Semester 1 Assignment 1 Solutions

Question 1

Simplify:

$$\frac{3a^2b^3}{2ab^2+4ab} \cdot \frac{b^2+4b+4}{6a^2b^5} = \frac{3a^2b^3}{2ab(b+2)} \cdot \frac{(b+2)^2}{3a^2b^3(2b^2)}$$

$$= \frac{b+2}{2ab(2b^2)} \quad \text{(Cancel common factors)}$$

$$= \frac{b+2}{4ab^3}$$

Answer: Option 2

Question 2

Simplify:

$$\frac{1+\frac{1}{x+1}}{x-\frac{4}{x}} = \frac{1 \cdot \frac{x+1}{x+1} + \frac{1}{x+1}}{x \cdot \frac{x}{x} - \frac{4}{x}}$$

[It is best to express the numerator and denominator of the given expression as a single unit.]

$$= \frac{\frac{x+1+1}{x+1}}{\frac{x^2-4}{x}}$$

$$= \frac{\frac{x+2}{x+1}}{\frac{(x-2)(x+2)}{x}}$$

$$= \frac{x+2}{x+1} \cdot \frac{x}{(x-2)(x+2)}$$

[Invert the denominator to multiply]

$$= \frac{x}{(x+1)(x-2)}$$

Option 3

Question 3

Simplify:

$$\frac{2^{-4}\cdot(2^{-1})^2}{(2^0+1)^{-1}} = \frac{2^{-4}\cdot 2^{-2}}{2^{-1}} = 2^{-4-2+1} = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$$

Option 2

Question 4

$$\left(\frac{a^2b^{-1}c^3}{a^3b^{-2}}\right)^2 = \frac{a^{2\cdot2}b^{(-1)(2)}c^{3\cdot2}}{a^{3\cdot2}b^{(-2)(2)}}$$

$$= \frac{a^4b^{-2}c^6}{a^6b^{-4}}$$

$$= a^{(4-6)}b^{(-2+4)}c^6$$

$$= a^{-2}b^2c^6$$

$$= \frac{b^2c^6}{a^2}$$

Option 1

Question 5

Solve:

$$\sqrt{2x+5} = 3$$

$$\left(\sqrt{2x+5}\right)^2 = 3^2$$

$$2x+5 = 9$$

$$2x = 4$$

$$x = 2$$

Option 3

Question 6

Solve: $2\left(\frac{x-1}{4}\right) - \frac{2x}{3} = \frac{4-3x}{12}$

$6(x-1) - 8x = 4-3x$ [Multiply both sides by 12]

$6x - 6 - 8x = 4 - 3x$

$-6 - 2x = 4 - 3x$

$x = 10$

Option 2

Question 7

The solution is similar to finding the number of permutations for eight unique objects, arranged four at a time.

$\therefore P(8,4) = \frac{8!}{(8-4)!} = \frac{8!}{4!} = 8.7.6.5 = 1680$ ways.

Option 4

Question 8

The order of selection is not important, therefore the solution is similar to finding the number of combinations of ten objects taken four at a time.

Hence, $C(10,4) = \frac{10!}{4!(10-4)!} = \frac{10!}{4!\,6!} = \frac{10.9.8.7}{4.3.2.1} = 210$ ways

Option 2

Question 9

The order in which each musician is selected is not important; therefore:

Violinist: $C(6,2) = 15$

Violist: $C(3,1) = 3$

Cellist: $C(2,1) = 2$

Use the multiplication rule of combinations: $15 \times 3 \times 2 = 90$ ways

Option 4

Question 10

In this question, order of selection is important. Therefore we use permutations:

Violinists: $P(6,2) = 30$
Violist: $P(3,1) = 3$
Cellist: $P(2,1) = 2$

Using the multiplication rule of permutations: $30 \times 3 \times 2 = 180$ ways.

Option 1

Question 11

$3 + 2 + 3 = 8$

$\text{Red} = \frac{3}{8} \times 160 = 60$

$\text{Blue} = \frac{2}{8} \times 160 = 40$

$\text{Yellow} = \frac{3}{8} \times 160 = 60$

Option 2

Question 12

$\frac{TQ}{PQ} = \frac{2}{3}$

$\therefore TQ = PQ \times \frac{2}{3}$

$= 45 \times \frac{2}{3}$

$= 30\text{ mm}$

$\therefore PT = 45 - 30$

$= 15\text{mm}$

OR

$\frac{PT}{PQ} = \frac{1}{3}$

$\therefore PT = 45 \times \frac{1}{3}$

$= 15\text{ mm}$

Option 3

Question 13

Note that the width of the frame is the same all the way around. Suppose the width of the frame is x cm. Since the picture is 30cm high and the picture and frame is 40 cm high it follows that

