Assignment 3 - Semester 1 Solutions

Question 1

We use the formula: $S = P \times (1+R)^T$

where $P = 3\,500$, $R = 7{,}5\% = 0{,}075$, $S = 4\,044.69$ and $T = ?$

Since 'T' is the unknown, it is always best to arrange the formula and make 'T' the subject.

$$S = P(1+R)^T$$

$$(1+R)^T = \frac{S}{P}$$

$$\log\left((1+R)^T\right) = \log\left(\frac{S}{P}\right)$$

$$T \log(1+R) = \log\left(\frac{S}{P}\right)$$

$$T = \frac{\log\left(\frac{S}{P}\right)}{\log(1+R)}$$

$$= \frac{\log\left(\frac{4044.69}{3\,500}\right)}{\log(1+0.075)}$$

$$= 2.0$$

$\therefore$ It will take 20 years for the investment to reach R4 044.69.

Answer: Option 2

Question 2

We use the formula: $S = P(1+RT)$, with $P = 5000$, $R = 0.125$, $T = 2$ and $S = ?$.

$$\therefore \quad S = 5000(1 + 0.125 \times 2)$$
$$= 6250$$

$\therefore$ lump sum payment amount is R6250.

Answer: Option 1

Question 3

We use the formula $S = P(1+RT)$ with $S = 4000$, $P = 2500$, $T = 5$ and $R = ?$

$$S = P(1+RT)$$

$$4000 = 2500(1 + R \times 5)$$

$$\frac{4000}{2500} = 1 + 5R$$

$$5R = \frac{4000}{2500} - 1$$

$$R = \frac{\frac{4000}{2500} - 1}{5}$$

$$= 0.12$$

$\therefore$ The desired rate is 12%

Answer: Option 1

Question 4

We use $S = P(1+RT)$, where $P = 6610$, $S = 11\,131.24$, $T = 12$ and $R = ?$

$$S = P(1+RT)$$

$$11\,131.24 = 6610(1 + R \times 12)$$

$$R = \frac{\left(\frac{11\,131.24}{6610}\right) - 1}{12}$$

$$= 0.057 = 5.7\%$$

The simple interest rate is 5.7%

Answer: Option 4

Question 5

We use $S = P(1+R)^T$, where $P = R\,3500$, $R = 0.075$, $T = 5$ and $S = ?$

$$\therefore S = P(1+R)^T$$
$$= 3\,500(1+0.075)^5$$
$$= 5\,024.70$$

Investment accumulates to R 5 024.70.

Answer: Option 3.

Question 6

We use $S = P(1+R)^T$, where $S = ?$, $P = 60\,000$, $R = 0.10/4 = 0.025$, $T = 4 \times 4 = 16$

$$\therefore S = 60\,000(1+0.025)^{16}$$
$$= 89\,070.34$$

$\therefore$ The investment accumulates to R 89 070.34 after 4 years.

Answer: Option 3

Question 7

We use $S = P(1+R)^T$, where $S = 100\,000$, $P = ?$, $R = 0.15/12 = 0.0125$, $T = 5 \times 12 = 60$

$$\therefore \quad 100\,000 = P(1+0.0125)^{60}$$

$$P = \frac{100\,000}{(1+0.0125)^{60}}$$

$$= 47\,456.76$$

$\therefore$ The couple should invest R47 456.76.

Answer: Option 2

Question 8

In this question we use the formula $S = P(1+R)^T$. There are three investments that differ in size and period. In all these investments, the future value is unknown.

