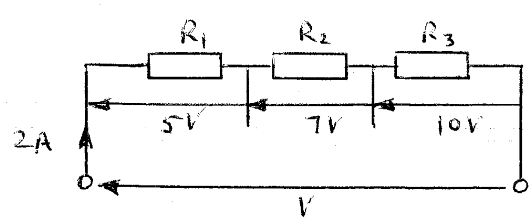
CHAPTER 36 SERIES AND PARALLEL NETWORKS

EXERCISE 169, Page 377

1. The p.d.'s measured across three resistors connected in series are 5 V, 7 V and 10 V, and the supply current is 2 A. Determine (a) the supply voltage, (b) the total circuit resistance and (c) the values of the three resistors.

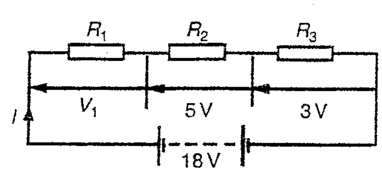


(a) Supply voltage, **V** = 5 + 7 + 10 = **22 V**

(b) Total circuit resistance, $R_T = \frac{V}{I} = \frac{22}{2} = \mathbf{11\ \Omega}$

(c) $R_1 = \frac{V_1}{I} = \frac{5}{2} = \mathbf{2.5\ \Omega}$, $R_2 = \frac{V_2}{I} = \frac{7}{2} = \mathbf{3.5\ \Omega}$ and $R_3 = \frac{V_3}{I} = \frac{10}{2} = \mathbf{5\ \Omega}$

2. For the circuit shown below, determine the value of V_1. If the total circuit resistance is 36 Ω determine the supply current and the value of resistors R_1, R_2 and R_3



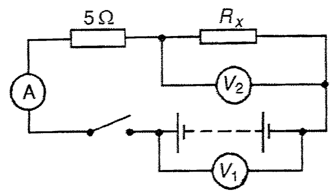
Supply voltage, $18 = V_1 + 5 + 3$

Hence, **voltage,** $V_1 = 18 - 5 - 3 = \mathbf{10\ V}$

Supply current, $\mathbf{I} = \frac{V}{R_T} = \frac{18}{36} = \mathbf{0.5\ A}$

$\mathbf{R_1} = \frac{V_1}{I} = \frac{10}{0.5} = \mathbf{20\ \Omega}$ $\mathbf{R_2} = \frac{V_2}{I} = \frac{5}{0.5} = \mathbf{10\ \Omega}$ $\mathbf{R_3} = \frac{V_3}{I} = \frac{3}{0.5} = \mathbf{6\ \Omega}$

3. When the switch in the circuit shown is closed the reading on voltmeter 1 is 30 V and that on voltmeter 2 is 10 V. Determine the reading on the ammeter and the value of resistor R_X

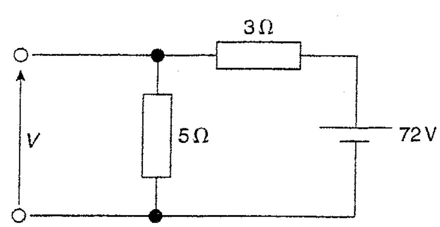


Voltage across 5 Ω resistor = $V_1 - V_2 = 30 - 10 = 20$ V

Hence, current in 5 Ω resistor, i.e. **reading on the ammeter** = $\frac{V_{5\Omega}}{5} = \frac{20}{5} = \mathbf{4\ A}$

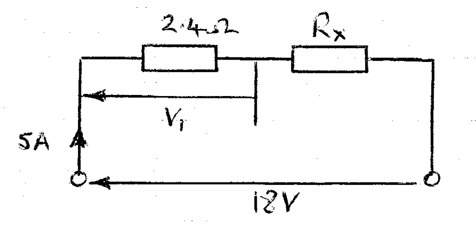
Total resistance, $R_T = \frac{V_T}{I} = \frac{30}{4} = 7.5\,\Omega$, hence $R_X = 7.5 - 5 = \mathbf{2.5\ \Omega}$

4. Calculate the value of voltage V in the diagram below.



Voltage, V = $\left(\frac{5}{5+3}\right)(72) = \mathbf{45\ V}$

5. Two resistors are connected in series across an 18 V supply and a current of 5 A flows. If one of the resistors has a value of 2.4 Ω determine (a) the value of the other resistor and (b) the p.d. across the 2.4 Ω resistor.



