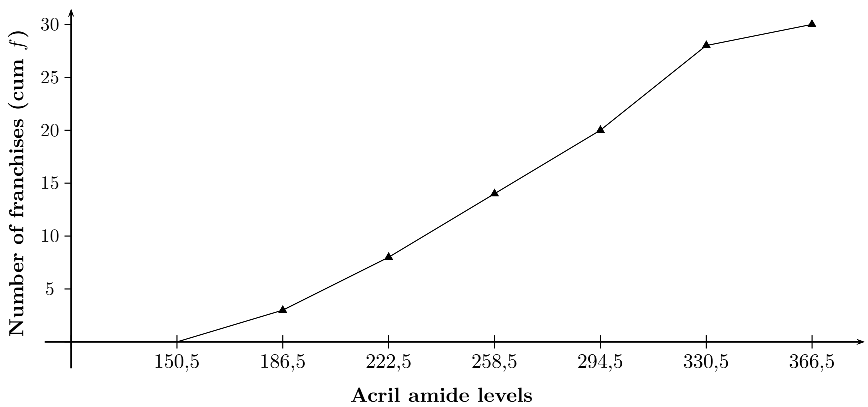
Pie chart for acryl amide levels



Activity 2.3:5

Upper limit	Cumulative frequency
$< 186{,}5$	3
$< 222{,}5$	$3 + 5 = 8$
$< 258{,}5$	$3 + 5 + 6 = 14$
$< 294{,}5$	$3 + 5 + 6 + 6 = 20$
$< 330{,}5$	$3 + 5 + 6 + 6 + 8 = 28$
$< 366{,}5$	$3 + 5 + 6 + 6 + 8 + 2 = 30$

Cumulative frequency polygon of acryl amide levels



Activity 2.3:6
The data is the following:

78	82	96	74	52	68	82	78	74	76
88	62	66	76	76	84	95	91	58	86

The stem-and-leaf diagram is as follows:

Stem	**Leaf**	**Frequency**
5	2 8	2
6	8 2 6	3
7	8 4 8 4 6 6 6	7
8	2 2 8 4 6	5
9	6 5 1	3

The sorted stem-and-leaf diagram is as follows:

Stem	**Leaf**	**Frequency**
5	2 8	2
6	2 6 8	3
7	4 4 6 6 6 8 8	7
8	2 2 4 6 8	5
9	1 5 6	3

Activity 2.4
The data is the following:

11 20 28 14 19 28 30 23 25 17 15 26 9 22 28

The data written in ascending order is as follows:

No.	**Data ordered from smallest to largest** (x_i)
1	9
2	11
3	14
4	15
5	17
6	19
7	20
8	22
9	23
10	25
11	26
12	28
13	28
14	28
15	30
	$\sum_{i=1}^{15} x_i = 315$

The number of observations is $n = 15$.

Activity 2.4:1
The mean is

$$\begin{aligned}\overline{x} &= \frac{\sum_{i=1}^{15} x_i}{n} \\ &= \frac{315}{15} \\ &= 21{,}0.\end{aligned}$$

Activity 2.4:2
The position of the median is

$$\begin{aligned}\frac{n+1}{2} &= \frac{15+1}{2} \\ &= 8.\end{aligned}$$

The median is the 8th value in the ordered data set.

The value of the median is 22.

Half of the reservations were fewer than 22 per day, while the other half of the reservations were more than 22 per day.

Activity 2.4:3
The mode is 28 because it is the value that occurs most often (three times).

To interpret, one can say that the number of observations that occurred most frequently was 28.

Activity 2.5
The sample is the following:

Time in minutes	**Number of people frequency** (f)
15,5 − 21,5	2
21,5 − 27,5	6
27,5 − 33,5	8
33,5 − 39,5	4
39,5 − 45,5	4
45,5 − 51,5	1
	25

The completed table is as follows:

Interval	**Frequency** (f)	**Midpoints** (x)	$f \times x$	**Cumulative frequency**
15,5 − 21,5	2	$(15{,}5 + 21{,}5) \div 2 = 18{,}5$	$2 \times 18{,}5 = 37$	2
21,5 − 27,5	6	$(21{,}5 + 27{,}5) \div 2 = 24{,}5$	$6 \times 24{,}5 = 147$	$2 + 6 = 8$
27,5 − 33,5	8	$(27{,}5 + 33{,}5) \div 2 = 30{,}5$	$8 \times 30{,}5 = 244$	$8 + 8 = 16$
33,5 − 39,5	4	$(33{,}5 + 39{,}5) \div 2 = 36{,}5$	$4 \times 36{,}5 = 146$	$16 + 4 = 20$
39,5 − 45,5	4	$(39{,}5 + 45{,}5) \div 2 = 42{,}5$	$4 \times 42{,}5 = 170$	$20 + 4 = 24$
45,5 − 51,5	1	$(45{,}5 + 51{,}5) \div 2 = 48{,}5$	$1 \times 48{,}5 = 48{,}5$	$24 + 1 = 25$
	$\sum f = 25$		$\sum fx = 792{,}5$	

Activity 2.5:1
The mean is

$$\begin{aligned}\overline{x} &= \frac{\sum fx}{n} \\ &= \frac{792{,}5}{25} \\ &= 31{,}7.\end{aligned}$$

The mean time to travel to work is 31,7 minutes.

Activity 2.5:2

To determine the median interval, we find $\frac{n}{2} = \frac{25}{2} = 12{,}5$. The interval where the cumulative frequency is equal to, or exceeds 12,5 for the first time is the interval 27,5 − 33,5 with a cumulative frequency of $f = 16$. Thus the median interval is the interval **27,5 − 33,5**.

Activity 2.5:3

The modal interval is the interval with the highest frequency (f). In this case it is the interval **27,5 − 33,5** with a frequency of $f = 8$.

Activity 2.6
The completed table is as follows:

No.	**Data ordered from smallest to largest** (x_i)	$(x-\overline{x})^2 = (x-21)^2$
1	9	$(9-21)^2 = (-12)^2 = 144$
2	11	$(11-21)^2 = (-10)^2 = 100$
3	14	$(14-21)^2 = (-7)^2 = 49$
4	15	$(15-21)^2 = (-6)^2 = 36$
5	17	16
6	19	4
7	20	1
8	22	1
9	23	4
10	25	16
11	26	25
12	28	49
13	28	49
14	28	49
15	30	81
	$\sum_{i=1}^{15} x_i = 315$	$\sum(x-\overline{x})^2 = 624$

The variance of the data is

$$\begin{aligned}S^2 &= \frac{\sum(x-\overline{x})^2}{n-1} \\ &= \frac{624}{14} \\ &= 44{,}57.\end{aligned}$$

The standard deviation is

$$\begin{aligned} S &= \sqrt{44{,}57} \\ &= 6{,}68. \end{aligned}$$

Activity 2.7:1
The completed table is as follows:

Interval	**Frequency** (f)	**Midpoint** (x)	$\sum(x-\overline{x})^2 f = (x-31{,}7)^2 f$
15,5 – 21,5	2	18,5	$(18{,}5-31{,}7)^2 \times 2 = 348{,}48$
21,5 – 27,5	6	24,5	$(24{,}5-31{,}7)^2 \times 6 = 311{,}04$
27,5 – 33,5	8	30,5	$(30{,}5-31{,}7)^2 \times 8 = 11{,}52$
33,5 – 39,5	4	36,5	$(36{,}5-31{,}7)^2 \times 4 = 92{,}16$
39,5 – 45,5	4	42,5	$(42{,}5-31{,}7)^2 \times 4 = 466{,}56$
45,5 – 51,5	1	48,5	$(48{,}5-31{,}7)^2 \times 1 = 282{,}24$
	$\sum f = 25$		$\sum(x-\overline{x})^2 f = 1\,512{,}00$

The variance of the data is

$$\begin{aligned} S^2 &= \frac{\sum(x-\overline{x})^2 f}{n-1} \\ &= \frac{1\,512}{24} \\ &= 63{,}00. \end{aligned}$$

The standard deviation is

$$\begin{aligned} S &= \sqrt{63{,}00} \\ &= 7{,}94. \end{aligned}$$

Activity 2.7:2(a)
The data in ascending order:

4 5 10 12 50 60 80

The first quartile, Q_1, is the $\frac{1}{4}(\underline{7}+1)$nd = $\underline{\text{2nd}}$ observation. Thus, $Q_1 = \underline{5}$.

The third quartile, Q_3, is the $\frac{3}{4}(\underline{7}+1)$th = $\underline{\text{6th}}$ observation. Thus, $Q_3 = \underline{60}$.

The quartile deviation is

$$\begin{aligned} Q_D &= \frac{Q_3 - Q_1}{2} \\ &= \frac{60-5}{2} \\ &= 27{,}5. \end{aligned}$$

Activity 2.7:2(b)
The data in ascending order:

12 15 16 22 22 23 26 27 29 30 31 48

The first quartile, Q_1, is the $\frac{1}{4}(12+1)$th $=$ 3,25th observation.

Q_1 lies a quarter of the distance between 16 and 22, measured from 16.

Q_1 is calculated as

$$\begin{aligned} Q_1 &= 16 + \frac{1}{4} \times (22 - 16) \\ &= 16 + 1{,}5 \\ &= 17{,}5. \end{aligned}$$

The third quartile, Q_3, is the $\frac{3}{4}(12+1)$th $=$ 9,75th observation.

Q_3 lies three quarters of the distance between 29 and 30, measured from 29.

Q_3 is calculated as

$$\begin{aligned} Q_3 &= 29 + \frac{3}{4} \times (30 - 29) \\ &= 29 + 0{,}75 \\ &= 29{,}75. \end{aligned}$$

The quartile deviation is

$$\begin{aligned} Q_D &= \frac{Q_3 - Q_1}{2} \\ &= \frac{29{,}75 - 17{,}5}{2} \\ &= 6{,}13. \end{aligned}$$

Activity 2.7:3
The sales of the three types of cereals are

	Type of cereal		
	A	**B**	**C**
Mean number of sales per week ($\overline{x}$)	88	56	100
Standard deviation (s)	16	15	25

The coefficient of variation for cereal A is

$$\begin{aligned} CV_A &= \frac{s}{\overline{x}} \times 100 \\ &= \frac{16}{88} \times 100 \\ &= 18{,}18\%. \end{aligned}$$

The coefficient of variation for cereal B is

$$\begin{aligned} CV_B &= \frac{s}{\overline{x}} \times 100 \\ &= \frac{15}{56} \times 100 \\ &= 26{,}79\%. \end{aligned}$$

The coefficient of variation for cereal C is

$$\begin{aligned} CV_C &= \frac{s}{x} \times 100 \\ &= \frac{25}{100} \times 100 \\ &= 25{,}00\%. \end{aligned}$$

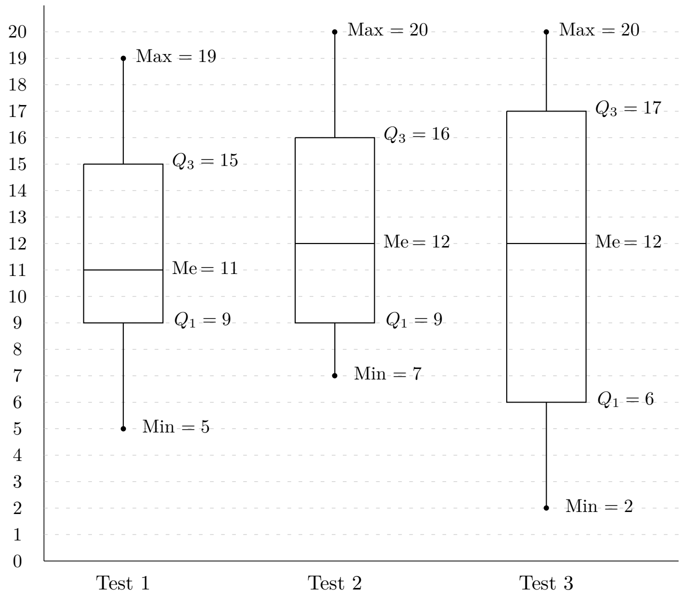
From the results we see that cereal A displays the least variation and is therefore the most consistent in sales.

Activity 2.8

Student test scores with marks out of 20 for each test:

	Test		
	1	**2**	**3**
Lowest value	5	7	2
Q_1	9	9	6
Me	11	12	12
Q_3	15	16	17
Highest value	19	20	20

Box-and-whisker diagram:



Activity 2.8:1

Test 3 showed the widest spread of data with the minimum of 2 out of 20 and the maximum of 20 out of 20.

Activity 2.8:2

The median divides an ordered data set into two equal parts. Test 1 has a median of 11 marks out of 20. Thus, in test 1, half of the students scored a mark of less than 11 out of 20.

Activity 2.8:3

In test 2, Q_1 is 9 out of 20 and Q_3 is 16 out of 20. Fifty percent of the data values falls between Q_1 and Q_3. Thus, 50% of the scores in test 2 falls between 9 and 16 out of 20.

Activity 2.8:4
For test 1, Q_3 is 15 out of 20. Seventy five percent of the data in a data set falls below Q_3. Thus, 75% of the students scored a mark of less than 15 out of 20 in test 1.

Activity 2.8:5
The highest value for test 1 is 19 out of 20 and the lowest mark is 5 out of 20. The range is 14 (19 - 5).

Activity 2.9:1

(a)	Median	C	A better measure of central tendency when the data are very skewed.
		E	The value that occupies the middle position of a group of values in a data set when the data values are arranged in ascending order.
(b)	Standard deviation	B	Measures how much the data differ from the mean.
		J	A value of zero means that there is no variation in the data.
(c)	Mode	F	Does not exist in many cases, and there may be more than one in others.
		I	The value that occurs most often in a data set (data value with the highest frequency).
(d)	Mean	D	Reliable since it reflects all the values in a data set.
		H	May be affected by extreme values that are not representative of the set as a whole.
(e)	Coefficient of variation	G	Compares two or more sets of data with different means, sample sizes or measurements units.
(f)	Range	A	The difference between the highest and the lowest value in a data set.

