Tutorial Letter 101/3/2014

Distribution Theory II

STA2603

Semesters 1 & 2

Department of Statistics

IMPORTANT INFORMATION:

This tutorial letter contains important information about your module and includes the assignment questions for both semesters.

BAR CODE

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1 INTRODUCTION

Dear Student,

We wish to welcome you to the module STA2603 (Distribution theory II), and hope that you will enjoy studying it. We shall do our best to make your study of this module successful.

This tutorial letter contains important information about how to study this module. We urge you to read it carefully and to keep it at hand when working through the study material, preparing the assignments, preparing for the examination and addressing questions to your lecturers.

In this tutorial letter, you will find the assignments as well as instructions on the preparation and submission of the assignments. This tutorial letter also provides all the information about your study material. Please study this information carefully.

1.1 Tutorial matter

1.1.1 Tutorial letters

The Department of Despatch will supply you with the following study material for this module:

- A study guide,
- this tutorial letter (Tutorial letter 101), as well as others which will be sent out during the semester (numbered 102, 103,... or 201, 202, ... and so on).

The Department of Despatch should have supplied to you the tutorial letter 101 and the study guide shortly after your registration. The other tutorial letters will be sent to you throughout the semester. Follow the instructions in the brochure entitled *my Studies @ Unisa* if you have not received some of the material that should have been sent to you.

Note that if you have access to the Internet, you can view, download and print the study guide and all the tutorial letters for the modules for which you are registered on the University's online campus, myUnisa, at http://my.unisa.ac.za.

Take note that every tutorial letter you will receive is important and you should read them all immediately and carefully. Some information contained in these tutorial letters may be urgent, while others may, for example, contain examination information. So, it is wise to keep them all in a file!

2 PURPOSE OF AND OUTCOMES FOR THE MODULE

STA2603 is one of the compulsory modules of the *major in Statistics* and is the "middle" module in distribution theory. Once you have completed *STA2603: Distribution theory II*, you will be properly prepared to continue with *STA3703: Distribution theory III*.

Most of the modules in the Statistics major have *prerequisite modules.* These are either statistics or mathematics modules carefully selected by the staff in the department. These are modules you should have completed before you will be allowed to register for that particular module. Some modules have *co-requisite* modules, meaning that you have to register simultaneously for the module (if you have not passed it at a previous stage). Statistics knowledge accumulates in a specific order and you will not be able to do the particular module if you do not have the necessary pre-knowledge. Achieving statistical knowledge can therefore be compared to the building of a brick wall - you have to start at the bottom and not “skip” anything, as it can have disastrous effects at a later stage. Another example is to imagine a non-existing spider’s web because the spider was only interested in the centerpiece of its web!

For this particular module, the pre-requisites are (STA1503 and MAT1512) or alternatively (STA1502 and DSC1620 and co-registration for STA2610) This means that you are assumed to be familiar with basic statistics and distribution theory at first year level, and you are also assumed to be familiar with basic calculus (differentiation and integration). If necessary, be ready to revise this knowledge to ensure success in this module!

2.1 Purpose

Students credited with this unit must be able to gain insight into the role that formal theory plays in data analytic methods and to discuss a wide variety of discrete and continuous distributions.

2.2 Outcomes

Qualifying students will be able to:

- understand the joint probability structure of two random variables (discrete and continuous case),
- calculate expectation and moment-generating functions; have insight into distributions of functions of random variables, extrema and order-statistics,
- prove the law of large numbers and the central limit theorem under fairly strong assumptions,
- comprehend how the chi-square-, t-, and F-distributions are derived from the normal distribution.

3 LECTURER AND CONTACT DETAILS

3.1 Lecturer

You are most welcome to contact your lecturer whenever you experience any difficulties with your studies. You may do this by writing a letter, by telephone, by fax, by electronic mail, or by seeing the lecturer in person. If you wish to see the lecturer in person, then in order to avoid disappointment, you are advised to make an appointment by telephone or letter in advance to make sure that the lecturer is available to help you.