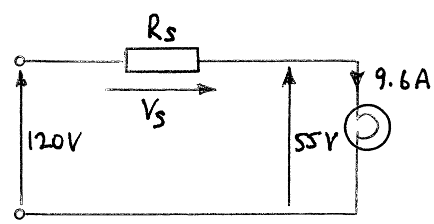
The circuit is shown above.

(a) Total resistance, $R_T = \frac{18}{5} = 3.6\ \Omega$, hence $R_X = 3.6 - 2.4 = \mathbf{1.2\ \Omega}$

(b) $V_1 = 5 \times 2.4 = \mathbf{12\ V}$

6. An arc lamp takes 9.6 A at 55 V. It is operated from a 120 V supply. Find the value of the stabilising resistor to be connected in series.

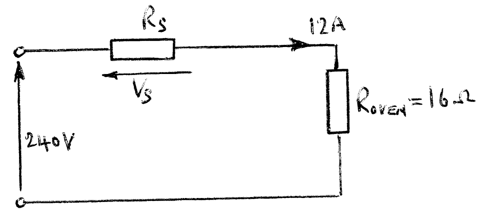
A circuit diagram is shown below.



The purpose of the stabilising resistor R_S is to cause a volt drop V_S – in this case equal to 120 – 55, i.e. 65 V. Hence, $R_S = \frac{V_S}{I} = \frac{65}{9.6} = \mathbf{6.77\ \Omega}$

7. An oven takes 15 A at 240 V. It is required to reduce the current to 12 A. Find (a) the resistor which must be connected in series, and (b) the voltage across the resistor.

(a) If the oven takes 15 A at 240 V, then resistance of oven, $R_{oven} = \frac{240}{15} = 16$ A



A circuit diagram is shown above.

If the current is reduced to 12 A then the total resistance of the circuit, $R_T = \frac{V}{I} = \frac{240}{12} = 20\ \Omega$

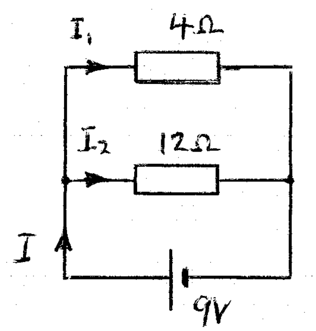
and $R_T = R_S + R_{oven}$

i.e. $20 = R_S + 16$ from which, **series resistor**, $R_S = 20 - 16 = \mathbf{4\ \Omega}$

(b) **Voltage across series resistor**, $V_S = I \times R_S = 12 \times 4 = \mathbf{48\ \Omega}$

EXERCISE 170, Page 383

1. Resistances of 4 Ω and 12 Ω are connected in parallel across a 9 V battery. Determine (a) the equivalent circuit resistance, (b) the supply current, and (c) the current in each resistor.



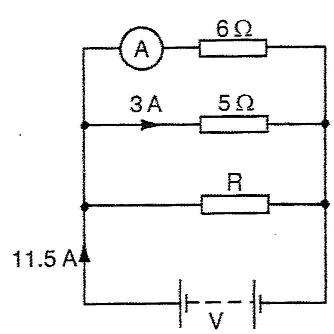
(a) Equivalent circuit resistance, $R_T = \frac{4 \times 12}{4+12} = \frac{48}{16} = \mathbf{3\ \Omega}$ (or use $\frac{1}{R_T} = \frac{1}{4} + \frac{1}{12}$)

(b) Supply current, $I = \frac{V}{R_T} = \frac{9}{3} = \mathbf{3\ A}$

(c) $I_1 = \frac{9}{4} = \mathbf{2.25\ A}$, $I_2 = \frac{9}{12} = \mathbf{0.75\ A}$ (or, by current division, $I_1 = \left(\frac{12}{4+12}\right)(3) = \mathbf{2.25\ A}$

and $I_2 = \left(\frac{4}{4+12}\right)(3) = \mathbf{0.75\ A}$)