Activity 2.9:2
The total amount paid on salaries is

$$\begin{aligned} 3 \times 152\,000 + 2 \times 190\,000 + 1 \times 175\,000 + 1 \times 290\,000 &= 456\,000 + 380\,000 + 175\,000 + 290\,000 \\ &= 1\,301\,000. \end{aligned}$$

In total R1 301 000 is paid on salaries.

The mean salary is calculated as

$$\begin{aligned} \frac{\text{total amount}}{\text{number of people}} &= \frac{1\,301\,000}{7} \\ &= 185\,857{,}14. \end{aligned}$$

The mean salary is R185 857,14.

Four people earned less than the mean salary of R185 857,14.

Activity 2.9:3
The median is 14 and remains 14.

Note that the mean will change and the mode will change from 19 to 15.

Component 3. Index numbers and transformations

Activity 3.1
The completed table is as follows:

Equipment	**Price (R)**		**Quantities**					
	2004	**2009**	**2004**	**2009**				
	p_{2004}	p_{2009}	q_{2004}	q_{2009}	$p_{2004}q_{2004}$	$p_{2004}q_{2009}$	$p_{2009}q_{2004}$	$p_{2009}q_{2009}$
Ballpoint pens	6,40	7,50	400	500	2 560,0	3 200	3 000	3 750
Pencils	6,50	6,50	150	300	975,0	1 950	975	1 950
Rulers	6,25	6,80	350	300	2 187,5	1 875	2 380	2 040
Total					5 722,5	7 025	6 355	7 740

Activity 3.1:1
The Laspeyres price index is

$$\begin{aligned} P_L(n) &= \frac{p_n q_0}{p_0 q_0} \times 100 \\ P_L(2009) &= \frac{p_{2009} q_{2004}}{p_{2004} q_{2004}} \times 100 \\ &= \frac{6\,355}{5\,722{,}5} \times 100 \\ &= 111{,}05. \end{aligned}$$

Activity 3.1:2
The Laspeyres quantity index is

$$\begin{aligned} Q_L(n) &= \frac{p_0 q_n}{p_0 q_0} \times 100 \\ Q_L(2009) &= \frac{p_{2004} q_{2009}}{p_{2004} q_{2004}} \times 100 \\ &= \frac{7\,025}{5\,722{,}5} \times 100 \\ &= 122{,}76. \end{aligned}$$

Activity 3.1:3
The Paasche price index is

$$\begin{aligned} P_P(n) &= \frac{p_n q_n}{p_0 q_n} \times 100 \\ P_P(2009) &= \frac{p_{2009} q_{2009}}{p_{2004} q_{2009}} \times 100 \\ &= \frac{7\,740}{7\,025} \times 100 \\ &= 110{,}18. \end{aligned}$$

Activity 3.1:4
The Paasche quantity index is

$$\begin{aligned} Q_P(n) &= \frac{p_n q_n}{p_n q_0} \times 100 \\ Q_P(2009) &= \frac{p_{2009} q_{2009}}{p_{2009} q_{2004}} \times 100 \\ &= \frac{7\,740}{6\,355} \times 100 \\ &= 121{,}79. \end{aligned}$$

Activity 3.1:5
The value index is

$$\begin{aligned} V &= \frac{p_n q_n}{p_0 q_0} \times 100 \\ &= \frac{p_{2009} q_{2009}}{p_{2004} q_{2004}} \times 100 \\ &= \frac{7\,740}{5\,722{,}5} \times 100 \\ &= 135{,}26. \end{aligned}$$

Activity 3.2
The completed table is as follows:

Year	**Actual expenses**	**CPI Base year = 1996**	$\frac{\textbf{actual expenses}}{\text{CPI}} \times 100$	**Deflated expenses (R)**
1995	116 000	98,0	$\frac{116\,000}{98{,}0} \times 100$	118 367,35
2000	125 500	108,5	$\frac{125\,500}{108{,}5} \times 100$	115 668,20
2005	146 500	121,5	$\frac{146\,500}{121{,}5} \times 100$	120 576,13

Activity 3.2:1
Taking the deflated expenses into account, the yearly expenses of a student in 1996 would have been R118 367,35 if it was R116 000 in 1995.

Activity 3.2:2
Taking the deflated expenses into account, the yearly expenses of a student in 1996 would have been R120 576,13 if it was R146 500 in 2005.

The cost of living had definitely risen since 1996.

Activity 3.3:1(a)
The exchange rate is R12,58 = €1. The number of euros that she receives can be interpreted as a ratio:

$$\begin{aligned} \text{rand} &: \text{euro} \\ 12{,}58 &: 1 \\ \frac{12{,}58}{12{,}58} &: \frac{1}{12{,}58} \\ 1 &: \frac{1}{12{,}58} \\ 1 \times 24\,500 &: \frac{1}{12{,}58} \times \frac{24\,500}{1} \\ 24\,500 &: \frac{24\,500}{12{,}58} \\ 24\,500 &: 1\,947{,}54 \end{aligned}$$

The amount of R24 500 is equal to €1 947,54.

Activity 3.3:1(b)
The exchange rate is R11,90 = €1. The amount in rand that she receives can also be interpreted as a ratio:

$$\begin{aligned} \text{euro} &: \text{rand} \\ 1 &: 11{,}90 \\ 1 \times 56 &: 11{,}90 \times 56 \\ 56 &: 666{,}40 \end{aligned}$$

The amount of €56 is equal to R666,40.

Activity 3.3:2(a)
Determine the gold price per fine ounce in rand value.
Write the exchange rate as a ratio:

$$\begin{aligned} \text{rand} &: \text{dollar} \\ 1 &: 0{,}135 \\ \frac{1}{0{,}135} &: \frac{0{,}135}{0{,}135} \\ 7{,}41 &: 1 \\ 7{,}41 \times 934 &: 1 \times 934 \\ 6\,920{,}94 &: 934{,}00 \end{aligned}$$

The gold price is $934,00 per fine ounce and $934,00 is equal to R6 920,94.

The gold price is thus R6 920,94 per fine ounce.

Activity 3.3:2(b)
The price of R6 920,94 per fine ounce can be written as a ratio:

$$\begin{aligned} \text{rand} &: \text{gram} \\ 6\,920{,}94 &: 31{,}10348 \\ \frac{6\,920{,}94}{31{,}10348} &: \frac{31{,}10348}{31{,}10348} \\ 222{,}51 &: 1 \end{aligned}$$

One gram of gold costs R222,51.

Activity 3.3:2(c)
There are 28,35 grams in one ounce.

The price of R222,51 for one gram of gold can be written as a ratio:

$$\begin{aligned} \text{rand} &: \text{gram} \\ 222{,}51 &: 1 \\ 222{,}51 \times 28{,}35 &: 1 \times 28{,}35 \\ 6\,308{,}16 &: 28{,}35 \end{aligned}$$

The price of one Kruger rand is R6 308,16.

Activity 3.3:3
The period is five years.

The growth rate from 2000 to 2005 is

$$\left(\left(\frac{GDP_{2005}}{GDP_{2000}}\right)^{\frac{1}{5}} - 1\right) \times 100 = \left(\left(\frac{635\,820}{517\,360}\right)^{\frac{1}{5}} - 1\right) \times 100$$
$$= 4{,}21\% \text{ per year.}$$

Component 4. Functions and representations of functions

Activity 4.1:1(a)
The cost is

$$\begin{aligned} C(1) &= 6\,500 + 4\,125 \times 1 \\ &= 10\,625. \end{aligned}$$

The cost in rand of renting a sailboat for one day is R10 625.

Activity 4.1:1(b)
To calculate the number of days, take the money that Nick has available and first subtract the fixed cost:

$$55\,000 - 6\,500 = 48\,500.$$

After the fixed cost of R6 500 is subtracted he has R48 500 left. Now divide this R48 500 by the cost of the boat per day:

$$\frac{48\,500}{4\,125} = 11{,}758.$$

He can afford to pay for eleven days for the boat. The cost is calculated as

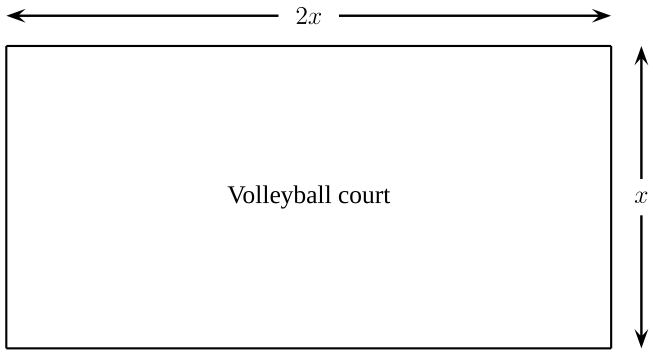
$$4\,125 \times 11 = 45\,375,$$

and

$$48\,500 - 45\,375 = 3\,125.$$

It will cost him R45 375 for the eleven days and he will have R3 125 left.

Activity 4.1:2(a)
The following diagram shows the dimensions of the volleyball court:



The area is given by the function $A(x) = 2x^2$.
(Remember: area = length × width = $x \times 2x$.)

Activity 4.1:2(b)
It is given that the width of the court is 12 metres. Thus, x is equal to 12.
The area of the court is calculated as

$$\begin{aligned} A(12) &= 2x^2 \\ &= 2 \times 12^2 \\ &= 288. \end{aligned}$$

The area is 288 m^2.
It costs R11,75 to repaint 1 m^2. Thus, the cost for 288 m^2 is calculated as

$$11{,}75 \times 288 = 3\,384.$$

It will cost R3 384 to repaint the whole court.

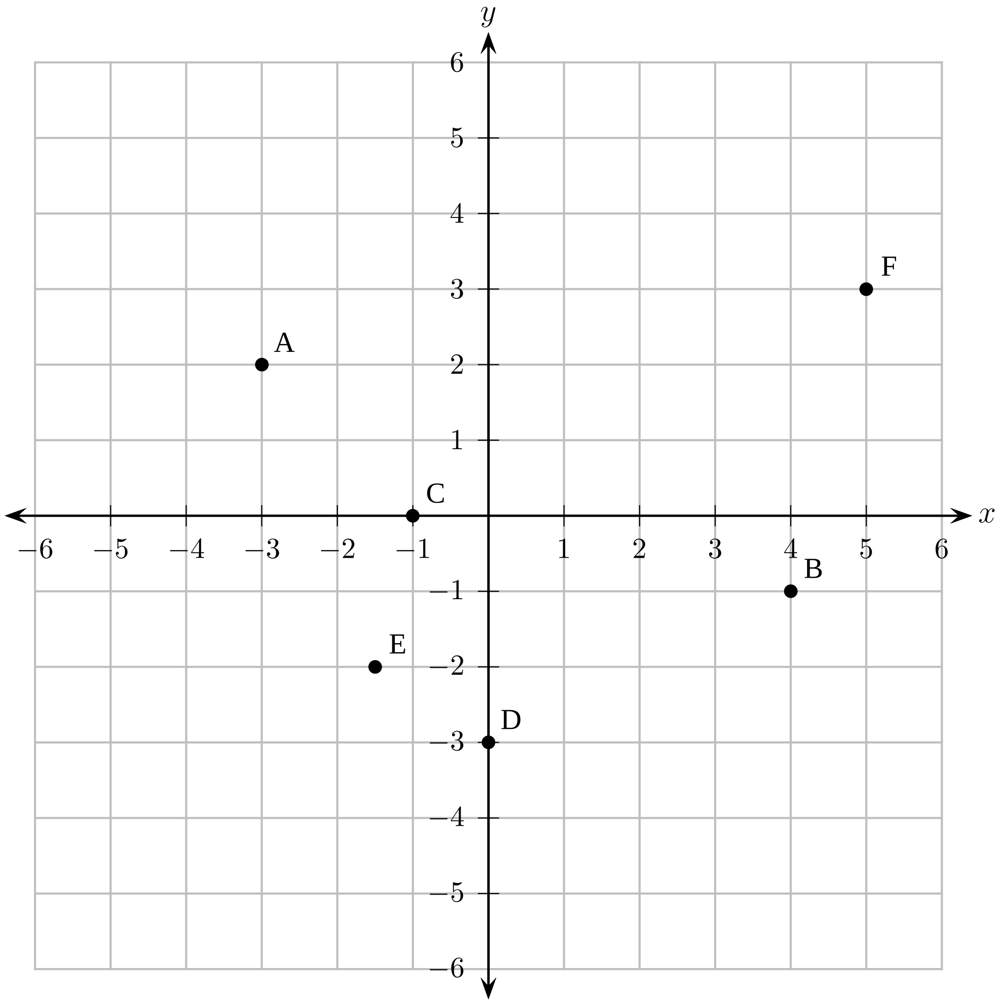
Activity 4.2:1(a)
The coordinate system is divided into four areas called quadrants.

Activity 4.2:1(b)
The coordinate system is created by the x- and y-axis. Coordinates $(x; y)$ are plotted on the coordinate system.

Activity 4.2:1(c)
The x-coordinate, which is the first number, tells us to go right from the origin if it is positive and left from the origin if it is negative.

Activity 4.2:1(d)
The y-coordinate, which is the second number, tells us to go up from the origin if it is positive and down from the origin if it is negative.

Activity 4.2:1(e)
The rectangular coordinate system is shown below.



Activity 4.2:2(a)

- The function

$$y = 2x + 4$$

is written in the form $y = f(x) = ax + b$ and where $a = 2$ (which is > 0) and $b = 4$ (which is > 0).