① $S = ?$ $P = 2000$ $R = 0.05/12$ $T = 6 \times 12 = 72$ [6 years]

② $S = ?$ $P = 4000$ $R = 0.05/12$ $T = 3 \times 12 = 36$ [from the end of 3rd year]

③ $S = ?$ $P = 5000$ $R = 0.05/12$ $T = 1 \times 12 = 12$ [from the end of the 5th year]

① $S = 2000 \times (1 + 0.05/12)^{72} = 2\,698.04$

② $S = 4000 \times (1 + 0.05/12)^{36} = 4\,645.89$

③ $S = 5000 \times (1 + 0.05/12)^{12} = \underline{5\,255.81}$

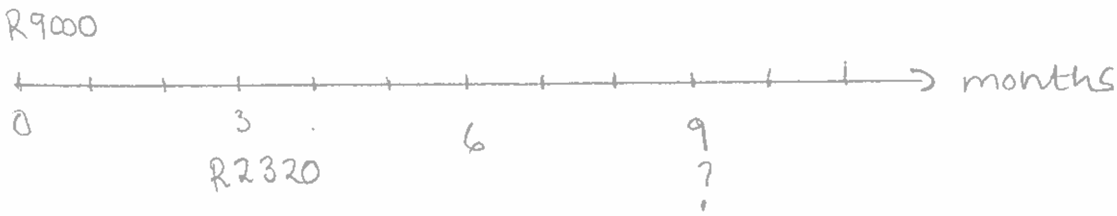
$12\,599.74$

$\therefore$ The accumulated sum is R12 599.74

Answer: Option 4.

Question 9

When on encounters questions like this one it is always best to draw a time line for that question. This question is represented by the time line below:



In this question we use the equation $S = P(1+R)^T$, were $P = 9000$ and $R = 0.10/4 = 0.025$

First we need to calculate the amount of the investment before the withdrawal, thus after 3 months. The payment period of this investment is quarterly, therefore every 3 months. In this case $T = 1$.

$\therefore S = 9000(1+0.025)^1 = 9\,225.$

There after we reduce the amount of the investment by the withdrawal amount, to get the new present value.

$P = 9225 - 2320 = 6905.$

From months 3 to 9, there are two payment periods. This is after month 6 and month 9. Therefore $T = 2$.

$\therefore S = 6905(1+0.025)^2 = 7\,254.57$

So Jaco has R7 254.57 after 9 months.

Answer: Option 1

Question 10

In this question we use the equation $P = R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$
where $P = ?$, $R = 1000$, $i = 0.09/12 = 0.0075$
and $n = 24$.

$$\therefore \quad P = 1000\left[\frac{((1+0.0075)^{24}-1)}{(0.0075(1+0.0075)^{24})}\right]$$

$$= 21\ 889.15$$

$\therefore$ Present value is R21 889.15

Answer: Option 2

Question 11

In this question we use the formula $P = R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$
where $P = ?$, $R = 1000$, $i = 0.12/4 = 0.03$; $n = 24$.

$$\therefore \quad P = 1000\left[\frac{(1+0.03)^{24}-1}{0.03(1+0.03)^{24}}\right]$$

$$= 16\ 935.54$$

The present value is R16 935.54.

Answer: Option 2

Question 12

The loan taken up by Michael is represented by the present value of an annuity where $R = 4000$; $i = 0.12/12 = 0.01$; $n = 12 \times 3 = 36$.

We use the formula:

$$P = R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= 4000\left[\frac{(1+0.01)^{36} - 1}{0.01(1+0.01)^{36}}\right]$$

$$= 120\ 430.02$$

The present value of the loan is R120 430 02.

To calculate the interest charges, first calculate the total payment for three years and subtract the loan amount.

$\therefore\ 36 \times 4000 - 120\ 430.02 = 23\ 569.98$

$\therefore$ The total interest paid is R23 569.98

Answer: Option 3

Question 13

In this question 1 payment is made immediately; thereafter 19 payment follow annually. Therefore we need to find the present value of an annuity for the 19 payments. Where $R = 500\ 000$, $n = 19$, $i = 0.09$.

We use $P = R\left[\frac{(1+i)^n - 1}{0.09(1+0.09)^n}\right]$

$$= 500\ 000\left[\frac{(1+0.09)^{19} - 1}{0.09(1+0.09)^{19}}\right] = 4\ 475\ 057.39$$

We then include the R500 000 that was payed immediately,

$500\ 000 + 4\ 475\ 057.39 = R4\ 975\ 057.39$.

