Tutorial Letter 201/1/2017

Elementary Quantitative Methods
QMI1500

Semester 1

Department of Decision Sciences

IMPORTANT INFORMATION:
Solutions to Assignment 01

Bar code

Define tomorrow.

UNISA university of south africa

Solutions to compulsory Assignment 01

Question 1

$$\begin{aligned}\frac{x^{\frac{2}{3}} \times x^{\frac{2}{5}}}{x^{-\frac{2}{4}}} &= x^{\frac{2}{3}+\frac{2}{5}+\frac{2}{4}} \\ &= x^{\frac{2\times5\times4+2\times3\times4+2\times3\times5}{60}} \\ &= x^{\frac{40+24+30}{60}} \\ &= x^{\frac{94}{60}} \\ &= x^{1\frac{17}{30}}\end{aligned}$$

Answer: Option 1

Question 2

The easiest way to get the solution for this question is to apply the concept of worker-days. 8 workers require 24 days to complete the job, therefore (8×24), i.e. 192 worker-days are required to complete the job.

If 18 workers are involved to complete a 192 worker-days job, then the number of days required to complete the paint job, will be

$$\frac{192}{18} = 10\frac{2}{3}$$

Answer: Option 2

Question 3

There are 4 parts to the meal, each with several choices.

Sandwich: 4 choices
Soup: 3 choices
Dessert: 2 choices
Drink: 5 choices

Thus the number of possible meals is

$$4 \times 3 \times 2 \times 5 = 120$$

Answer: Option 3

Question 4

You can get 0, 1, 2, 3, 4 or 5 toppings from the 11 choices. Therefore the number of possible choices for the first pizza is as follows:

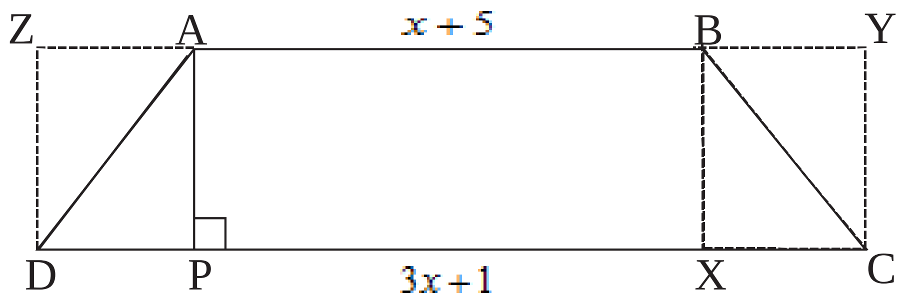
$${}_{11}C_0 + {}_{11}C_1 + {}_{11}C_2 + {}_{11}C_3 + {}_{11}C_4 + {}_{11}C_5 = 1 + 11 + 55 + 165 + 330 + 462 = 1\,024$$

The same applies for the second pizza. Thus the number of choices available for two pizzas is

$$1\,024 \times 1\,024 = 1\,048\,576$$

Answer: Option 4

Question 5



In order to answer this question, we first find the area of the trapezium $ABCD$ in terms of x. To find the area, label the diagram as above.

Find the areas of ZYCD, ZAD and BYC.

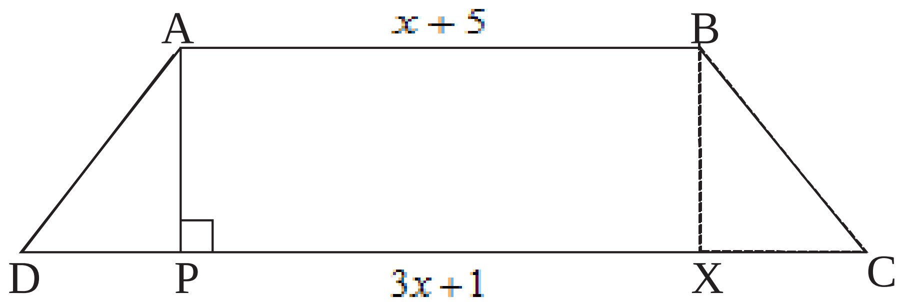
But Area of ZAD=Area of APD and Area of BYC=Area of BCX

$$\begin{aligned} \text{Area of } ABCD &= \text{Area of } ZYCD - \text{Area of } ZAD - \text{Area of } BYC \\ &= \text{Area of } ZYCD - \text{Area of } APD - \text{Area of } BYX \end{aligned}$$