- To determine the x-intercept, set $y = f(x) = 0$ and solve for x:

$$\begin{aligned} 2x + 4 &= 0 \\ 2x &= -4 \\ \frac{2x}{2} &= \frac{-4}{2} \\ x &= -2. \end{aligned}$$

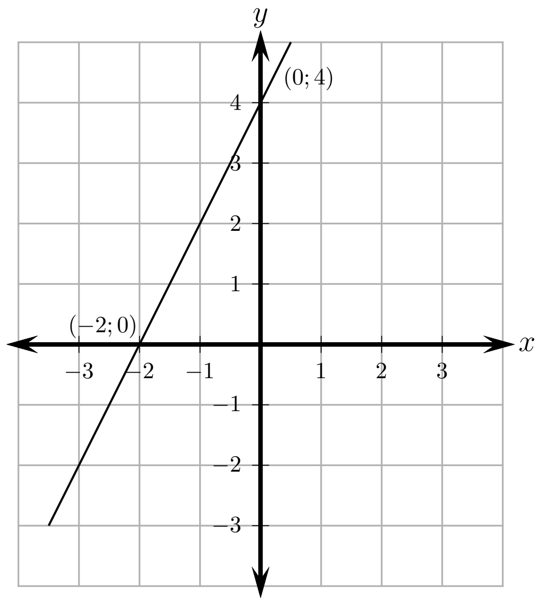
The x-intercept is at point $(-2; 0)$.

- To determine the y-intercept, set $x = 0$ to find $f(0)$:

$$\begin{aligned} y &= f(0) \\ &= 2 \times 0 + 4 \\ &= 0 + 4 \\ &= 4. \end{aligned}$$

The y-intercept is at point $(0; 4)$.

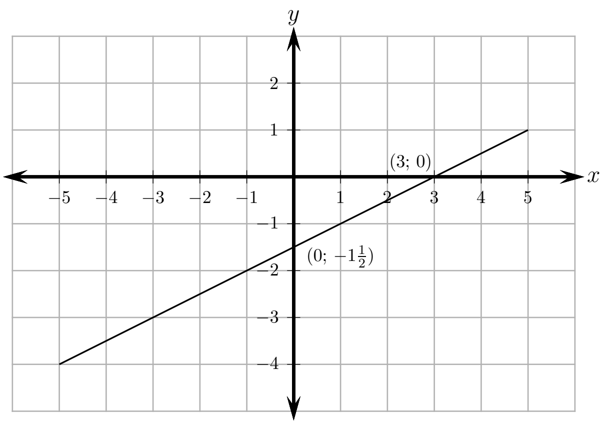
- The graph of the above function is a line passing through the points $(-2; 0)$ and $(0; 4)$ as shown below.



Since the slope, a, is (positive/negative), the function $f(x)$ (increases/decreases) if you move from left to right. The value of the y-intercept, b, is (positive/negative).

Activity 4.2:2(b)

- The function
$$x - 2y = 3$$
rewritten to the form $y = f(x) = ax + b$ is
$$\begin{aligned} x - 2y &= 3 \\ x - 2y + 2y - 3 &= 3 + 2y - 3 \\ x - 3 &= 2y \\ 2y &= x - 3 \\ \frac{2y}{2} &= \frac{x}{2} - \frac{3}{2} \\ y &= \frac{1}{2}x - 1\frac{1}{2}. \end{aligned}$$
In this case $a = \frac{1}{2}$ (which is > 0) and $b = -1\frac{1}{2}$ (which is < 0).
- To determine the x-intercept, set $y = f(x) = 0$ and solve for x:
$$\begin{aligned} \frac{1}{2}x - 1\frac{1}{2} &= 0 \\ \frac{1}{2}x &= 1\frac{1}{2} \\ \frac{1}{2}x &= \frac{3}{2} \\ \frac{2}{1} \times \frac{1}{2}x &= \frac{2}{1} \times \frac{3}{2} \\ x &= 3. \end{aligned}$$
The x-intercept is at point (3; 0).
- To determine the y-intercept, set $x = 0$ to find $f(0)$:
$$\begin{aligned} y &= f(0) \\ &= \frac{1}{2} \times 0 - 1\frac{1}{2} \\ &= 0 - 1\frac{1}{2} \\ &= -1\frac{1}{2}. \end{aligned}$$
The y-intercept is at point $\left(0; -1\frac{1}{2}\right)$.
- The graph of the above function is a line passing through the points (3; 0) and $\left(0; -1\frac{1}{2}\right)$ as shown below.



Since the slope, a, is (positive/negative), the function $f(x)$ (increases/decreases) if you move from left to right. The value of the y-intercept, b, is (positive/negative).

Activity 4.2:2(c)

- The function

$$2x + 5y = 10$$

rewritten to the form $y = f(x) = ax + b$ is

$$\begin{aligned} 2x + 5y &= 10 \\ 5y &= -2x + 10 \\ \frac{5y}{5} &= -\frac{2}{5}x + \frac{10}{5} \\ y &= -\frac{2}{5}x + 2. \end{aligned}$$

In this case $a = -\frac{2}{5}$ (which is < 0) and $b = 2$ (which is > 0).

- To determine the x-intercept, set $y = f(x) = 0$ and solve for x:

$$\begin{aligned} -\frac{2}{5}x + 2 &= 0 \\ -\frac{2}{5}x &= -\frac{2}{1} \\ -\frac{5}{2} \times -\frac{2}{5}x &= -\frac{5}{2} \times -\frac{2}{1} \\ x &= 5. \end{aligned}$$

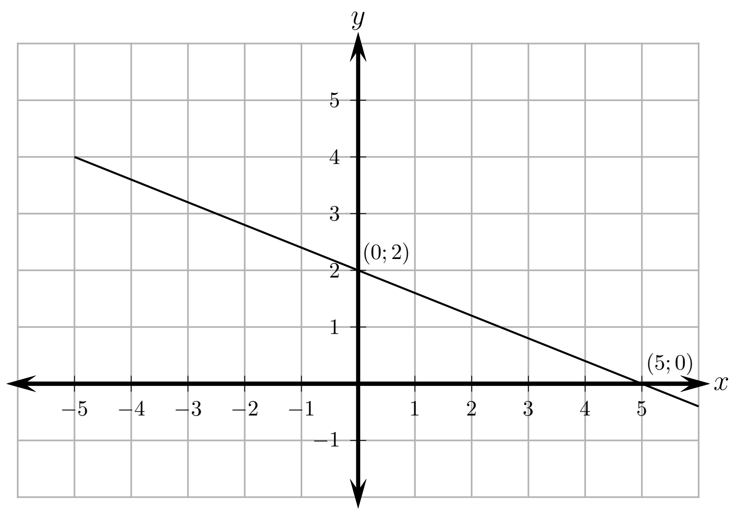
The x-intercept is at point (5; 0).

- To determine the y-intercept, set $x = 0$ to find $f(0)$:

$$\begin{aligned} y &= f(0) \\ &= -\frac{2}{5} \times 0 + 2 \\ &= 0 + 2 \\ &= 2 \end{aligned}$$

The y-intercept is at point (0; 2).

- The graph of the above function is a line passing through the points (5; 0) and (0; 2) as shown below.



Since the slope, a, is (positive/negative), the function $f(x)$ (increases/decreases) if you move from left to right. The value of the y-intercept, b, is (positive/negative).

Activity 4.2:2(d)

- The function

$$-3y = 15 + 6x$$

rewritten to the form $y = f(x) = ax + b$ is:

$$\begin{aligned} -3y &= 15 + 6x \\ -3y &= 6x + 15 \\ \frac{-3y}{-3} &= \frac{6}{-3}x + \frac{15}{-3} \\ y &= -2x - 5. \end{aligned}$$

In this case $a = -2$ (which is < 0) and $b = -5$ (which is < 0).

- To determine the x-intercept, set $y = f(x) = 0$ and solve for x:

$$\begin{aligned} -2x - 5 &= 0 \\ -2x &= 5 \\ \frac{-2}{-2}x &= \frac{5}{-2} \\ x &= -2\frac{1}{2}. \end{aligned}$$

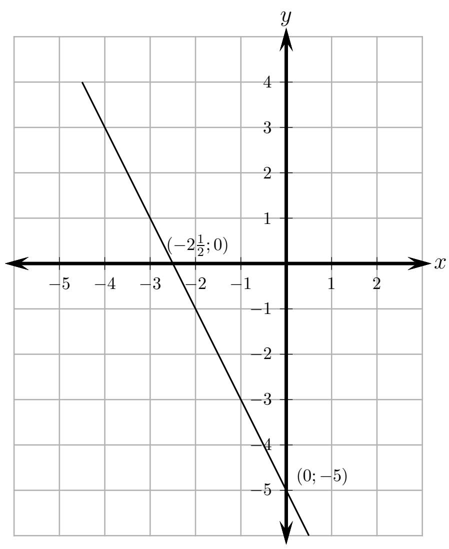
The x-intercept is at point $\left(-2\frac{1}{2}; 0\right)$.

- To determine the y-intercept, set $x = 0$ to find $f(0)$:

$$\begin{aligned} y &= f(0) \\ &= -2 \times 0 - 5 \\ &= 0 - 5 \\ &= -5 \end{aligned}$$

The y-intercept is at point $(0; -5)$.

- The graph of the above function is a line passing through the points $\left(-2\frac{1}{2}; 0\right)$ and $(0; -5)$ as shown below.



Since the slope, a, is (positive/<u>negative</u>), the function $f(x)$ (increases/<u>decreases</u>) if you move from left to right. The value of the y-intercept, b, is (positive/<u>negative</u>).

Activity 4.2:3(a)
The change in y-values is 4.

The change in x-values is 4.

The size of the slope is calculated as

$$\begin{aligned}\text{size of slope} &= \frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}} \\ &= \frac{4}{4} \\ &= 1.\end{aligned}$$

The line is (ascending/descending) from left to right.

The value of a is thus (positive (> 0)/negative (< 0)).

The slope is thus $a = 1$.

Activity 4.2:3(b)
The change in y-values is 6.

The change in x-values is 2.

The size of the slope is calculated as

$$\begin{aligned}\text{size of slope} &= \frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}} \\ &= \frac{6}{2} \\ &= 3.\end{aligned}$$

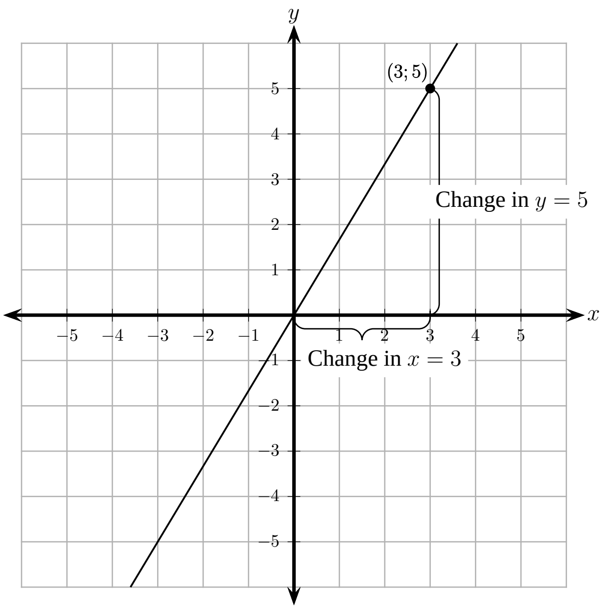
The line is (ascending/descending) from left to right.

The value of a is thus (positive (> 0)/negative (< 0)).

The slope is thus $a = -3$.

Activity 4.2:4(a)
The line cuts the y-axes at the point 2, thus the y-intercept is $y = 2$.

Activity 4.2:4(b)



The change in y-values is 5.

The change in x-values is 3.

The size of the slope is calculated as

$$\begin{aligned} \text{size of slope} &= \frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}} \\ &= \frac{5}{3} \\ &= 1\frac{2}{3}. \end{aligned}$$

The line is (ascending/descending) from left to right.

The value of a is thus (positive (>0)/negative (<0)).

The slope is thus $a = 1\frac{2}{3}$ and the value of b is zero.

The equation for the line is $y = 1\frac{2}{3}x$.

Activity 4.2:4(c)
The points are $(x_1;\, y_1) = (-2;\, 0)$ and $(x_2;\, y_2) = (0;\, 2)$. The slope of the line is calculated as

$$\begin{aligned} \text{slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2-0}{0-(-2)} \\ &= \frac{2}{2} \\ &= 1. \end{aligned}$$

The general expression is $y = ax + b$ which reduces to $y = 1x + b$ or $y = x + b$.

The line passes through the point (0; 2). It is the point where the value of x equals zero and where the y-intercept is found. The value of b is 2.

This can be tested. Substitute (0; 2) into $y = x + b$ to obtain the value of b:

$$\begin{aligned} y &= x + b \\ 2 &= 0 + b \\ b &= 2. \end{aligned}$$

The equation of the line is then $y = x + 2$.

Activity 4.2:4(d)
Consider

$$y = 3 - x.$$

The intercept on the x-axis (where $y = 0$) is

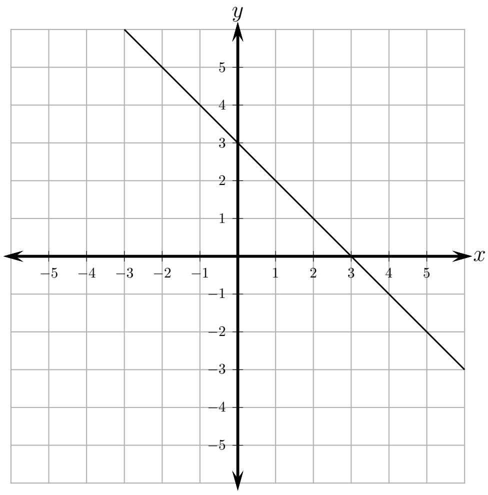
$$\begin{aligned} 0 &= 3 - x \\ x &= 3. \end{aligned}$$

The coordinates of the x-intercept are (3; 0).

