UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1503

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LINEAR ALGEBRA

Duration 2 Hours

100 Marks

EXAMINERS .
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Closed book examination.

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This paper consists of 3 pages

ANSWER ALL THE QUESTIONS

QUESTION 1

(a) **(i)** Find the solution set of the following linear equation

$$7x - 5y = 3$$ (2)

(ii) Reduce the following matrix to row-echelon form

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$ (3)

(b) Show that if A is an $m \times n$ matrix and $A(BA)$ is defined, then B is an $n \times m$ matrix (5)

(c) Show that the following matrices are inverses of each other

$$\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$ (5)

(d) Let

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 3 \\ 1 & -1 \\ -1 & 0 \end{bmatrix}$$

(i) Check as to whether or not AB and BA are defined (explain fully) (4)

(ii) If defined, what will their sizes be? (2)

(iii) Compute AB and BA if defined (4)

[25]

QUESTION 2

(a) **(i)** Compute $det(A)$, where A is given by

$$A = \begin{bmatrix} a+1 & a \\ a & a-1 \end{bmatrix} \qquad (3)$$

(ii) Let A, B, C be $n \times n$ matrices such that $det(A) = -1, det(B) = 2$ and $det(C) = 3$ Then evaluate $det(B^2C^{-1}AB^{-1}C^T)$ (5)

(b) Given that

$$A = \begin{bmatrix} 1 & a & b \\ -a & 1 & c \\ -b & -c & 1 \end{bmatrix}$$

then find $det(A)$ (5)

(c) **(i)** If B is a square matrix such that $B^n = 0$ where 0 is a zero matrix, then show that B is not invertible (2)

(ii) Find $det(C)$ if $C^2 = I$, where I is an identity matrix (2)

(d) Solve by Cramer's rule

$$\begin{cases} 3x + 4y = 9 \\ 2x - y = -1 \end{cases} \qquad (8)$$

[25]

QUESTION 3

(a) Consider the vectors $\underline{u} = (1, 0, 2)$ and $\underline{v} = (1, 1, 0)$

(i) Determine the orthogonal projection $proj_{\underline{u}}\underline{v}$ (4)

(ii) Calculate the area of the parallelogram bounded by $\underline{v}$ and $\underline{u}$ (6)

(iii) Determine the perimeter of the parallelogram bounded by $\underline{v}$ and $\underline{u}$ (5)

(b) Find an equation of the plane containing the point $(1, 1, 1)$ and perpendicular to the line passing through the points $(2, 1, 1)$ and $(1, 1, 0)$ (6)

(c) Determine whether the point $(0, -4, 6)$ lies on the plane whose parametric equations are $x = 2 - t, y = 2 - 3t, z = 4 + t$ (4)

[25]

QUESTION 4

(a) Use De Moivre's theorem to express $\sin 3\theta$ in terms of powers of $\sin\theta$ and $\cos\theta$ (10)

(b) Determine the cube roots of 8 in polar form (10)

[TURN OVER]

(c) Let a and b be real numbers such that $(a - ib)^2 = 4i$

(i) Prove that $a^2 - b^2 = 0$ (2)

(ii) Verify that $ab = 2$ (3)

[25]

TOTAL MARKS: [100]