$$x + 30 + x = 40$$

$$2x + 30 = 40$$

$$2x = 10$$

$$x = 5.$$

Hence the width of the frame is 5cm. The width of the picture and frame, in cm, is

$$2x + 25 = 10 + 25 = 35$$

Hence,

$$\text{area of frame} = \text{area of picture and frame} - \text{area of picture}$$

$$= (40 \times 35)\,\text{cm}^2 - (30 \times 25)\,\text{cm}^2$$

$$= 1400\,\text{cm}^2 - 750\,\text{cm}^2$$

$$= 650\,\text{cm}^2.$$

Option 1

Question 14

After 4 hours the number of litres of water in the tank is

$$4 \times 8500 = 34000.$$

Since 1 litre is $0.001\,\text{m}^3$, it follows that the volume of water in the tank, in m^3 is:

$$34000 \times 0.001 = 34.$$

Suppose that after 4 hours the level of water is x m from the bottom of the tank. Then the volume of water is

$$\text{Volume} = l \times b \times h$$

$$= (5 \times 4 \times x)\,\text{m}^3$$

$$= 20x\ \text{m}^3$$

Hence

$$20x = 34$$

$$x = \frac{34}{20}$$

$$= 1.7.$$

Thus the level of the water is 1.70m from the bottom of the tank and since the height of the tank is 3m it follows that the level is $(3 - 1.7)$ m, ie. 1.3 m from the top of the tank.

Option 4

Question 15

First find the area of the base, which is:

$$\text{area of square} = s^2$$

$$= 3^2$$

$$= 9\ cm^2$$

Then multiply area of the base by the height of the cube.

$$\text{Volume} = \text{area of base} \times \text{height}.$$

$$= 9 \times 3$$

$$= 27\ cm^3$$

To convert from cm^3 to mm^3

$$1\ cm^3 = 1\,000\ mm^3$$

$$\therefore \quad 27\ cm^3 = 27\,000\ mm^3.$$

Answer: Option 2

Question 16

First calculate the total area of rectangular surface, in cm^2.

$4m = 400cm$ and $3m = 300cm$.

$\therefore$ Total surface area $= 300cm \times 400cm$

$= 120000 cm^2$

Calculate the area of each tile:

Area of tile $= 10cm \times 10cm$

$= 100cm^2$

To get the number of tiles, divide total surface area by the area of each tile.

$\therefore \frac{120\,000 cm^3}{100cm^3 \text{ per tile}} = 1\,200$ tiles.

Option 2.

Question 17

The base of the tank measures 25m by 12m. Thus the area of the base is: $25m \times 12m = 300m^2$

Two of the walls measure 25m by 6m and the two measure 12m by 6m. Therefore the total area of the walls is:

$$2 \times (25 \times 6) + 2 \times (12 \times 6) = 300 + 144 = 444 m^2$$

Hence the total surface area of the tank is;

$$300 + 444 = 744 m^2$$

The cost of painting is R26 per square metre. Therefore the cost of painting $744m^2$ is:

$$744 \times 26 = 19\,344$$

Total cost is R19 344.

Option 4.

Question 18

let the equivalent amount of euros be x;

$$\therefore \quad x = \frac{10\,000}{10.68}$$

$$= 936.33$$

∴ Moloko can buy 936.33 euros with R10 000.

Answer Option 1

Question 19

$(16.99+2) \times 16.89 = 320.74$ (round-off). She will pay R320.74 if she buys from the UK publisher.

Option 1

Question 20

$(32.50+3) \times 12.43 = £441.27$ (round-off). She will pay R441.27 if she buys from the USA publisher.

Option 3

Question 21

$$I = \frac{\text{Number of farms in 2001}}{\text{Number of farms in 1996}}(100) = \frac{246\,927}{276\,548}(100) = 89.29$$

Question 22

Option 2

$$P_L(n) = \frac{\sum p_n q_0}{\sum p_0 q_0}(100) = \frac{85 \times 500 + 45 \times 1200}{75 \times 500 + 40 \times 1200}(100)$$

$$= \frac{96\,500}{85\,500}(100)$$

$$= 112.9$$

Option 2

Question 23

$$P_p(n) = \frac{\sum P_n q_n}{\sum P_0 q_n}(100) = \frac{85(520) + 45(1300)}{75(520) + 40(1300)}(100)$$

$$= \frac{102\,700}{91\,000}(100)$$

$$= 112.9.$$

Option 3

Question 24

$$\text{Real income in 2012} = \frac{\text{takehome pay in 2012}}{\text{CPI in 2012}} \times 100$$

$$= \frac{R25000}{107.6} \times 100$$

$$= R23\ 234.20$$

Option 1

Question 25

$$\text{Real income in 2017} = \frac{\text{take home pay in 2017}}{\text{CPI in 2017}} \times 100$$

$$= \frac{R41\ 200}{119.0} \times 100$$

$$= R34\ 621.85$$

Option 4.