2. For the circuit shown determine (a) the reading on the ammeter, and (b) the value of resistor R.



(a) V = 3 × 5 = 15 V. Hence, **ammeter reading**, $I_{6\,\Omega} = \frac{V}{6} = \frac{15}{6} = \mathbf{2.5\ A}$

(b) $I_R = 11.5 - 3 - 2.5 = 6$ A hence, $R = \frac{V}{I} = \frac{15}{6} = \mathbf{2.5\ \Omega}$

3. Find the equivalent resistance when the following resistance's are connected (a) in series (b) in Parallel (i) 3 Ω and 2 Ω (ii) 20 kΩ and 40 kΩ (iii) 4 Ω, 8 Ω and 16 Ω (iv) 800 Ω, 4 kΩ and 1500 Ω

(a)(i) Total resistance, $\mathbf{R_T} = 3 + 2 = \mathbf{5\ \Omega}$

(ii) Total resistance, $\mathbf{R_T} = 20 + 40 = \mathbf{60\ k\Omega}$

(iii) Total resistance, $\mathbf{R_T} = 4 + 8 + 16 = \mathbf{28\ \Omega}$

(iv) Total resistance, $\mathbf{R_T} = 800 + 4000 + 1500 = \mathbf{6300\ \Omega}$ or $\mathbf{6.3\ k\Omega}$

(b)(i) Total resistance, R_T is given by: $\frac{1}{R_T} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$ from which, $\mathbf{R_T} = \frac{6}{5} = \mathbf{1.2\ \Omega}$

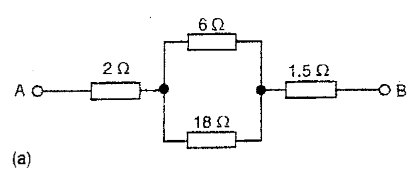
(ii) Total resistance, R_T is given by: $\frac{1}{R_T} = \frac{1}{20} + \frac{1}{40} = \frac{3}{40}$ from which, $\mathbf{R_T} = \frac{40}{3} = \mathbf{13.33\ k\Omega}$

(iii) Total resistance, R_T is given by: $\frac{1}{R_T} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{7}{16}$ from which, $\mathbf{R_T} = \frac{16}{7} = \mathbf{2.29\ \Omega}$

(iv) Total resistance, R_T is given by: $\frac{1}{R_T} = \frac{1}{800} + \frac{1}{4000} + \frac{1}{1500} = \frac{13}{6000}$

from which, $\mathbf{R_T} = \frac{6000}{13} = \mathbf{461.54\ \Omega}$

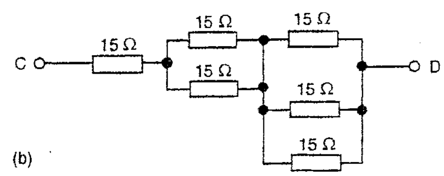
4. Find the total resistance between terminals A and B of circuit (a) shown below.



Resistance of parallel branches, $R_p = \frac{6 \times 18}{6 + 18} = 4.5\ \Omega$

Total circuit resistance, $\mathbf{R_T} = 2 + 4.5 + 1.5 = \mathbf{8\ \Omega}$

5. Find the equivalent resistance between terminals C and D of circuit (b) shown below.



Resistance of first parallel branches, $R_{P_1} = \dfrac{15 \times 15}{15 + 15} = 7.5\ \Omega$

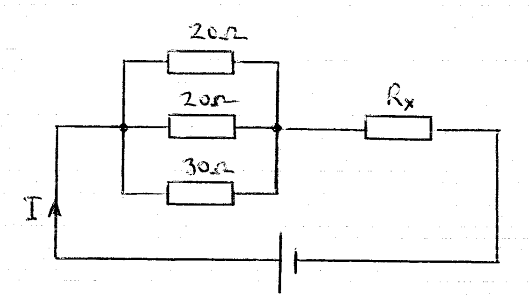
Resistance of second parallel branches, R_{P_2} is given by: $\dfrac{1}{R_{P_2}} = \dfrac{1}{15} + \dfrac{1}{15} + \dfrac{1}{15} = \dfrac{3}{15} = \dfrac{1}{5}$

i.e. $R_{P_2} = 5\ \Omega$

Total circuit resistance, $\mathbf{R_T} = 15 + 7.5 + 5 = \mathbf{27.5\ \Omega}$

6. Resistors of 20 Ω, 20 Ω and 30 Ω are connected in parallel. What resistance must be added in series with the combination to obtain a total resistance of 10 Ω. If the complete circuit expends a power of 0.36 kW, find the total current flowing.