Hence

$$\begin{aligned} 66 &= [(3x+1) \times 6] - [6 \times \frac{1}{2}(3x+1) - (x+5)] \\ 66 &= 18x + 6 - 9x - 3 + 3x + 15 \\ 66 &= 12x + 18 \\ 48 &= 12x \\ 4 &= x \end{aligned}$$

Alternative



In order to answer this question, we first find the area of the trapezium $ABCD$ in terms of x. To find the area, label the diagram as above.

Find the areas of $ABXP$, DAP and CBX.

Hence

$$\text{Area of } ABCD = \text{Area of } ABXP + \text{Area of } DAP + \text{Area of } CBX.$$

Now

$$PX = AB = x + 5.$$

Hence

$$DC = DP + PX + XC$$

i.e.

$$3x + 1 = DP + x + 5 + XC$$

i.e.

$$DP + XC = (3x + 1) - (x + 5)$$

Now

$$\begin{aligned} \text{Area } ABCD &= (x+5) \times 6 + \frac{1}{2}DP \times 6 + \frac{1}{2}XC \times 6 \\ &= (x+5) \times 6 + \frac{1}{2} \times 6(DP + XC) \\ &= (x+5) \times 6 + 3((3x+1) - (x+5)) \\ &= 6x + 30 + 3(2x - 4) \\ &= 6x + 30 + 6x - 12 \\ &= 12x + 18 \end{aligned}$$

Hence

$$66 = 12x + 18$$

i.e.

$$12x = 66 - 18$$

i.e.

$$12x = 48$$

i.e.

$$x = 4.$$

Answer: Option 2

Question 6

Consider the hole as a cylinder with height 4 cm and base which has diameter 3.5 cm.

The surface area of the square nut with the hole

$$\begin{aligned} &= \text{surface area of the square nut without the hole} \\ &- \text{twice the area of the base of the cylinder} \\ &+ \text{area of the outer surface of the cylinder} \end{aligned}$$

Hence the surface area

$$\begin{aligned} &= 2ab + 2bc + 2ac - 2\pi r^2 + 2\pi \times r \times h \\ &= [2 \times 6 \times 6 + 2 \times 6 \times 4 + 2 \times 4 \times 6] - [2 \times \frac{22}{7} \times (\frac{1}{2}(3.5))^2] + [2 \times \frac{22}{7} \times \frac{1}{2}(3.5) \times 4] \text{ cm}^2 \\ &= (168 - 19.25 + 44) \text{ cm}^2 \\ &= 192.75 \text{ cm}^2 \end{aligned}$$

Answer: Option 4

Question 7

$$\text{Range} = 180 - 68 = 112$$

Thus the width of the class intervals is

$$\frac{112}{8} = 14$$

Answer: Option 1

Question 8

The mode is the number with most occurrences. In this sample, the number 12 appears three times, which is more than any other number.

Answer: Option 2

Question 9

For the average or mean, add all numbers and divide by the sample size:

$$\frac{130}{10} = 13$$

Answer: Option 3

Question 10

The median is the middle number in the sample. First order the numbers as follows:

$$5, 8, 8, 10, 10, 12, 12, 12, 23, 30$$

The median is

$$\frac{10 + 12}{2} = 11$$

Answer: Option 2

Question 11

For variance use the formula $S^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$ where x_i is the ith observation in the sample.

The mean was calculated as 13 in question 9.

$$\begin{aligned} \therefore\ S^2 &= \frac{(10-13)^2 + (12-13)^2 + (12-13)^2 + (8-13)^2 + (30-13)^2 + (12-13)^2 + (8-13)^2 + (10-13)^2 + (5-13)^2 + (23-13)^2}{10-1} \\ &= 58.22 \end{aligned}$$

Answer: Option 3

Question 12

Standard deviation is the square root of variance. Hence

$$\begin{aligned} \text{Standard deviation} &= \sqrt{58.22} \\ &= 7.63 \end{aligned}$$