The intercept on the y-axis (where $x = 0$) is

$$\begin{aligned} y &= 3 - 0 \\ y &= 3. \end{aligned}$$

The coordinates of the y-intercept are (0; 3).
This is represented by the third graph:



Activity 4.2:5
The equation $2y = 5x + 7$ expressed in the form $y = ax + b$ is

$$\begin{aligned} 2y &= 5x + 7 \\ \frac{2y}{2} &= \frac{5}{2}x + \frac{7}{2} \\ y &= 2\frac{1}{2}x + 3\frac{1}{2}. \end{aligned}$$

The value of a is $2\frac{1}{2}$ and the value of b is $3\frac{1}{2}$.

The value of a gives the slope of the line. Thus, the line has a slope of $2\frac{1}{2}$.

Any line parallel to $2y = 5x + 7$ also has a slope of $2\frac{1}{2}$.

The required straight line parallel to $2y = 5x + 7$ passes through $\left(0; -3\frac{1}{2}\right)$.

Thus, the line has as y-intercept $y = -3\frac{1}{2}$.

Therefore, the equation of the line is $y = 2\frac{1}{2}x - 3\frac{1}{2}$.

Activity 4.2:6
1. E
2. C
3. B
4. A
5. D

Activity 4.3:1(a)
The intercepts on the x-axis are $x = -5$ and $x = -1$, the points are $(-5; 0)$ and $(-1; 0)$.

Activity 4.3:1(b)
The intercept on the y-axis is at $c = 5$. That is the point $(0; 5)$.

Activity 4.3:1(c)
At the vertex $x = -3$.

Activity 4.3:1(d)
The minimum value of the function is the value of the function at the point where $x = -3$, that is $y = f(-3) = -4$.

Activity 4.3:1(e)
The value of a is positive ($a > 0$) because the quadratic function opens upward.

Activity 4.3:1(f)
The discriminant $b^2 - 4ac > 0$ because two separate x-intercepts exist, namely $x = -1$ and $x = -5$.

Activity 4.3:2
The quadratic functions are the following:

(a) $y = f(x) = 4x^2 - 8x + 3$

(b) $y = f(x) = -x^2 + 6x + 7$

(c) $y = f(x) = x^2 + 5$

(d) $y = f(x) = -x^2 - 6x - 9$

(e) $y = f(x) = -x^2 + 2x - 6$

(f) $y = f(x) = x^2 + 3x$

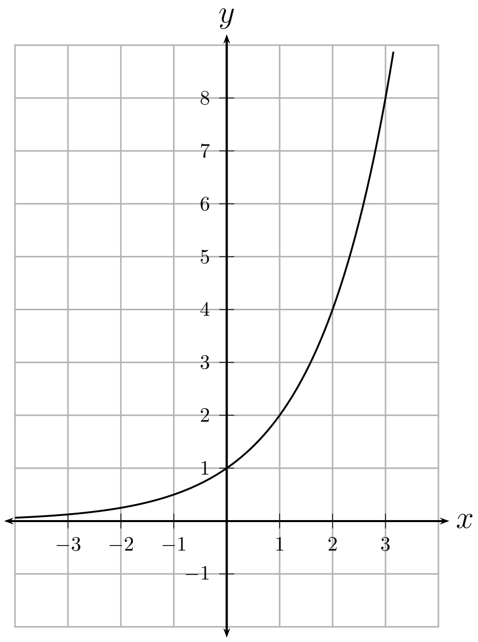
The completed table is

Coefficients	Min. or max. value?	At the vertex $x_m = -\frac{b}{2a}$	At the vertex $y = f\left(-\frac{b}{2a}\right)$	$b^2 - 4ac$	x-intercepts (if any)	$c = f(0)$	Corresponding graph
(a) $a = 4$ $b = -8$ $c = 3$	Minimum because $a > 0$	$x_m = 1$	$f(1) = -1$	16	$x = \frac{1}{2}$ and $x = 1\frac{1}{2}$	$f(0) = 3$	C
(b) $a = -1$ $b = 6$ $c = 7$	Maximum because $a < 0$	$x_m = 3$	$f(3) = 16$	64	$x = -1$ and $x = 7$	$f(0) = 7$	E
(c) $a = 1$ $b = 0$ $c = 5$	Minimum because $a > 0$	$x_m = 0$	$f(0) = 5$	$-20 < 0$	No intercepts on x-axis	$f(0) = 5$	A
(d) $a = -1$ $b = -6$ $c = -9$	Maximum because $a < 0$	$x_m = -3$	$f(-3) = 0$	0	Vertex touches x-axis at $x = -3$	$f(0) = -9$	D
(e) $a = -1$ $b = 2$ $c = -6$	Maximum because $a < 0$	$x_m = 1$	$f(1) = -5$	$-20 < 0$	No intercepts on x-axis	$f(0) = -6$	F
(f) $a = 1$ $b = 3$ $c = 0$	Minimum because $a > 0$	$x_m = -1\frac{1}{2}$	$f(-1\frac{1}{2}) = -2\frac{1}{4}$	9	$x = 0$ and $x = -3$	$f(0) = 0$	B

Activity 4.4
The values of the function $y = 2^x$ for the values of $x = -2, -1, 0, 1, 2$ and 3 is the following:

x	$y = 2^x$
-2	$2^{-2} = 0{,}25$
-1	$2^{-1} = 0{,}5$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$

The graphical representation of the function $y = 2^x$ is as follows:



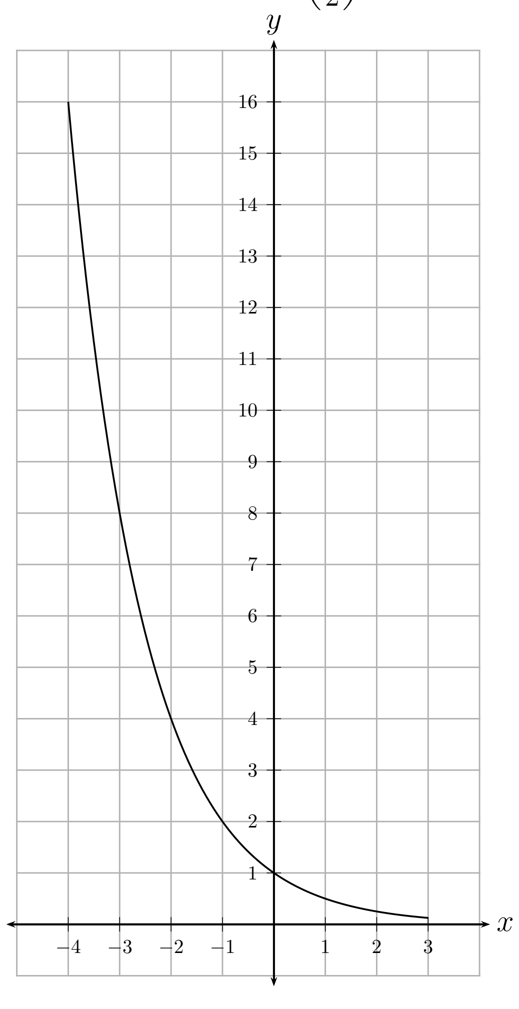
Activity 4.5:1(a)

The completed table:

x	$y = \left(\frac{1}{2}\right)^x$
-4	$\left(\frac{1}{2}\right)^{-4} = (2^{-1})^{-4} = 2^{-1\times-4} = 2^4 = 16$
-3	$\left(\frac{1}{2}\right)^{-3} = (2^{-1})^{-3} = 2^{-1\times-3} = 2^3 = 8$
-2	$\left(\frac{1}{2}\right)^{-2} = (2^{-1})^{-2} = 2^{-1\times-2} = 2^2 = 4$
-1	$\left(\frac{1}{2}\right)^{-1} = (2^{-1})^{-1} = 2^{-1\times-1} = 2^1 = 2$
0	$\left(\frac{1}{2}\right)^{0} = 1$
1	$\left(\frac{1}{2}\right)^{1} = \frac{1}{2} = 0{,}5$
2	$\left(\frac{1}{2}\right)^{2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = 0{,}25$
3	$\left(\frac{1}{2}\right)^{3} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} = 0{,}125$

Activity 4.5:1(b)

The graphical representation of the function $y = \left(\frac{1}{2}\right)^x$ is as follows:



Activity 4.5:2(a)
All graphs go through the point (0; 1).

Activity 4.5:2(b)
All are increasing graphs. If you move from left to right on the x-axis, the functions are increasing.

Activity 4.5:2(c)
All are decreasing graphs. If you move from left to right on the x-axis, the functions are decreasing.

Activity 4.5:2(d)
All negative and positive values of x are included. The mathematical way of writing this, is $(-\infty; \infty)$. In words: minus infinity to positive infinity.

Activity 4.5:2(e)
All positive values of y, or $(0; \infty)$.

Activity 4.6:1(a)
Set the expression $\log_a 1$ equal to x and solve for x:

$$\begin{aligned} \log_a 1 &= x \\ a^x &= 1 \\ a^x &= a^0. \end{aligned}$$

Therefore,

$$x = 0$$

giving

$$\log_a 1 = 0.$$

Activity 4.6:1(b)
Set the expression $\log_7 7$ equal to x and solve for x:

$$\begin{aligned} \log_7 7 &= x \\ 7^x &= 7 \\ 7^x &= 7^1. \end{aligned}$$

Therefore,

$$x = 1$$

giving

$$\log_7 7 = 1.$$

Activity 4.6:1(c)
Set the expression $\log_5 \sqrt{5}$ equal to x and solve for x:

$$\begin{aligned} \log_5 \sqrt{5} &= x \\ 5^x &= \sqrt{5} \\ 5^x &= 5^{\frac{1}{2}}. \end{aligned}$$

Therefore,

$$x = \frac{1}{2}$$

giving

$$\log_5 \sqrt{5} = \frac{1}{2}.$$

Activity 4.6:1(d)

Set the expression $\log_6 \frac{1}{36}$ equal to x and solve for x:

$$\begin{aligned} \log_6 \frac{1}{36} &= x \\ 6^x &= \frac{1}{36} \\ 6^x &= \frac{1}{6^2} \\ 6^x &= 6^{-2}. \end{aligned}$$

Therefore,

$$x = -2$$

giving

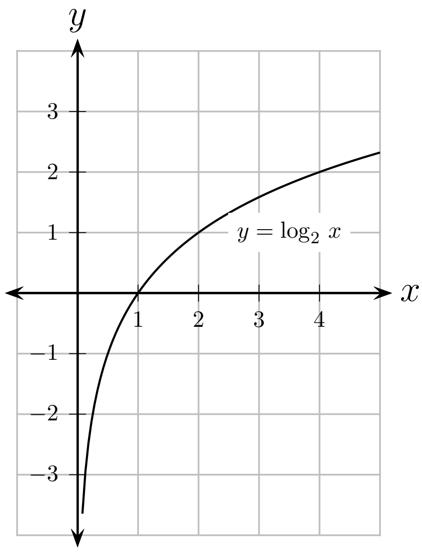
$$\log_6 \frac{1}{36} = -2.$$

Activity 4.6:2(a)

The function $y = f(x) = \log_2 x$ calculated for the values $x = \frac{1}{4}, \frac{1}{2}$, 1, 2 and 4 is the following:

x	$\log_2 x$	$y = \log_2 x$
$\frac{1}{4}$	$\log_2 \frac{1}{4}$	Because $\frac{1}{4} = 2^{-2}$, it follows from $y = \log_2 \frac{1}{4}$ that $2^y = \frac{1}{4}$ $2^y = 2^{-2}$. Thus, $y = -2$.
$\frac{1}{2}$	$\log_2 \frac{1}{2}$	Because $\frac{1}{2} = 2^{-1}$, it follows from $y = \log_2 \frac{1}{2}$ that $2^y = \frac{1}{2}$ $2^y = 2^{-1}$. Thus, $y = -1$.
1	$\log_2 1$	Because $1 = 2^0$, it follows from $y = \log_2 1$ that $2^y = 1$ $2^y = 2^0$. Thus, $y = 0$.
2	$\log_2 2$	Because $2 = 2^1$, it follows from $y = \log_2 2$ that $2^y = 2$ $2^y = 2^1$. Thus, $y = 1$.
4	$\log_2 4$	Because $4 = 2^2$, it follows from $y = \log_2 4$ that $2^y = 4$ $2^y = 2^2$. Thus, $y = 2$.