The circuit is shown below.



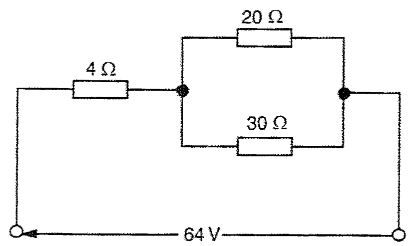
For the parallel branch, $\dfrac{1}{R_P} = \dfrac{1}{20} + \dfrac{1}{20} + \dfrac{1}{30}$ from which, $R_P = 7.5\ \Omega$

Hence, **resistance to be added in series,** $R_X = R_T - R_P = 10 - 7.5 = \mathbf{2.5\ \Omega}$

Power, $P = I^2R$ hence $0.36 \times 10^3 = I^2(10)$

from which, **total current flowing, I** = $\sqrt{\frac{360}{10}} = \sqrt{36} = \mathbf{6\ A}$

7. (a) Calculate the current flowing in the 30 Ω resistor shown in the circuit below



(b) What additional value of resistance would have to be placed in parallel with the 20 Ω and 30 Ω resistors, to change the supply current to 8 A, the supply voltage remaining constant.

(a) Total resistance, $R_T = 4 + \frac{20 \times 30}{20 + 30} = 4 + 12 = 16\ \Omega$

Hence, total current, $I = \frac{V}{R_T} = \frac{64}{16} = 4\ A$

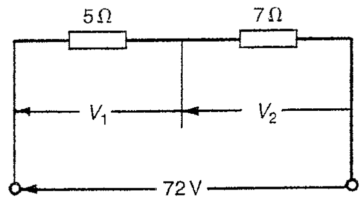
and, by current division, $I_{30\,\Omega} = \left(\frac{20}{20+30}\right)(4) = \mathbf{1.6\ A}$

(b) If I = 8 A then new total resistance, $R_{T_2} = \frac{64}{8} = 8\ \Omega$ and the resistance of the parallel branch will be: 8 – 4 = 4 Ω

i.e. $\frac{1}{4} = \frac{1}{20} + \frac{1}{30} + \frac{1}{R_X}$ where R_X is the additional resistance to be placed in parallel

from which, $\frac{1}{R_X} = \frac{1}{4} - \frac{1}{20} - \frac{1}{30}$ from which, $\mathbf{R_X = 6\ \Omega}$

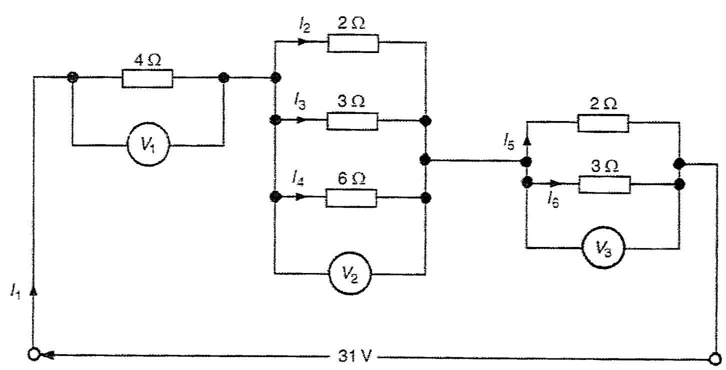
8. For the circuit shown below, find (a) V_1, (b) V_2, without calculating the current flowing



(a) **Voltage,** $\mathbf{V_1} = \left(\frac{5}{5+7}\right)(72) = \mathbf{30\ V}$ by voltage division

(b) **Voltage,** $\mathbf{V_2} = \left(\frac{7}{5+7}\right)(72) = \mathbf{42\ V}$

9. Determine the currents and voltages indicated in the circuit below.



$$\frac{1}{R_{P_1}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} \quad \text{from which,} \quad R_{P_1} = 1\ \Omega$$