Therefore the bank needs R4 975 057.39 in the bank in order to guarantee payment.

Answer: Option 1

Question 14

In this question we have two investments that run parallel. The first is that of $P = 1\,500\,000$, $R = 0.08/4 = 0.02$, $T = 4 \times 20 = 80$. We need to find the future value of this investment using compound interest formula:

$$S = P(1+R)^T = 1\,500\,000(1+0.02)^{80} = 7\,313\,158.73$$

The second investment is that of R30 000 quarterly payments for 20 years, where $R = 30\,000$, $i = 0.08/4 = 0.02$; $n = 4 \times 20 = 80$. Therefore, we need to find the future value of the annuity.

$$S = R\left[\frac{(1+i)^n - 1}{i}\right] = 30\,000\left[\frac{(1+0.02)^{80} - 1}{0.02}\right] = 5\,813\,158.73$$

∴ The amount accumulated over twenty years is the sum of the two investments.

∴ R7 313 158.73 + R5 813 158.73 = R13 126 317.46

Answer: Option 4

Question 15

In this question there are two investments. The first is that of $P = 50\,000$, $R = 0.06/12 = 0.005$; $T = (5 \times 12) + 1 = 61$ (months). We therefore use tc compound interest formula to find the future value:

$$S = P(1+R)^T = 50\,000(1+0.005)^{61} = 67\,779.72$$

The second investment is an annuity were $R = 2000$; $i = 0.06/12 = 0.005$, $n = 5 \times 12$. Therefore we find the future value of the annuity:

$$S = R\left[\frac{(1+i)^n - 1}{i}\right] = 2000\left[\frac{(1+0.005)^{60} - 1}{0.005}\right] = 139\,540.06$$

The amount accumulated over five years is the sum of the two investments.

∴ R67 779.72 + R139 540.06 = R207 319.78

Answer: Option 4

Question 16

In this question we seek to find the monthly payment 'R' of an ordinary annuity. Therefore $R = ?$; $P = R\ 1200\ 000$; $i = 0.09/12 = 0.0075$, $n = 12 \times 20 = 240$.

We use the formula:

$$P = R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$\therefore R = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

$$= 1200000 \left[\frac{(0.0075(1+0.0075)^{240})}{((1+0.0075)^{240} - 1)}\right]$$

$$= 10\ 796.71$$

Oupa will get paid R10 796.71 per month.

Answer: Option 3

Question 17

In this question we seek to find the monthly payment of a loan, where $R = ?$; $P = 2250\ 000$; $i = 0.06/12 = 0.005$; $n = 30 \times 12 = 360$.

We use the formula: $P = R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

$$R = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

$$= 2\ 250\ 000 \left[\frac{(0.005(1+0.005)^{360})}{((1+0.005)^{360} - 1)}\right]$$

$$= 13\ 489.89$$

The monthly payment is R13 489.89

Answer: Option 3

Question 18

In this question we need to find the monthly payment on the loan. Where $P = 1400\,000$; $n = 15 \times 12 = 180$; $i = 0.105/12 = 0.00875$

We use the formula: $P = R\left[\dfrac{(1+i)^n - 1}{i(1+i)^n}\right]$

$$R = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

$$= 1400\,000 \left[\frac{0.00875(1+0.00875)^{180}}{(1+0.00875)^{180} - 1}\right]$$

$$= 15\ 475.58$$

$\therefore$ The monthly payment is R15 475.58

Answer: Option 4

Question 19

In this question we are given a down-payment of R6000, and $R = 300$ $n = 3 \times 12 = 36$; $i = 0.0125$.. Therefore we need to find the present value of the loan $P = ?$, using the annuity formula.

$$P = R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = 300\left[\frac{((1+0.0125)^{36} - 1)}{(0.0125(1+0.0125)^{36})}\right]$$

$$= 8654.18$$

To get the total cost of the coffee machine we will add the loan amount to the down payment amount.