Activity 4.6:2(b)
The graph of the function $y = f(x) = \log_2 x$:



Component 5. Linear systems

Activity 5.1
Simplifying gives

$$\begin{aligned} -7k + 78 &= 3k - 72 \\ -7k + 78 - 3k &= 3k - 72 - 3k \\ -10k + 78 &= -72 \\ -10k + 78 - 78 &= -72 - 78 \\ -10k &= -150 \\ \frac{-10k}{-10} &= \frac{-150}{-10} \\ k &= 15. \end{aligned}$$

Activity 5.2
Simplifying gives

$$\begin{aligned} 2x + 3 &= 6 - (2x - 3) \\ 2x + 3 &= 6 - 2x + 3 \\ 2x + 3 &= 9 - 2x \\ 2x + 3 + 2x &= 9 - 2x + 2x \\ 4x + 3 &= 9 \\ 4x + 3 - 3 &= 9 - 3 \\ 4x &= 6 \\ \frac{4x}{4} &= \frac{6}{4} \\ x &= \frac{3}{2} \\ &= 1\frac{1}{2}. \end{aligned}$$

Activity 5.3:1
Simplifying gives

$$\begin{aligned}
\frac{4-5x}{6}-\frac{1-2x}{3} &= \frac{13}{42}\\
\frac{42}{1}\times\frac{4-5x}{6}-\frac{42}{1}\times\frac{1-2x}{3} &= \frac{42}{1}\times\frac{13}{42}\\
\frac{\cancel{42}^{\,7}}{1}\times\frac{4-5x}{\cancel{6}_1}-\frac{\cancel{42}^{14}}{1}\times\frac{1-2x}{\cancel{3}_1} &= \frac{\cancel{42}^{\,1}}{1}\times\frac{13}{\cancel{42}_{\,1}}\\
7\times(4-5x)-14\times(1-2x) &= 13\\
28-35x-14+28x &= 13\\
14-7x &= 13\\
14-7x-14 &= 13-14\\
-7x &= -1\\
\frac{-7x}{-7} &= \frac{-1}{-7}\\
x &= \frac{1}{7}.
\end{aligned}$$

Activity 5.3:2
Simplifying gives

$$\begin{aligned}
\frac{3}{4}d-2 &= \frac{1}{3}d+3\\
\frac{12}{1}\times\frac{3}{4}d+12\times -2 &= \frac{12}{1}\times\frac{1}{3}d+12\times 3\\
\frac{\cancel{12}^{\,3}}{1}\times\frac{3}{\cancel{4}_1}d-24 &= \frac{\cancel{12}^{\,4}}{1}\times\frac{1}{\cancel{3}_1}d+36\\
3\times 3d-24 &= 4\times 1d+36\\
9d-24 &= 4d+36\\
9d-24-4d &= 4d+36-4d\\
5d-24 &= 36\\
5d-24+24 &= 36+24\\
5d &= 60\\
\frac{5d}{5} &= \frac{60}{5}\\
d &= 12.
\end{aligned}$$

Activity 5.4
Simplifying gives

$$\begin{aligned}
\frac{8}{x} &= 2\\
\frac{\cancel{x}^{\,1}}{1}\times\frac{8}{\cancel{x}_1} &= \frac{x}{1}\times\frac{2}{1}\\
1\times 8 &= x\times 2\\
8 &= 2x\\
2x &= 8\\
\frac{2x}{2} &= \frac{8}{2}\\
x &= 4.
\end{aligned}$$

Activity 5.5:1
Rearranging gives

$$\begin{aligned} 2x+3 &= 6-(2x-3) \\ 2x+3 &= 6-2x+3 \\ 2x+3 &= 9-2x \\ 2x+3-9+2x &= 9-2x-9+2x \\ 4x-6 &= 0. \end{aligned}$$

This is the form: $ax+b=0.$

Thus, a is 4 and b is -6 in the equation $ax+b=0$.
The value of x is calculated as

$$\begin{aligned} x &= -\frac{b}{a} \\ &= \frac{-(-6)}{4} \\ &= \frac{3}{2} \\ &= 1\frac{1}{2}. \end{aligned}$$

Activity 5.5:2
The equation

$$\frac{4-5x}{6}-\frac{1-2x}{3}=\frac{13}{42}$$

was rewritten to

$$14-7x=13$$

in Activity 5.3.1. Now,

$$\begin{aligned} 14-7x-13 &= 13-13 \\ -7x+1 &= 0. \end{aligned}$$

This is the form: $ax+b=0.$

Thus, a is -7 and b is 1 in the equation $ax+b=0$.
The value of x is calculated as

$$\begin{aligned} x &= -\frac{b}{a} \\ &= \frac{-1}{-7} \\ &= \frac{1}{7}. \end{aligned}$$

Activity 5.6:1
Simplifying gives

$$\begin{aligned}
\frac{1}{2}x + \frac{1}{3}x + x &= 44 \\
\frac{6}{1} \times \frac{1}{2}x + \frac{6}{1} \times \frac{1}{3}x + 6 \times x &= 6 \times 44 \\
3x + 2x + 6x &= 264 \\
11x &= 264 \\
\frac{11x}{11} &= \frac{264}{11} \\
x &= 24.
\end{aligned}$$

The number is 24.
To check:

One half of 24 is 12,
one third of 24 is 8 and
12 + 8 + 24 = 44.

Activity 5.6:2
The total distance that he travels is x.
This can be represented by the equation

$$\frac{5}{8}x + \frac{1}{4}x + 15 = x.$$

Simplifying gives

$$\begin{aligned}
\frac{5}{8}x + \frac{1}{4}x + 15 &= x \\
\frac{8}{1} \times \frac{5}{8}x + \frac{8}{1} \times \frac{1}{4}x + 8 \times 15 &= 8 \times x \\
5x + 2x + 120 &= 8x \\
7x + 120 &= 8x \\
7x + 120 - 8x &= 8x - 8x \\
-x + 120 &= 0 \\
-x &= -120 \\
x &= 120.
\end{aligned}$$

The total distance that he travels is 120 km.
To check:

$$\frac{5}{8} \times \frac{120}{1} = \frac{600}{8} = 75,$$
$$\frac{1}{4} \times \frac{120}{1} = \frac{120}{4} = 30 \text{ and}$$
$$75 + 30 + 15 = 120.$$

Activity 5.7:1
Equation 1: $3x - 2y = 1$.

Make y the subject of the first equation:

$$\begin{aligned}3x-2y-3x &= 1-3x\\ -2y &= -3x+1\\ \frac{-2y}{-2} &= \frac{-3}{-2}x+\frac{1}{-2}\\ y &= \frac{3}{2}x-\frac{1}{2}.\end{aligned}$$

Equation 2: $2x-y=1$.
Make y the subject of the second equation:

$$\begin{aligned}2x-y-2x &= 1-2x\\ -y &= -2x+1\\ \frac{-1y}{-1} &= \frac{-2}{-1}x+\frac{1}{-1}\\ y &= 2x-1.\end{aligned}$$

To solve for x, set the two ys equal to each other:

$$\begin{aligned}\frac{3}{2}x-\frac{1}{2} &= 2x-1\\ \frac{3}{2}x-2x-\frac{1}{2} &= 2x-1-2x\\ \frac{3}{2}x-\frac{4}{2}x-\frac{1}{2} &= -1\\ -\frac{1}{2}x-\frac{1}{2} &= -1\\ -\frac{1}{2}x-\frac{1}{2}+\frac{1}{2} &= -1+\frac{1}{2}\\ -\frac{1}{2}x &= -\frac{1}{2}\\ -\frac{2}{1}\times-\frac{1}{2}x &= -\frac{2}{1}\times-\frac{1}{2}\\ x &= 1.\end{aligned}$$

Substitute the value of $x=1$ into equation 1:

$$\begin{aligned}y &= \frac{3}{2}x-\frac{1}{2}\\ &= \frac{3}{2}\times\frac{1}{1}-\frac{1}{2}\\ &= \frac{3}{2}-\frac{1}{2}\\ &= \frac{2}{2}\\ &= 1.\end{aligned}$$

The solution is $x=1$ and $y=1$. It can also be written as $(1;1)$.

Activity 5.7:2
Equation 1: $2x+3y=7$.

Make x the subject of the **first** equation:

$$\begin{aligned} 2x + 3y &= 7 \\ 2x + 3y - 3y &= 7 - 3y \\ 2x &= 7 - 3y \\ \frac{2x}{2} &= \frac{7}{2} - \frac{3}{2}y \\ x &= \frac{7}{2} - \frac{3}{2}y. \end{aligned}$$

Equation 2: $x + 2y = 9$.
To solve for y, substitute x into the **second** equation:

$$\begin{aligned} x - y &= 2 \\ \left(\frac{7}{2} - \frac{3}{2}y\right) - y &= 2 \\ \frac{2}{1} \times \left(\frac{7}{2} - \frac{3}{2}y\right) + 2 \times -y &= 2 \times 2 \\ 7 - 3y - 2y &= 4 \\ 7 - 5y &= 4 \\ 7 - 5y - 7 &= 4 - 7 \\ -5y &= -3 \\ \frac{-5y}{-5} &= \frac{-3}{-5} \\ y &= \frac{3}{5}. \end{aligned}$$

To solve for x, substitute $y = \frac{3}{5}$ into $x = \frac{7}{2} - \frac{3}{2}y$:

$$\begin{aligned} x &= \frac{7}{2} - \frac{3}{2}y \\ &= \frac{7}{2} - \frac{3}{2} \times \frac{3}{5} \\ &= \frac{7}{2} - \frac{9}{10} \\ &= \frac{35}{10} - \frac{9}{10} \\ &= \frac{26}{10} \\ &= \frac{13}{5} \\ &= 2\frac{3}{5}. \end{aligned}$$

The solution is $x = 2\frac{3}{5}$ and $y = \frac{3}{5}$, or $\left(2\frac{3}{5}; \frac{3}{5}\right)$.

Activity 5.7:3
The equations are

$$2x + 3y = 4 \qquad (1)$$

and

$$x - 2y = -5 \qquad (2)$$

Multiply every term in equation (2) by 2:

$$2 \times x - 2 \times 2y = 2 \times -5.$$

Call this equation (3):

$$2x - 4y = -10. \qquad (3)$$

Subtract (3) from (1):

$$\begin{aligned} 2x + 3y &= 4 & (1) \\ -2x + 4y &= 10 & -(3) \\ \hline 7y &= 14 \end{aligned}$$

giving

$$y = 2.$$

Substitute $y = 2$ into (1):

$$\begin{aligned} 2x + 3y &= 4 \\ 2x + 3 \times 2 &= 4 \\ 2x + 6 &= 4 \\ 2x + 6 - 6 &= 4 - 6 \\ 2x &= -2 \\ \frac{2x}{2} &= \frac{-2}{2} \\ x &= -1. \end{aligned}$$

The solution is $x = -1$ and $y = 2$, or $(-1; 2)$.

Activity 5.7:4
Equation 1:

$$\begin{aligned} x + 2y &= -5 \\ 2y &= -x - 5 \\ y &= -\frac{1}{2}x - \frac{5}{2}. \end{aligned}$$

Determine the ***y*-intercept**. If $x = 0$, then

$$\begin{aligned} y &= -\frac{1}{2} \times 0 - \frac{5}{2} \\ &= -2\frac{1}{2}. \end{aligned}$$

The point is $(0; -2\frac{1}{2})$. Call this point A.

Determine the **x-intercept**. If $y = 0$, then

$$\begin{aligned} 0 &= -\frac{1}{2}x - \frac{5}{2} \\ \frac{1}{2}x &= -\frac{5}{2} \\ \frac{2}{1} \times \frac{1}{2}x &= \frac{2}{1} \times -\frac{5}{2} \\ x &= -5. \end{aligned}$$

The point is $(-5; 0)$. Call this point B.

Equation 2:

$$\begin{aligned} 3x - 2y &= -3 \\ -2y &= -3x - 3 \\ y &= \frac{3}{2}x + \frac{3}{2}. \end{aligned}$$

Determine the **y-intercept**. If $x = 0$, then

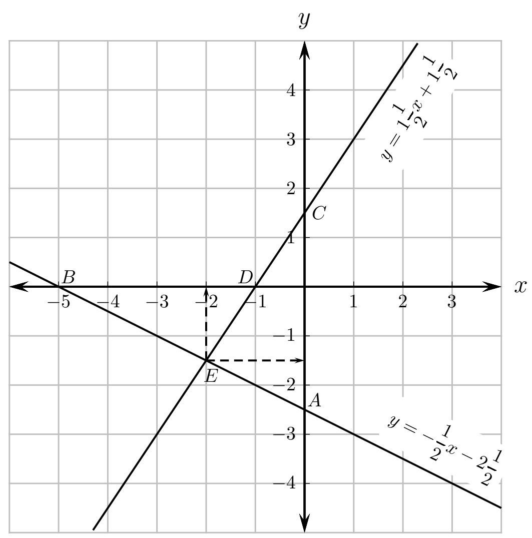
$$\begin{aligned} y &= \frac{3}{2} \times 0 + \frac{3}{2} \\ &= 1\frac{1}{2}. \end{aligned}$$

The point is $\left(0; 1\frac{1}{2}\right)$. Call this point C.
Determine the **x-intercept**. If $y = 0$, then

$$\begin{aligned} 0 &= \frac{3}{2}x + \frac{3}{2} \\ -\frac{3}{2}x &= \frac{3}{2} \\ -\frac{2}{3} \times -\frac{3}{2}x &= -\frac{2}{3} \times \frac{3}{2} \\ x &= -1. \end{aligned}$$

The point is $(-1; 0)$. Call this point D.

The solution is given in the following graph where the two lines intersect. That is at point E with coordinates $\left(-2; -1\frac{1}{2}\right)$.



Activity 5.8:1
If the price of a pen is R11 more than the price of a pencil, then

$$\begin{aligned} \text{price of pencil} + \text{R11} &= \text{price of pen} \\ R + 11 &= P \\ R - P &= -11 \\ P - R &= 11. \end{aligned}$$

Name this Equation 1.
The mathematical expression for the second equation is

$$\begin{aligned} P + 3 &= 2(R + 3) \\ P + 3 &= 2R + 6 \\ P + 3 - 2R &= 6 \\ P - 2R &= 6 - 3 \\ P - 2R &= 3. \end{aligned}$$

Name this Equation 2.

Activity 5.8:2
Let p be the price of 1 kg potatoes.
Let a be the price of 1 kg apples.

The two equations are

$$6p + 3a = 60$$

and

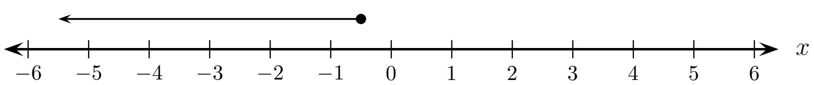
$$5p + 2a = 45.$$

Activity 5.9:1
To obtain x on the left-hand side, the inequality has to be rewritten as

$$\begin{aligned} 3x - 11x &\geq 4 \\ -8x &\geq 4 \\ \frac{-8x}{-8} &\leq \frac{4}{-8} \\ x &\leq -\frac{1}{2}. \end{aligned}$$

The sign changed from $\geq$ to $\leq$ because we divided by a negative number.