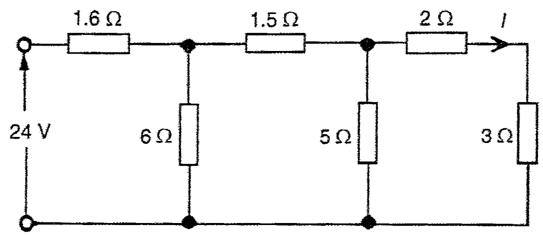
$$R_{P_2} = \frac{2 \times 3}{2+3} = 1.2\ \Omega$$

Hence, total resistance, $R_T = 4 + 1 + 1.2 = 6.2\ \Omega$

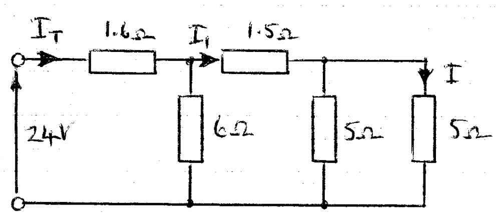
$\mathbf{I_1} = \frac{31}{6.2} = \mathbf{5\ A}$, $\mathbf{V_1} = I_1(4) = 5 \times 4 = \mathbf{20\ V}$, $\mathbf{V_2} = 5 \times 1 = \mathbf{5\ V}$ and $\mathbf{V_3} = 5 \times 1.2 = \mathbf{6\ V}$

$\mathbf{I_2} = \frac{V_2}{2} = \frac{5}{2} = \mathbf{2.5\ A}$, $\mathbf{I_3} = \frac{5}{3} = \mathbf{1.67\ A}$, $\mathbf{I_4} = \mathbf{0.83\ A}$, $\mathbf{I_5} = \frac{V_3}{2} = \frac{6}{2} = \mathbf{3\ A}$ and $\mathbf{I_6} = \frac{6}{3} = \mathbf{2\ A}$

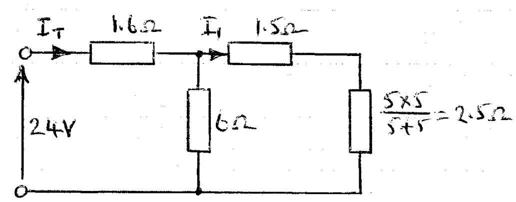
10. Find the current I in the circuit below.



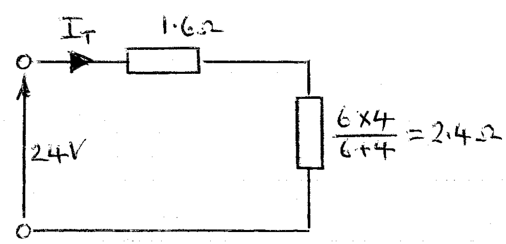
The circuit is reduced step by step as shown in diagrams (a) to (d) below.



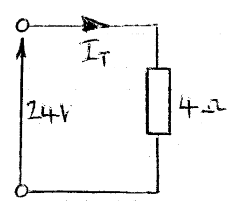
(a)



(b)



(c)



(d)

From (d), $I_T = \frac{24}{4} = 6$ A

From (b), $I_1 = \left(\frac{6}{6+4}\right)(6) = 3.6$ A

and from (a), $I = \left(\frac{5}{5+5}\right)(3.6) = \mathbf{1.8\ A}$

EXERCISE 171, Page 385

1. If four identical lamps are connected in parallel and the combined resistance is 100 Ω, find the resistance of one lamp.

If each lamp has a resistance of R then:

$$\frac{1}{100} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{4}{R} \quad \text{and} \quad R = 4 \times 100 = \mathbf{400\ \Omega = resistance\ of\ a\ lamp}$$

2. Three identical filament lamps are connected (a) in series, (b) in parallel across a 210 V supply. State for each connection the p.d. across each lamp.

(a) In **series**, p.d. across each lamp $= \frac{210}{3} = \mathbf{70\ V}$

(b) In **parallel**, p.d. across each lamp = **210 V**

EXERCISE 172, Page 385

Answers found from within the text of the chapter, pages 3374to 385.

EXERCISE 173, Page 386

1. (a) **2.** (c) **3.** (c) **4.** (c) **5.** (a) **6.** (d) **7.** (b) **8.** (c) **9.** (d) **10.** (d)
