Semester 1 - Assignment 2 - Solutions

Question 1

The monthly basic charge is R150.

If x is the total number of kWh consumed per household, and the charge per kWh is 50c, therefore the total charge is $0.5x$.

If $y(x)$ represents the utility function, then

$$y(x) = 0.5x + 150.$$

Option 4

Question 2

Let x represent the total kWh consumed by all households in the municipality.

The basic collection revenue is now $700\ 000 \times R150 = R105\ 000\ 000$

$$\therefore R(x) = 0.5x + 105\ 000\ 000$$

Option 3

Question 3

Fixed costs of production can be seen when zero units are produced, in other words, when $x = 0$

$$\therefore C(0) = 89.9(0) + 120\ 000$$
$$= 120\ 000$$

Fixed cost is R120 000

Option 4.

Question 4

$x = 1000$

$$C(1000) = 89.9(1000) + 120\ 000 = 209\ 900.$$

$\therefore$ Total cost of producing 1000 books is R209 900

Option 2

Question 5

From previous question, the total cost of producing 1000 book is R209 900.

But average cost $= \dfrac{\text{Total Cost}}{\text{Number of units}}$

$\qquad\qquad = \dfrac{R209\,900}{1000}$

$\qquad\qquad = R209.90$

Option 3

Question 6

From the given graph, one can choose two points, $(1, 200)$ and $(5, 600)$. These two points are used to determine the slope of the line.

$m = \dfrac{600 - 200}{5 - 1} = \dfrac{400}{4} = 100.$

∴ we have $y = 100x + c.$

We substitute $(1, 200)$ to get:

$\qquad 200 = 100(1) + c$

$\qquad c = 100$

∴ The the equation of the line is $y = 100x + 100$

Option 3

Question 7

The equation of the line is $y = 100x + 100$, to predict sales for the sixth year, substitute $x = 6$ into the equation.

∴ $y = 100(6) + 100 = 600 + 100 = 700.$

The value of y is in Thousands of Rands

∴ Predicted sales is R700 000

Option 2

Question 8

The corresponding points are; 2006: $(0, 7.9)$ and 2012: $(6, 18.8)$

∴ we need to derive the equation of the line through the two points. We find the slope first:

$$m = \frac{18.8 - 7.9}{6 - 0} = \frac{10.9}{6} = 1.82$$

∴ $y = 1.82x + C$. Substitute $(0, 7.9)$ to get

$$7.9 = 1.82(0) + C$$

$$C = 7.9$$

∴ $y = 1.82x + 7.9$

Option 1

Question 9

The year 2011 corresponds with year number 5. From the given table, annual sales is R 17 billions

∴ Option 2

Question 10

A parabola can be symmetrical abouts its y-axis and not the x-axis.

Option 2

Question 11

$(1; 5)$

$$y(1) = 2(1-3)^2 + 1 = 2(2)^2 + 1 = 9$$

∴ $y(1) \neq 5$.

Option 4.

Question 12

$$y(x) = 2(x-3)^2 + 1$$

To find x-intercepts, let $y(x) = 0$

$$\therefore \quad 0 = 2(x-3)^2 + 1$$
$$0 = 2(x^2 - 6x + 9) + 1$$
$$0 = 2x^2 - 12x + 18 + 1$$
$$0 = 2x^2 - 12x + 19$$
$$= x^2 - 6x + 9.5 \qquad \text{divide 2 on both sides.}$$

To solve the equation we need to find 2 numbers, that are factors of the number 9.5; and when multiplied together will give the value $[+9.5]$ and their summation will result in the number $[-6]$. However, these numbers do not exist. Therefore, the equation does not have a solution and there are no x-intercepts.

Option 1

Question 13

To find the vertex of $y(x)$, we first need to simplify the function. From question 12 above,

$$y(x) = 2(x-3)^2 + 1$$
$$= 2x^2 - 12x + 19$$

Thereafter, we need to find the first derivative $y'(x)$ and solve for $y'(x) = 0$.

$$y'(x) = 4x - 12 = 0$$
$$4x = 12$$
$$x = 3 \quad \text{(divide 4 on both sides)}$$

$\therefore$ the x-value of the vertex is $x = 3$; we need to find $y(3)$.

$$\therefore y(3) = 2(3-3)^2 + 1 = 1 \qquad \therefore \text{vertex is } (3, 1) \quad \text{Option 2}$$

Question 14.

The profit function $P(x)$ is a downward facing parabola. Therefore the maximum point will be at its vertex. To find the vertex we first need to find the first derivative $P'(x)$ and solve for $P'(x) = 0$.

$$P(x) = -0.04x^2 + 240x - 10\,000$$
$$P'(x) = -0.08x + 240 = 0$$
$$x = \frac{-240}{-0.08}$$
$$= 3000$$

$\therefore$ In order to maximize profits, the company needs to sell 3000 units.

Option 3.