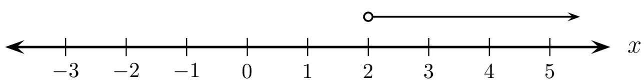
It is represented on the number line as follows:



Activity 5.9:2(a)
To obtain x on the left-hand side, the inequality has to be rewritten as

$$\begin{aligned} 2x-6 &> 2(1-x) \\ 2x-6 &> 2-2x \\ 2x-6+2x &> 2-2x+2x \\ 4x-6 &> 2 \\ 4x-6+6 &> 2+6 \\ 4x &> 8 \\ \frac{4x}{4} &> \frac{8}{4} \\ x &> 2. \end{aligned}$$

This is represented by graph C.



Activity 5.9:2(b)
To obtain x on the left-hand side, the inequality has to be rewritten as

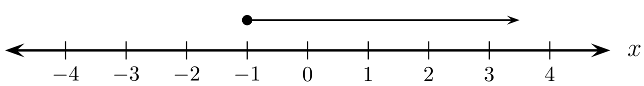
$$\begin{aligned} \frac{x}{4} &> 16 \\ \frac{4}{1}\times\frac{x}{4} &> 4\times 16 \\ x &> 64. \end{aligned}$$

No graph fits this scenario.

Activity 5.9:2(c)
To obtain x on the left-hand side, the inequality has to be rewritten as

$$\begin{aligned} \frac{x+3}{2} &\leq 2x+3 \\ \frac{2}{1}\times\left(\frac{x+3}{2}\right) &\leq 2\times(2x+3) \\ x+3 &\leq 4x+6 \\ x+3-4x &\leq 6 \\ -3x &\leq 6-3 \\ -3x &\leq 3 \\ \frac{-3x}{-3} &\geq \frac{3}{-3} \\ x &\geq -1. \end{aligned}$$

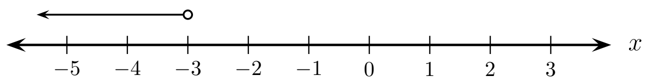
This is represented by graph A.



Activity 5.9:2(d)
To obtain x on the left-hand side, the inequality has to be rewritten as

$$\begin{aligned} -4x &> 12 \\ \frac{-4x}{-4} &< \frac{12}{-4} \\ x &< -3. \end{aligned}$$

This is represented by graph B.



Activity 5.9:2(e)
To obtain x on the left-hand side, the inequality has to be rewritten as

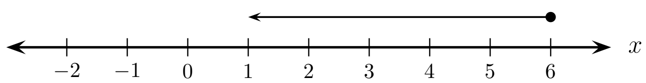
$$\begin{aligned} 2(x+5)-1 &\leq x+5 \\ 2x+10-1 &\leq x+5 \\ 2x+9 &\leq x+5 \\ 2x-x+9 &\leq 5 \\ x &\leq 5-9 \\ x &\leq -4. \end{aligned}$$

No graph fits this scenario.

Activity 5.9:2(f)
To obtain x on the left-hand side, the inequality has to be rewritten as

$$\begin{aligned} 2(7x-3) &\leq 12x+6 \\ 14x-6 &\leq 12x+6 \\ 14x-6-12x &\leq 6 \\ 2x &\leq 6+6 \\ 2x &\leq 12 \\ \frac{2x}{2} &\leq \frac{12}{2} \\ x &\leq 6. \end{aligned}$$

This is represented by graph D.



Activity 5.10
Firstly, rewrite the inequality to obtain y on the left-hand side:

$$\begin{aligned} 2x-3y &< 6 \\ -3y &< -2x+6 \\ \frac{-3y}{-3} &> \frac{-2x}{-3}+\frac{6}{-3} \\ y &> \frac{2}{3}x-2. \end{aligned}$$

The sign changed from $<$ to $>$ because we divide by a negative number.

Secondly, graph the inequality and find the solution region.

To get the intercepts on the two axes, replace the inequality sign temporarily by =.

The y-intercept is obtained when $x = 0$, thus

$$y = \frac{2}{3} \times 0 - 2 = -2.$$

The point is $(0;\, -2)$.
The x-intercept is obtained when $y = 0$, thus

$$\begin{aligned} 0 &= \frac{2}{3}x - 2 \\ \frac{2}{3}x &= 2 \\ x &= 3. \end{aligned}$$

The point is $(3;\, 0)$.
The = is not part of the inequality, therefore the line is dashed.

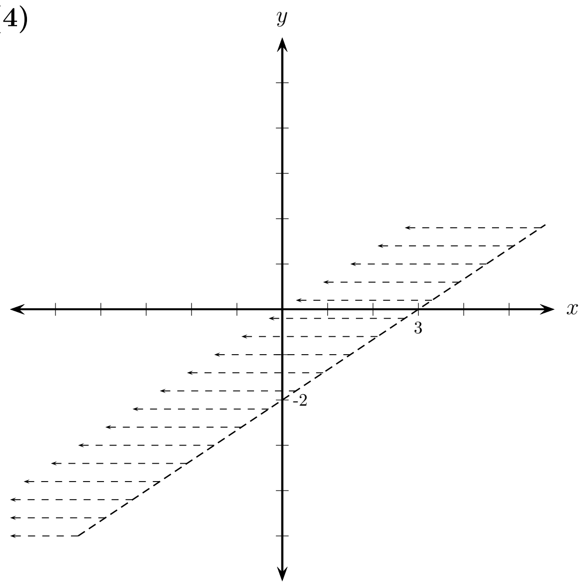
Select the point $(0; 0)$ that is not on the line. Substitute it into the inequality to see if it satisfies the inequality:

$$0 > \frac{2}{3}x \times 0 - 2 \quad \text{or} \quad 0 > -2,$$

which is true.

Therefore, all the points on the same side of the line as the point $(0; 0)$ satisfy the inequality. Colour or use lines to indicate the region of all $(x;\, y)$ points that satisfy the inequality. The correct graph is (4).

(4)



Activity 5.11
The system of inequalities is

$$\begin{aligned} x - y &\leq -2 \quad (1) \\ x - y &\geq 2. \quad (2) \end{aligned}$$

Firstly, rewrite the inequalities to obtain y on the left-hand side.
For inequality (1) it is

$$\begin{aligned} x - y &\leq -2 \\ -y &\leq -x - 2 \\ \frac{-y}{-1} &\geq \frac{-x}{-1} - \frac{2}{-1} \\ y &\geq x + 2. \end{aligned}$$

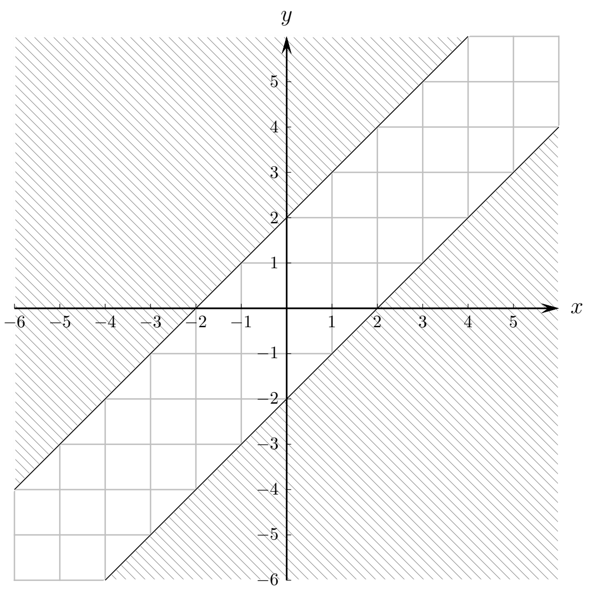
For inequality (2) it is

$$\begin{aligned} x - y &\geq 2 \\ -y &\geq -x + 2 \\ \frac{-y}{-1} &\leq \frac{-x}{-1} + \frac{2}{-1} \\ y &\leq x - 2. \end{aligned}$$

Secondly, graph each inequality using the table to assist you.

Inequality	***x*-intercept**	***y*-intercept**	**Line type**	**Point (0; 0)**
$x - y \leq -2$ or $y \geq x + 2$	$(-2;\ 0)$	$(0;\ 2)$	Solid	False
$x - y \geq 2$ or $y \leq x - 2$	$(2;\ 0)$	$(0;\ -2)$	Solid	False

The graphical representation of the inequalities is given below.



Thirdly, there is no place where the individual solutions overlap. (Note that the lines $y \geq x + 2$ and $y \leq x - 2$ never intersect, being parallel lines with different y-intercepts.) Since there is no intersection, there is **no solution**.

Activity 5.12:1
The system of inequalities is

$$\begin{aligned} 2x+3y-6 &\geq 0 \quad (1)\\ x-3y+6 &\geq 0 \quad (2)\\ x-4 &\leq 0 \quad (3)\\ x &\geq 0\\ y &\geq 0. \end{aligned}$$

Firstly, rewrite the first two inequalities to obtain y on the left-hand side and the third inequality to obtain x on the left-hand side.
For inequality (1) it is

$$\begin{aligned} 2x+3y-6 &\geq 0\\ 3y &\geq -2x+6\\ y &\geq -\frac{2}{3}x+2. \end{aligned}$$

For inequality (2) it is

$$\begin{aligned} x-3y+6 &\geq 0\\ -3y &\geq -x-6\\ y &\leq \frac{1}{3}x+2. \end{aligned}$$

For inequality (3) it is

$$\begin{aligned} x-4 &\leq 0\\ x &\leq 4. \end{aligned}$$

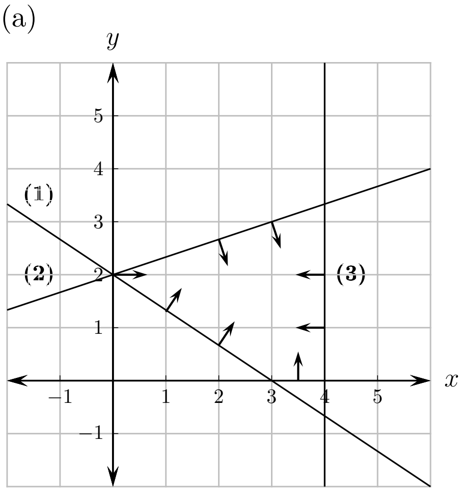
This is a line parallel to the y-axis through the point $x = 4$.

Secondly, graph each inequality.

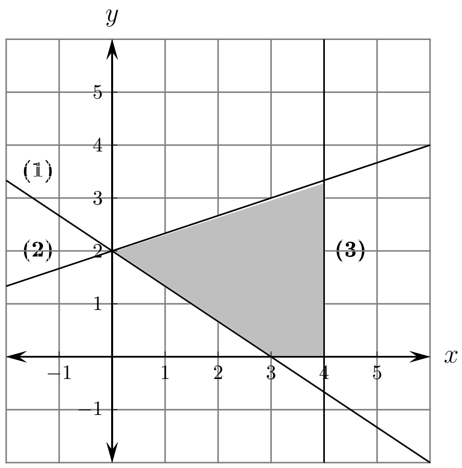
Inequality	**x-intercept**	**y-intercept**	**Line type**	**Point $(0; 0)$**
$2x+3y-6 \geq 0$ or $y \geq -\frac{2}{3}x+2$	(3; 0)	(0; 2)	Solid	False
$x-3y+6 \geq 0$ or $y \leq \frac{1}{3}x+2$	(−6; 0)	(0; 2)	Solid	True

Thirdly, we use arrows in graph (a) to indicate the region of all the $(x; y)$ points that satisfy the system of inequalities. The last two inequalities imply the first quadrant, including the axes. The solution region is shown in graph (b) as the shaded part. It is also closed or bounded because there are lines on all the sides of the solution region.

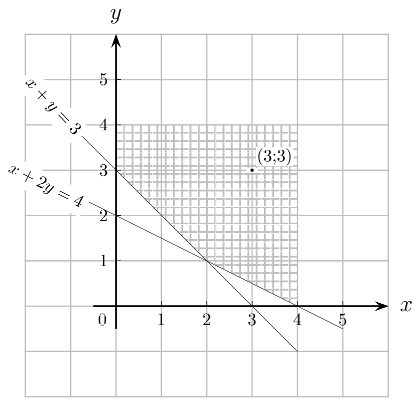
(a)



(b)



Activity 5.12:2



* The equation $x + y = 3$ can be rewritten to $y = -x + 3$.

It is a solid line, thus the $=$ sign will be included in the inequality.

Take a point that lies in the solution region, for example (3; 3) and substitute it into $y = -x + 3$, but omit the $=$ sign. Obtain the inequality sign that will satisfy the inequality. Since

$$\begin{aligned} 3 &\geq -3 + 3 \\ 3 &\geq 0, \end{aligned}$$

the $\geq$ sign satisfies this inequality.

Hence

$$y \geq -x + 3.$$

* The equation $x + 2y = 4$ can be rewritten to $y = -\frac{1}{2}x + 2$.

It is a solid line, thus the $=$ sign will be included in the inequality.

Take a point that lies in the solution region, for example (3; 3) and substitute it into $y = -\frac{1}{2}x + 2$, but omit the $=$ sign. Obtain the inequality sign that will satisfy the inequality. Since

$$\begin{aligned} 3 &\geq -\frac{1}{2} \times 3 + 2 \\ 3 &\geq \frac{1}{2}, \end{aligned}$$

the $\geq$ sign satisfies this inequality.