$\therefore$ R6000 + R 8 654.18 = R 14 654.18

Answer: Option 2

Question 20

In order to find the total interest paid by Thandi, we first need to calculate the monthly payment (R). We use the formula for the present value of an ordinary annuity. Where $P = 300\,000$, $n = 5 \times 12 = 60$, $i = 0.12/12 = 0.01$; $R = ?$

but $$P = R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$\therefore \quad R = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

$$= 300\,000\left[\frac{(0.01(1+0.01)^{60})}{((1+0.01)^{60} - 1)}\right]$$

$$= 6673.33$$

$\therefore$ Thandi pays R6673.33 for 60 months. This totals R6673.33 × 60 = R400 399.80.

We subtract the total payment from the original purchase price of R300 000.

$\therefore$ Total interest paid is R400 399.80 - R300 000 = R100 399.80

Answer: Option 1

Question 21

In order to answer the question, one first needs to find the present value of an ordinary annuity where R = 20 000, $n = 20 \times 12 = 240$, $i = 0.09/12 = 0.0075$.

$$\therefore P = R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= 20\,000\left[\frac{(1+0.0075)^{240} - 1}{0.0075(1+0.0075)^{240}}\right]$$

$$= 2\,222\,899.08.$$

$\therefore$ The (maximum) loan amount that they can afford is R2 222 899.08

To get the maximum price of a house that they can afford, we need to add the maximum loan amount to the down payment.

R600 000 + R 2 222 899.08 = R2 822 899.08

Answer: Option 2

Question 22

In this question we need to find the yearly payment (R) using the formula for the present value of an ordinary annuity; where $P = 500\,000$, $i = 0.08$ and $n = 5$.

$$P = R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$\therefore R = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

$$= 500000\left[\frac{(0.08(1+0.08)^5)}{((1+0.08)^5 - 1)}\right] = 125\,228.23$$

$\therefore$ The yearly payment is R 125 228.23.

Answer: Option 1

Question 23

The original purchase price is R 2000 000, therefore a 20% downpayment is 20% x R 2000 000 = R 400 000. Therefore loan amount is R2000 000 - R400 000 = R1 600 000.

To solve the problem we first need to find the monthly installment on the home loan, Using the formula for the present value of an ordinary annuity; where P = 1600 000, i = 0.09/12 = 0.0075; and n = 30x12 = 360.

$$\therefore \quad P = R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$R = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

$$= 1600\,000 \left[\frac{(0.0075(1+0.0075)^{360})}{((1+0.0075)^{360} - 1)}\right]$$

$$= 12\,873.96.$$

So the monthly payment is R12 873.96.

Thereafter we need to find the outstanding balance of the home loan after 10 years. This is equal to the present value of an annuity with n = 20x12 = 240; i = 0.09/12 = 0.0075; and R = 12 873.96.

$$P = 12\,873.96 \left[\frac{((1+0.0075)^{240} - 1)}{(0.0075(1+0.0075)^{240})}\right]$$

$$= 1\,430\,875.69$$

The oustanding balanced owed to the bank is R1 430 875.69.
If they sell at R 3800 000, they will remain with
R3 800 000 - R1430 875.69 = R2 369 124.31

Answer: Option 3

Question 24

In this question we seek to find the present value of an ordinary annuity where $R = 2500$, $i = 0.12/12 = 0.01$; $n = 1 \times 12$, and $P = ?$

$$\therefore P = R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= 2500\left[\frac{((1+0.01)^{12} - 1)}{(0.01(1+0.01)^{12})}\right]$$

$$= 28137.69.$$

$\therefore$ Jacob's parents need to deposit R 28137.69

Answer: Option 4

Question 25

To find the purchase price of the furniture, we first need to find the balance of the loan using the present value of an ordinary annuity where $R = 753.20$, $i = 0.12/12 = 0.01$; $n = 2 \times 12 = 24$.

$$\therefore P = R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= 753.20\left[\frac{((1+0.01)^{24} - 1)}{(0.01(1+0.01)^{24})}\right]$$

$$= 16\,000.52$$

To get the total purchase price we need to add the down payment.

Therefore $16\,000.52 + 4000 = 20\,000.52$

Answer; Option 3