Answer: Option 3

Question 13

Coefficient of variation $= CV = \frac{S}{\bar{x}}$

S was calculated in question 12 as 7.63 and $\bar{x}$ was calculated in question 9 as 13.

$$\begin{aligned} CV &= \frac{7.63}{13} \\ &= 0.587 \end{aligned}$$

Answer: Option 1

Question 14

The dispersion around the mean

Answer: Option 2

Question 15

	1995		2005			
Item	**Price ($)**	**Quantity**	**Price ($)**	**Quantity**	P_0q_0	P_nq_0
Bread, white (loaf)	0.77	50	1.98	55	38.50	99.00
Eggs (dozen)	1.85	26	2.98	20	48.10	77.48
Milk (litre) white	0.88	102	1.98	130	89.76	201.96
Apples, red delicious (500 g)	1.46	30	1.75	40	43.80	52.50
Orange juice (355 ml, concentrate)	1.58	40	1.70	41	63.20	68.00
Coffee, 100% ground roast (400 g)	4.40	12	4.75	12	52.80	57.00
					336.16	555.94
Laspeyres:	165.4					

The Laspeyres weighted price index for 2005 is 165.4, found by

$$P_L = \frac{\sum p_n q_0}{\sum p_0 q_0}(100) = \frac{\$555.94}{\$336.16}(100) = 165.4$$

Answer: Option 3

Question 16

	1995		2005			
Item	**Price ($)**	**Quantity**	**Price ($)**	**Quantity**	P_0q_n	P_nq_n
Bread, white (loaf)	0.77	50	1.98	55	42.35	108.90
Eggs (dozen)	1.85	26	2.98	20	37.00	59.60
Milk (litre) white	0.88	102	1.98	130	114.40	257.40
Apples, red delicious (500 g)	1.46	30	1.75	40	58.40	70.00
Orange juice (355 ml, concentrate)	1.58	40	1.70	41	64.78	69.70
Coffee, 100% ground roast (400 g)	4.40	12	4.75	12	52.80	57.00
					369.73	622.60
Paasche:	168.4					

The Paasche weighted price index for 2005 is 168.4, found by

$$P_p = \frac{\sum p_n q_n}{\sum p_0 q_n}(100) = \frac{\$622.60}{\$369.73}(100) = 168.4$$

Answer: Option 4

Question 17

Item	**2000 Price** $p_0(\$)$	**2000 Quantity Sold thousands** q_0	p_0q_0 ($) **thousands**	**2005 Price** $p_t(\$)$	**2005 Quantity Sold thousands** q_t	p_tq_t ($) **thousands**
Ties (each)	10	1 000	10 000	12	900	10 800
Suits (each)	300	100	30 000	400	120	48 000
Shoes (pair)	100	500	50 000	120	500	60 000
			90 000			118 800

Total sales in May 2005 were $118 800 000, and the comparable figure for 2000 is $90 000 000. (See the above table) Thus, the index of value of May 2005 is 132.0 (using 2000 as base year with $CPI = 100$). The value of apparel sales in 2005 was 132 percent of the 2000 sales. To put it another way, the value of apparel sales increased 32.0 percent from May 2000 to May 2005.

$$V = \frac{\sum p_t q_t}{\sum p_0 q_0}(100) = \frac{118\,800}{90\,000}(100) = 132.0$$

Answer: Option 4

Question 18

$$\begin{aligned}\text{Real income in 1998} &= \frac{\text{take-home pay in 1998}}{CPI \text{ in 1998}} \times 100 \\ &= \$\frac{25\,000}{107.6} \times 100 \\ &= \$23\,234.20\end{aligned}$$

Answer: Option 3

Question 19

$$\begin{aligned}\text{Real income in 2003} &= \frac{\text{take-home pay in 2003}}{CPI \text{ in 2003}} \times 100 \\ &= \$\frac{41\,200}{119.0} \times 100 \\ &= \$34\,621.85\end{aligned}$$

Answer: Option 3

Question 20

An article costs \$15 and the exchange rate is \$1.00 = R6.50.

$\therefore$ the article costs $15 \times \text{R}6.50 = \text{R}97.50$

$\therefore$ the number of articles bought is

$$\frac{2\,535}{97.50} = 26$$

Answer: Option 4