Hence

$$y \geq -\frac{1}{2}x + 2.$$

The last two inequalities imply the first quadrant including the axes. The correct system of linear inequalities is option (c):

$$\begin{aligned} y &\geq -x + 3 \\ y &\geq -\frac{1}{2}x + 2 \\ x &\geq 0 \\ y &\geq 0. \end{aligned}$$

Component 6. An application of differentiation

Activity 6.1:1

Function $f(x)$	**Derivative** $f'(x)$
$f(x) = x^5$	$f'(x) = 5x^4$
$f(x) = x^{-4}$	$f'(x) = -4 \times x^{-4-1} = -4x^{-5}$
$f(x) = x^{\frac{2}{3}}$	$f'(x) = \frac{2}{3}x^{\frac{2}{3}-\frac{3}{3}} = \frac{2}{3}x^{-\frac{1}{3}}$
$f(x) = \sqrt{x} = x^{\frac{1}{2}}$	$f'(x) = \frac{1}{2}x^{\frac{1}{2}-\frac{2}{2}} = \frac{1}{2}x^{-\frac{1}{2}}$
$f(x) = 2x^4$	$f'(x) = 2 \times 4x^3 = 8x^3$
$f(x) = -6x^2$	$f'(x) = -6 \times 2x^1 = -12x$
$f(x) = \frac{3}{2}x^{\frac{2}{3}}$	$f'(x) = \frac{3}{2} \times \frac{2}{3}x^{\frac{2}{3}-\frac{3}{3}} = x^{-\frac{1}{3}}$
$f(x) = \frac{-x}{2} = -\frac{1}{2}x$	$f'(x) = -\frac{1}{2} \times x^0 = -\frac{1}{2} \times 1 = -\frac{1}{2}$
$f(x) = 12x^2 - 24x + 8$	$f'(x) = 12 \times 2x^1 - 24 \times 1x^0 + 0$ $= 24x - 24 \times 1 = 24x - 24$
$f(x) = \frac{-x^4}{2} + 3x^3 - 2x$ $= -\frac{1}{2}x^4 + 3x^3 - 2x$	$f'(x) = -\frac{1}{2} \times 4x^3 + 3 \times 3x^2 - 2$ $= -2x^3 + 9x^2 - 2$
$f(x) = 12x + 6\sqrt{x} - \frac{4}{x}$ $= 12x + 6x^{\frac{1}{2}} - 4x^{-1}$	$f'(x) = 12 + 6 \times \frac{1}{2}x^{\frac{1}{2}-\frac{2}{2}} - 4 \times -1x^{-1-1}$ $= 12 + 3x^{-\frac{1}{2}} + 4x^{-2}$ $= 12 + \frac{3}{\sqrt{x}} + \frac{4}{x^2}$

$x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$

$x^{-2} = \frac{1}{x^2}$

Activity 6.1:2(a)
The profit function is

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= -0{,}02x^2 + 400x - (100x + 200\,000) \\ &= -0{,}02x^2 + 400x - 100x - 200\,000 \\ &= -0{,}02x^2 + 300x - 200\,000. \end{aligned}$$

The derivative of $P(x)$ is

$$\begin{aligned} P'(x) &= -0{,}02 \times 2x^{2-1} + 300x^{1-1} \\ &= -0{,}04x + 300. \end{aligned}$$

The **marginal profit when 5 000** sound systems are produced and sold, is

$$\begin{aligned} P'(5\,000) &= -0{,}04 \times 5\,000 + 300 \\ &= 100. \end{aligned}$$

This means that when 5 000 sound systems are produced and sold, the profit will increase by R100 if one additional sound system is produced. Is this indeed the case? Calculate P(5 000) and P(5 001), that is the profit at 5 000 and 5 001 sound systems:

$$\begin{aligned} P(5\,000) &= -0{,}02 \times 5\,000^2 + 300 \times 5\,000 - 200\,000 \\ &= 800\,000, \\ P(5\,001) &= -0{,}02 \times 5\,001^2 + 300 \times 5\,001 - 200\,000 \\ &= 800\,099{,}98 \\ &\approx 800\,100. \end{aligned}$$

It is indeed the case. When 5 000 sound systems are produced and sold, the profit is R800 000 and when 5 001 sound systems are produced and sold, the profit is R800 100, an increase of R100.

The **marginal profit when 7 500** sound systems are produced and sold, is

$$\begin{aligned} P'(7\,500) &= -0{,}04 \times 7\,500 + 300 \\ &= 0. \end{aligned}$$

When 7 500 sound systems are produced and sold, the marginal profit is R0 which indicates that 7 500 is the number of sound systems that should be produced to maximise the profit.

The **marginal profit when 8 000** sound systems are produced and sold, is

$$\begin{aligned} P'(8\,000) &= -0{,}04 \times 8\,000 + 300 \\ &= -20. \end{aligned}$$

The correct option:

When 8 000 sound systems are produced and sold, the marginal profit is (positive/negative). This indicates that the profit will (increase/decrease) by R20 if one additional sound system is produced.

Activity 6.1:2(b)
The x-value where the profit is maximised is

$$\begin{aligned} P'(x) &= 0 \\ -0{,}04x + 300 &= 0 \\ -0{,}04x &= -300 \\ \frac{-0{,}04x}{-0{,}04} &= \frac{-300}{-0{,}04} \\ x &= 7\,500. \end{aligned}$$

The maximum profit is obtained when 7 500 items are produced and sold.

Activity 6.2:1(a)
The cost function is

$$C(x) = 0{,}0001x^3 - 0{,}08x^2 + 40x + 5\,000.$$

The marginal cost to manufacture x calculators is

$$\begin{aligned} C'(x) &= 0{,}0001 \times 3 \times x^{3-1} - 0{,}08 \times 2 \times x^{2-1} + 40 \times x^{1-1} + 0 \\ &= 0{,}0003 \times x^2 - 0{,}16 \times x^1 + 40 \times x^0 \\ &= 0{,}0003x^2 - 0{,}16x + 40. \end{aligned}$$

Activity 6.2:1(b)
The marginal cost to manufacture 400 calculators is

$$\begin{aligned} C'(400) &= 0{,}0003 \times 400^2 - 0{,}16 \times 400 + 40 \\ &= 24. \end{aligned}$$

Interpretation: At a production level of <u>400</u> calculators, the cost to manufacture one additional calculator, is <u>R24</u>. This is checked as follows:

For 400 calculators:

$$\begin{aligned} C(400) &= 0{,}0001 \times 400^3 - 0{,}08 \times 400^2 + 40 \times 400 + 5\,000 \\ &= 14\,600. \end{aligned}$$

The cost to manufacture 400 calculators is R14 600.

For 401 calculators:

$$\begin{aligned} C(401) &= 0{,}0001 \times 401^3 - 0{,}08 \times 401^2 + 40 \times 401 + 5\,000 \\ &= 14\,624{,}04. \end{aligned}$$

The cost to manufacture 401 calculators is <u>R14 624,04</u>.
The difference in cost is

$$\begin{aligned} C(401) - C(400) &= 14\,624{,}04 - 14\,600 \\ &= 24{,}04 \\ &\approx 24. \end{aligned}$$

It is indeed true.

Activity 6.2:1(c)
For 500 calculators:

$$\begin{aligned} C'(500) &= 0{,}0003 \times 500^2 - 0{,}16 \times 500 + 40 \\ &= 35. \end{aligned}$$

The marginal cost to manufacture 500 calculators is R35.
Interpretation:
At a production level of 500 calculators, the cost to manufacture one additional calculator, is R35.

Activity 6.2:2
The cost function is

$$C(x) = 500 + 5x.$$

The marginal cost function is $C'(x) = 5$ and

$$\begin{aligned} C'(100) &= 5, \\ C'(500) &= 5, \text{ and} \\ C'(1\,000) &= 5. \end{aligned}$$

Interpretation:
The function, $C(x) = 500 + 5x$, is a straight line. Thus, the marginal cost is always equal to 5. At any production level, it will cost R5 to produce one additional box of nougat bars.

Component 7. Mathematics of finance

Activity 7.1
The following is given:

$$\begin{aligned} P &= 5\,000 \\ R &= 7{,}5\% = \frac{7{,}5}{100} = 0{,}075 \\ T &= 5 \text{ years} \\ S &= ? \end{aligned}$$

The **first** method is to calculate I and then $S = P + I$.

The interest is

$$\begin{aligned} I &= PRT \\ &= 5\,000 \times 0{,}075 \times 5 \\ &= 1\,875. \end{aligned}$$

The interest is R1 875.

The amount at the end of the period is

$$\begin{aligned} S &= P + I \\ &= 5\,000 + 1\,875 \\ &= 6\,875. \end{aligned}$$

The amount is R6 875.

The **second** method is to calculate $S = P(1 + RT)$.

The amount is

$$\begin{aligned} S &= P(1+RT) \\ &= 5\,000 \times (1 + 0{,}075 \times 5) \\ &= 6\,875. \end{aligned}$$

The amount is R6 875.

Activity 7.2

The following is given:

$$\begin{aligned} S &= 6\,000 \\ R &= 10\% = \frac{10}{100} = 0{,}10 \\ T &= 4{,}5 \text{ years} \\ P &= ? \end{aligned}$$

The formula for the future value is

$$S = P(1+RT).$$

Therefore, the principal is calculated as

$$\begin{aligned} P &= \frac{S}{1+RT} \\ &= \frac{6\,000}{1+0{,}1\times 4{,}5} \\ &= 4\,137{,}93. \end{aligned}$$

The principal is R4 137,93.

The interest is

$$\begin{aligned} I &= S-P \\ &= 6\,000-4\,137{,}93 \\ &= 1\,862{,}07. \end{aligned}$$

The interest is R1 862,07.

The principal amount to be invested is R4 137,93 and the interest on the investment is R1 862,07.

Activity 7.3

The following is given:

$$\begin{aligned} I &= \text{interest } = 3\,100-2\,500 = 600 \\ R &= 0{,}06 \\ P &= 2\,500 \\ T &= ? \end{aligned}$$

From $I = PRT$ follows that

$$\begin{aligned} T &= \frac{I}{P\times R} \\ &= \frac{600}{2\,500\times 0{,}06} \\ &= 4. \end{aligned}$$

The time is 4 years.

Activity 7.4:1(a)

The actual amount of money received by the borrower, after the discount is deducted from the face/future value, is <u>R8 000</u>.

Activity 7.4:1(b)

The face/future value is

$$\begin{aligned} S &= \frac{P}{1-dT} \\ &= \frac{8\,000}{1-0{,}12\times \frac{3}{4}} \\ &= 8\,791{,}21. \end{aligned}$$

The face/future value is R8 791,21.

Activity 7.4:1(c)
The discount that will be subtracted from the face/future value is

$$\begin{aligned} D &= SdT \\ &= 8\,791{,}21 \times 0{,}12 \times \frac{3}{4} \\ &= 791{,}21, \end{aligned}$$

or

$$\begin{aligned} D &= S - P \\ &= 8\,791{,}21 - 8\,000 \\ &= 791{,}21. \end{aligned}$$

The discount is R791,21.

Activity 7.4:1(d)
The equivalent simple interest rate of the loan is

$$\begin{aligned} R &= \frac{I}{P \times T} \\ &= \frac{791{,}21}{8\,000 \times \frac{3}{4}} \\ &= 0{,}1319. \end{aligned}$$

The simple interest rate is $0{,}1319 \times 100 = 13{,}2\%$.

Activity 7.4:2(a)
The following is given:

$$\begin{aligned} S &= 3\,500 \\ d &= 0{,}14 \\ T &= \frac{6}{12} = \frac{1}{2} \text{ year.} \end{aligned}$$

The discount is

$$\begin{aligned} D &= SdT \\ &= 3\,500 \times 0{,}14 \times \frac{1}{2} \\ &= 245. \end{aligned}$$

The discount is R245.

Activity 7.4:2(b)
The amount that she receives now (present value) is

$$\begin{aligned} P &= S(1 - dT) \\ &= 3\,500 \times (1 - 0{,}14 \times \frac{1}{2}) \\ &= 3\,255, \end{aligned}$$

or

$$\begin{aligned} P &= S - D \\ &= 3\,500 - 245 \\ &= 3\,255. \end{aligned}$$

The amount is R3 255.

Activity 7.4:2(c)
The following is given:

$$P = 3\,255$$
$$I = 245$$
$$T = \frac{1}{2} \text{ year.}$$

From $I = PRT$ it follows that

$$\begin{aligned} R &= \frac{I}{P \times T} \\ &= \frac{245}{3\,255 \times \frac{1}{2}} \\ &= 0{,}1505. \end{aligned}$$

The equivalent simple interest rate is $0{,}1505 \times 100 = 15{,}05\% \approx 15\%$.

Activity 7.5

Term	**Interest rate per year**	**Period/ interval**	**Interest rate per period/interval**	**Number of periods/intervals**
5 years	8%	yearly/annually	per year: $\frac{0{,}08}{1} = 0{,}08$	no. of years: $5 \times 1 = 5$
18 months	$7\frac{1}{2}\%$	biannually/ half-yearly	per half year: $\frac{0{,}075}{2} = 0{,}0375$	no. of half years: $1{,}5 \times 2 = 3$
6 years	10%	monthly	per month: $\frac{0{,}10}{12}$	no. of months: $6 \times 12 = 72$
2 and a half years	$14\frac{1}{2}\%$	yearly/annually	per year: $\frac{0{,}145}{1} = 0{,}145$	no. of years: $2{,}5 \times 1 = 2{,}5$
9 months	15%	quarterly	per quarter: $\frac{0{,}15}{4} = 0{,}0375$	no. of quarters: $\frac{9}{12} \times 4 = 3$
3 and a half years	18%	monthly	per month: $\frac{0{,}18}{12} = 0{,}015$	no. of months: $3{,}5 \times 12 = 42$
5 and a half years	16%	quarterly	per quarter: $\frac{0{,}16}{4} = 0{,}04$	no. of quarters: $5{,}5 \times 4 = 22$

Activity 7.6
The following is given:

$$\begin{aligned} P &= 9\,000 \\ R &= 0{,}12 \\ T &= 4 \text{ years} \\ S &= ? \end{aligned}$$

From this we find

$$\begin{aligned} S &= P(1+R)^T \\ &= 9\,000 \times (1+0{,}12)^4 \\ &= 9\,000 \times 1{,}12^4 \\ &= 14\,161{,}67. \end{aligned}$$

After four years the total amount is R14 161,67.