Question 15

The revenue function is given by:
$$R(x) = (200 + x)(300 - x)$$
$$= 60\,000 + 100x - x^2$$
$$= -x^2 + 100x + 60\,000$$

Option 3

Question 16

See question 15.

$R(x) = -x^2 + 100x + 60\,000$. $R(x)$ is downward facing parabola. Therefore the maximum point will be at its vertex. To find the vertex we find the first derivative $R'(x)$ and solve $R'(x) = 0$.

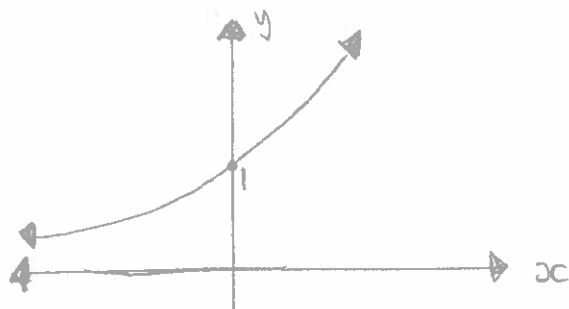$$R'(x) = -2x + 100 = 0$$
$$x = \frac{-100}{-2}$$
$$= 50 \quad [\text{passengers above } 200]$$

$\therefore 200 + 50 = 250$ sign-ups will result in maximum profit.
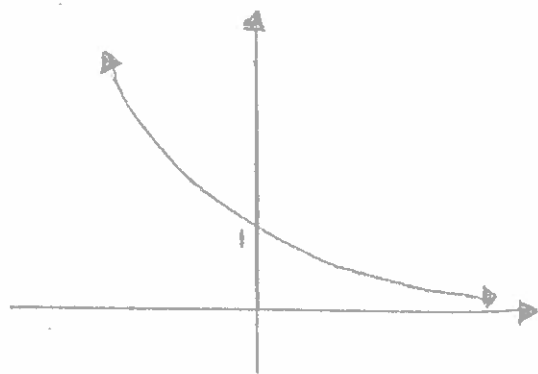
Option 2

Question 17

First represent the function graphically:

Option 3: The graph cuts the negative x-axis. This incorrect because the negative x-axis is a vertical assymptote. Therefore the graph never touches the negative x-axis.

Question 18

First represent the function graphically:

Option 1: The graph climbs steeply to the right. This is incorrect because this graph climbs steeply to the left.

Question 19

$$3x - 2y = 4 \quad ---①$$
$$-6x + 4y = 7 \quad ---②$$

To solve for the system of equations, multiply ① × 2 to get ③. Then sum ③ + ② and solve.

$$6x - 4y = 8 \quad ---③ \Rightarrow ① \times 2$$
$$-6x + 4y = 7 \quad ---②$$
$$\overline{}$$
$$0 = 15 \qquad\qquad ③ + ②$$

The false statement "$0 = 15$" signifies that the system of equations has no solution.

Answer: Option 4.

Question 20

Let x be the number of bicycles and tet y be the number of tricycles.

$$\therefore \quad x + y = 7 \quad --①$$
$$2x + 3y = 19 \quad --②$$

Rearrange equation ① and substitute into equation ②

$$y = 7 - x$$

$$\therefore \quad 2x + 21 - 3x = 19$$
$$-x = -2$$
$$\therefore \quad x = 2$$

Calculate for y:

$$y = 7 - x$$
$$= 7 - 2$$
$$= 5$$

∴ There are 5 tricycles and 2 bicycles.

Answer: Option 3

Question 21

Let x represent the price of milkshakes, and y represents the price for a fruit juice.

∴
$$y = x + 2 \quad - - - ①$$
$$3y + 5x = 78 \quad - - - ②$$

Substitute ① into ②:

$$3(x + 2) + 5x = 78$$
$$3x + 6 + 5x = 78$$
$$8x = 72$$
$$∴ x = 9$$

Substitute x into ①:
$$y = x + 2$$
$$y = 9 + 2$$
$$= 11.$$

∴ The price for milkshake is R9 and for fruit juice is R11

Answer: Option 1

Question 22

The new volume (x) of mixture must contain 50% of salt, therefore:

$$0.69 = 0.5x$$

$$x = \frac{0.69}{0.5}$$

$$= 1.38$$

∴ The volume of mixture is 1.38 ℓ.

The amount of water (y) to be added is:

$$y = x - 1.00$$

$$= 1.38 - 1.00$$

$$= 0.38$$

0.38 ℓ of water is equivalent to 380 mℓ.

Therefore 380 mℓ of water must be added to make the mixture of 50% salt.

Answer: Option 2

Question 23

$$3 < x < 6$$

Answer: Option 3

Question 24.

$$x > -10$$

Answer: Option 2

Question 25

$40x + 10y \geqslant 2400;\ 10x + 15y \geqslant 2100;\ 5x + 15y \geqslant 1500;\ x \geqslant 0;$ and $y \geqslant 0$

Answer: Option 4