If you cannot get through to the lecturer by phone, PLEASE call the departmental secretary. She will be able to tell you when the lecturer will be available, or forward your call to another lecturer involved in this module who may be able to help you, or you can leave a message with her asking the lecturer to call you back!

Your STA2603 lecturer in 2014 will be Dr. E. Rapoo. The phone number and office number of the lecturer will be sent to you in a separate tutorial letter. In the meanwhile, you can send any queries to the e-mail address

rapooe@unisa.ac.za,

to the fax number (012) 429 8129, or the mail address

The Lecturer (STA2603)
Department of Statistics
University of South Africa
PO Box 392
UNISA
0003

There is also an e-mail link to the lecturer from the module's myUnisa page.

Please do not include your enquiries with your assignments as this will cause unnecessary delays.

3.2 Department

The contact details of the department will be sent out in a later tutorial letter.

3.3 University

If you need to contact the University about matters not related to the content of this module, please consult the publication *My Studies @ Unisa* that you received with your study material. This brochure contains information on how to contact the University (e.g. to whom you can write for different queries, important telephone and fax numbers, addresses and details of the times certain facilities are open).

Always have your student number at hand when you contact the University.

4 MODULE RELATED RESOURCES

4.1 Prescribed books

The prescribed book for this module is

Rice JA, Mathematical Statistics and data analysis (2007), 3rd edition, Cengage.

The following 5 chapters are relevant for this module:

Chapter 2: Random variables
Chapter 3: Joint distributions
Chapter 4: Expected values
Chapter 5: Limit theorems
Chapter 6: Distributions derived from the normal distribution

You have to buy this book. Please consult the list of official booksellers and their addresses listed in my Studies @ Unisa. If you have any difficulties in obtaining books from these bookshops, please see *my Studies @ Unisa* for more information.

4.2 Recommended books

There are no recommended books for this module.

4.3 Electronic Reserves (e-Reserves)

There are no e-Reserves for this module.

5 STUDENT SUPPORT SERVICES FOR THE MODULE

For information on the various student support systems and services available at Unisa (e.g. student counseling, tutorial classes, language support), please consult the publication *my Studies @ Unisa* that you received with your study material.

5.1 Contact with Fellow Students

5.1.1 Study Groups

It is advisable to have contact with fellow students. One way to do this is to form study groups. **Please consult the publication *my Studies@Unisa* to find out how to obtain the addresses of students in your region.**

5.2 myUnisa

If you have access to a computer that is linked to the internet, you can quickly access resources and information at the University. The *myUnisa* learning management system is Unisa's online campus that will help you to communicate with your lecturers, with other students and with the administrative departments of Unisa – all through the computer and the internet.

Joining *myUnisa* will offer you the following benefits:

- You have access to the additional resources on this module.
- You will be able to immediately download all your study material from this site, in electronic format.

- You can use the discussion forum to communicate with your fellow students.
- You can contact your lecturer through the e-mail link of your *myUnisa* module page.

For this module, the lecturer will use announcements and FAQs (frequently asked questions) throughout the semester. You will also be able to access self assessment quizzes, which will help you know how well you understand the study material.

To go to the *myUnisa* website, start at the main Unisa website, http://www.unisa.ac.za, and then click on the "Login to *myUnisa*" link on the right-hand side of the screen. This will take you to the *myUnisa* website. You can also go there directly by typing in http://my.unisa.ac.za. On the website you will find general Unisa related information, plus a module site for each module you are registered for. Please consult the publication *my Studies @ Unisa* which you received with your study material for more information on *myUnisa*.

6 MODULE-SPECIFIC STUDY PLAN

The semester during which you study at UNISA consists of 15 weeks between the last day of registration and the beginning of the examination period, during which time you need to study and understand the contents of the module, complete and submit six assignments, and then prepare for the examination. Therefore it is important that you create a timetable for planning your studies for this module, and all the other modules you take this semester or year.
Please start studying as soon as you receive your study material. Note that if you are registered for Semester 1, then all your assignments need to be submitted by end of April and you will write your examination in May-June; and if you are registered for Semester 2, then your assignments need to be submitted by early October and you will write your examination in October-November.