The interest earned is

$$\begin{aligned} \text{interest} &= S-P \\ &= 14\,161{,}67 - 9\,000{,}00 \\ &= 5\,161{,}67. \end{aligned}$$

The interest earned is R5 161,67.

Activity 7.7:1
The following is given:

$$\begin{aligned} S &= 7\,500 \\ R &= \frac{0{,}08}{4} = 0{,}02 \\ T &= 1{,}5 \times 4 = 6 \text{ quarters} \\ P &= ? \end{aligned}$$

The present value is

$$\begin{aligned} P &= \frac{S}{(1+R)^T} \\ &= \frac{7\,500}{(1+0{,}02)^6} \\ &= \frac{7\,500}{1{,}02^6} \\ &= 6\,659{,}79. \end{aligned}$$

The amount borrowed, is R6 659,79.

Activity 7.7:2

For George the following is given:

$$\begin{aligned} P &= 200\,000 \\ R &= \frac{0{,}13}{4} = 0{,}0325 \\ T &= 5 \times 4 = 20 \text{ quarters} \\ S &= ? \end{aligned}$$

The amount available for George after five years is

$$\begin{aligned} S &= P(1+R)^T \\ &= 200\,000 \times (1+0{,}0325)^{20} \\ &= 379\,167{,}58. \end{aligned}$$

The amount for George is R379 167,58.

For Jim the following is given:

$$\begin{aligned} P &= 200\,000 \\ R &= 0{,}1375 \\ T &= 5 \text{ years} \\ S &= ? \end{aligned}$$

The amount available for Jim after five years is

$$\begin{aligned} S &= P(1+RT) \\ &= 200\,000 \times (1+0{,}1375 \times 5) \\ &= 337\,500{,}00. \end{aligned}$$

The amount for Jim is R337 500,00.

The total amount of money they have to pool after five years is

$$379\,167{,}58 + 337\,500{,}00 = 716\,667{,}58.$$

In total they have R716 667,58.

At this point we know the following:

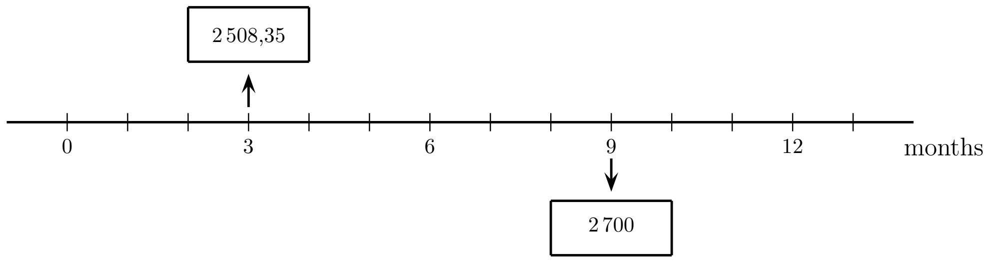
$$\begin{aligned} P &= 716\,667{,}58 \\ R &= \frac{0{,}12}{12} = 0{,}01 \\ T &= 5 \times 12 = 60 \text{ months} \\ S &= ? \end{aligned}$$

The total amount after ten years is

$$\begin{aligned} S &= P(1+R)^T \\ &= 716\,667{,}58 \times (1+0{,}01)^{60} \\ &= 1\,301\,967{,}63. \end{aligned}$$

The final amount is R1 301 967,63.

Activity 7.8:1
The completed time line is



@ 15% per annum, compounded quarterly

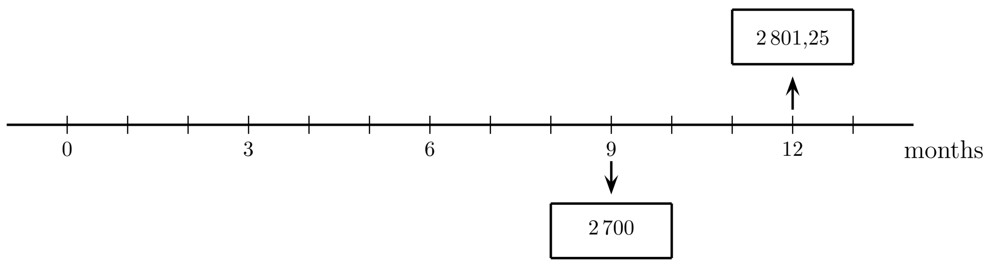
The payment is moved backwards six months.

That is two quarters. The amount that is due, is

$$\begin{aligned} P &= \frac{S}{(1+R)^T} \\ &= \frac{2\,700}{\left(1+\frac{0{,}15}{4}\right)^2} \\ &= 2\,508{,}35. \end{aligned}$$

The amount due is R2 508,35.

Activity 7.8:2
The completed time line is:



@ 15% per annum, compounded quarterly

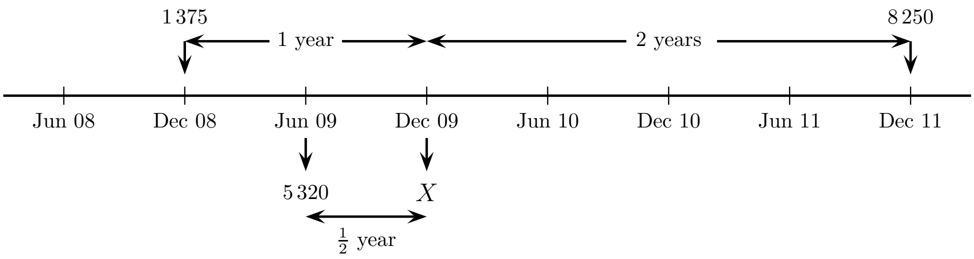
The payment is moved forward three months.

That is one quarter. The amount due is

$$\begin{aligned} S &= P(1+R)^T \\ &= 2\,700 \times \left(1+\frac{0{,}15}{4}\right)^1 \\ &= 2\,801{,}25. \end{aligned}$$

The amount due is R2 801,25.

Activity 7.9
The time line with debts above the line and payments below:



The **common date** is December 2009. Move the payment and the debts to this date.

The value of the R1 375 **debt** incurred in December 2008, in December 2009 is

$$\begin{aligned} S &= P \times (1+R)^T \\ &= 1\,375 \times (1+0{,}17)^1 \\ &= 1\,608{,}75. \end{aligned}$$

The value of the debt is R1 608,75.

The value of the R8 250 **debt** payable in December 2011, in December 2009 is

$$\begin{aligned} P &= \frac{S}{(1+R)^T} \\ &= \frac{8\,250}{(1+0{,}17)^2} \\ &= 6\,026{,}74. \end{aligned}$$

The value of the debt is R6 026,74.

The value of the R5 320 **payment** made in June 2009, in December 2009 is

$$\begin{aligned} S &= P \times (1+R)^T \\ &= 5\,320 \times (1+0{,}17)^{0{,}5} \\ &= 5\,754{,}46. \end{aligned}$$

The value of the payment is R5 754,46.

To find the amount that will settle the debt in December 2009, all payments must be set equal to the outstanding debt. Suppose the payment that will settle the remaining debt is X. Then

$$\begin{aligned} \text{payments} &= \text{debts} \\ X + 5\,754{,}46 &= 1\,608{,}75 + 6\,026{,}74 \\ X + 5\,754{,}46 &= 7\,635{,}49 \\ X &= 7\,635{,}49 - 5\,754{,}46 \\ &= 1\,881{,}03. \end{aligned}$$

Thus, the balance due at the end of December 2009, will be R1 881,03.

Activity 7.10

For the future value of the annuity calculation, the following is given:

$$\begin{aligned} R &= 750 \\ i &= \frac{0{,}13}{4} = 0{,}0325 \\ n &= 4{,}5 \times 4 = 18 \text{ quarters} \\ S &= ? \end{aligned}$$

From this follows that

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ &= R \times \left[\frac{(1+i)^n - 1}{i}\right] \\ &= 750 \times \left[\frac{(1+0{,}0325)^{18} - 1}{0{,}0325}\right] \\ &= 750 \times \left[\frac{1{,}0325^{18} - 1}{0{,}0325}\right] \\ &= 17\,962{,}29. \end{aligned}$$

The accumulated amount is R17 962,29.

Activity 7.11

For the size of the deposit calculation, when the future value of the annuity is given, we have the following information:

$$\begin{aligned} S &= 200\,000 \\ i &= \frac{0{,}16}{4} = 0{,}04 \\ n &= 10 \times 4 = 40 \text{ quarters} \\ R &= ? \end{aligned}$$

From this follows that

$$\begin{aligned} R &= S \times \left[\frac{i}{(1+i)^n - 1}\right] \\ &= 200\,000 \times \left[\frac{0{,}04}{(1+0{,}04)^{40} - 1}\right] \\ &= \frac{200\,000 \times 0{,}04}{1{,}04^{40} - 1} \\ &= 2\,104{,}70. \end{aligned}$$

The deposit every quarter will be R2 104,70.

Activity 7.12

For the calculation of the present value of an annuity, the following information is given:

$$\begin{aligned} R &= 1\,350 \\ i &= 0{,}12 \\ n &= 7 \text{ years} \\ P &= ? \end{aligned}$$

The present value is calculated as

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ &= R \times \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \\ &= 1\,350 \times \left[\frac{(1+0{,}12)^7 - 1}{0{,}12(1+0{,}12)^7}\right] \\ &= 1\,350 \times \left[\frac{1{,}12^7 - 1}{0{,}12(1{,}12)^7}\right] \\ &= 6\,161{,}07. \end{aligned}$$

The present value of the annuity is R6 161,07.

Activity 7.13

For the calculation of the size of the payment, when the present value of the annuity is given, we have the following information:

$$\begin{aligned} P &= 5\,600 - 1\,250 = 4\,350 \\ i &= \frac{0{,}10}{2} = 0{,}05 \\ n &= 4 \times 2 = 8 \text{ half years} \\ R &= ? \end{aligned}$$

The half-yearly payments are

$$\begin{aligned} R &= P \times \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \\ &= 4\,350 \times \left[\frac{0{,}05(1+0{,}05)^8}{(1+0{,}05)^8 - 1}\right] \\ &= \frac{4\,350 \times 0{,}05 \times 1{,}05^8}{1{,}05^8 - 1} \\ &= 673{,}04. \end{aligned}$$

The payment every six months is R673,04.

Activity 7.14

The following is given:

$$\begin{aligned} P &= 25\,000 \\ i &= \frac{0{,}08}{4} = 0{,}02 \\ n &= 2 \times 4 = 8 \text{ quarters} \\ R &= ? \end{aligned}$$

The quarterly payments are

$$\begin{aligned} R &= P \times \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \\ &= 25\,000 \times \left[\frac{0{,}02(1+0{,}02)^8}{(1+0{,}02)^8 - 1}\right] \\ &= 3\,412{,}74. \end{aligned}$$

The quarterly payments are R3 412,74 each.

The interest due at the end of each quarter is calculated as $I = PRT$, where P is the outstanding principal at the beginning of the quarter, R is the interest rate per year, and T is the time in years. In this case it is quarters of years, therefore $T = 0{,}25$. The interest is calculated as

$$\begin{aligned} I &= P \times R \times T \\ &= P \times 0{,}08 \times 0{,}25. \end{aligned}$$

The completed table is:

Quar- -ter	**Outstanding principal at beginning of quarter**	**Interest due at end of quarter (simple)** $I = PRT^*$	**Payment**	**Principal repaid**
1	25 000,00	25 000,00 × 0,08 × 0,25 = 500,00	3 412,74	3 412,74 − 500,00 = 2 912,74
2	25 000,00 − 2 912,74 = 22 087,26	22 087,26 × 0,08 × 0,25 = 441,75	3 412,74	3 412,74 − 441,75 = 2 970,99
3	22 087,26 − 2 970,99 = 19 116,27	19 116,27 × 0,08 × 0,25 = 382,33	3 412,74	3 412,74 − 382,33 = 3 030,41
4	19 116,27 − 3 030,41 = 16 085,85	16 085,85 × 0,08 × 0,25 = 321,72	3 412,74	3 412,74 − 321,72 = 3 091,02
5	16 085,85 − 3 091,02 = 12 994,83	12 994,83 × 0,08 × 0,25 = 259,90	3 412,74	3 412,74 − 259,90 = 3 152,84
6	12 994,83 − 3 152,84 = 9 841,98	9 841,98 × 0,08 × 0,25 = 196,84	3 412,74	3 412,74 − 196,84 = 3 215,90
7	9 841,98 − 3 215,90 = 6 626,08	6 626,08 × 0,08 × 0,25 = 132,52	3 412,74	3 412,74 − 132,52 = 3 280,22
8	6 626,08 − 3 280,22 = 3 345,87	3 345,87 × 0,08 × 0,25 = 66,92	3 412,74	3 412,74 − 66,92 = 3 345,82
	Total	2 301,96	27 301,92	24 999,96

After 15 months, five payments have been made – there are three months in one quarter, therefore $15 \div 3 = 5$.

Determine the number of payments that remains as

$$\begin{aligned} \text{no. of payments that remains} &= \text{initial no. of payments} - \text{no. of payments already made} \\ &= 8 - 5 \\ &= 3. \end{aligned}$$

The present value of the loan after 15 months (when three payments remain) is

$$\begin{aligned} P &= R \times \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \\ &= 3\,412{,}74 \times \left[\frac{(1+0{,}02)^3 - 1}{0{,}02(1+0{,}02)^3}\right] \\ &= 9\,841{,}94. \end{aligned}$$

The present value is R9 841,94.

The new interest rate is $(9 \div 4)\% = 2{,}25\%$ per quarter.

The new payments are now

$$\begin{aligned} R &= P \times \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \\ &= 9\,841{,}94 \left[\times \frac{0{,}0225(1+0{,}0225)^3}{(1+0{,}0225)^3 - 1}\right] \\ &= 3\,429{,}37. \end{aligned}$$

The new payments are R3 429,37.