6.1 Suggested time table

The following time tables are provided as a starting point for your personal schedule.

SEMESTER 1	**Study units for preparing your assignments**	**From**	**To**
Assignment 1	Pre-knowledge of probability theory	Registration	17 Feb
Assignment 2	Unit 2	17 Feb	25 Feb
Assignment 3	Unit 3	25 Feb	18 March
Assignment 4	Units 4, 5, 6	18 March	8 April
Assignments 5 & 6	The whole module	8 April	22 April
Exam	Prepare for the examination	22 April	Exam

SEMESTER 2	**Study units for preparing your assignments**	**From**	**To**
Assignment 1	Pre-knowledge of probability theory	Registration	4 August
Assignment 2	Unit 2	4 August	12 August
Assignment 3	Unit 3	12 August	2 Sept
Assignment 4	Units 4, 5, 6	2 Sept	23 Sept
Assignments 5 & 6	The whole module	23 Sept	6 Oct
Exam	Prepare for the examination	6 Oct	Exam

6.2 How to study this module

6.2.1 An overview of the module

The outcomes of the module are listed in Section 2.2 of this tutorial letter. To pass this module, you must achieve these outcomes.

To do this, you will need to study and work through the material in the study guide and the prescribed textbook, until you are able to understand and apply the concepts and principles involved. The study guide contains activities and problems, which are there to help you ensure that you have mastered the material. Another way to find out how you are doing is through the assignments that you are supposed to submit throughout the semester. The lecturer will mark your work and give individual feedback to you.

For even more help in case you need it, please join myUnisa — on the module web page at myUnisa, there will be more resources available. These will be explained on the web page.

The final decision on whether you have mastered the module outcomes well enough comes from your final mark for the module, which is calculated from your semester mark and the examination mark. (How exactly this is done is explained later on.)

Note that the examination date is fixed, and it is your duty to make sure that you are ready to write the examination when it comes! In Statistics, it is often very hard to catch up again with the work if you fall behind, since you need to understand previous material thoroughly before learning new things.

Although you do need to take responsibility for your studies, remember that you are not alone. Your lecturer is there to help you, and you can also contact your fellow students and use Unisa's student support systems. Details of all of these are listed elsewhere in this tutorial letter!

6.2.2 Guidelines for studying this module

Guidelines of what you should do while studying for this module are therefore as follows:

- There is quite a bit of work to be done in the 15 weeks of study time. Make a timetable for yourself, to make sure you know what amount of work you need to do by what time to keep up to date with the work.
- Work through the study guide. This includes doing the activities, and working on more exercises from the textbook if you feel you need more practice.
- You will need to use a calculator for this module. Make sure you know how to use your calculator! You will be allowed to bring a non-programmable calculator to the examination.
- Submit the assignments by their due dates. The due dates of the assignments are chosen in such a way that you will need to work steadily through the semester. When you receive back your marked assignments, make note and take advantage of the lecturer's feedback on your work.

- Prepare well for the examination.

7 MODULE PRACTICAL WORK AND WORK-INTEGRATED LEARNING

There are no practicals for this module.

8 ASSESSMENT

Assessment is the process where the lecturer assesses your work by comparing it to the module outcomes and the related assessment criteria. The assessment in this module consists of formative assessment and summative assessment.

Formative assessment means assessment of your work while you are still studying. This is particularly important in distance learning since it might sometimes be the only way you can get feedback on how you are doing, while you can still benefit from it. In this module, formative assessment is through the assignments. The lecturer marks your work and gives you individual feedback on how you are doing, as well as suggestions for improvement. Make sure to take advantage of the lecturer's feedback! In addition to your marked assignment, all students will also receive comments on the assignment in a tutorial letter sent out after the closing date of each assignment. You will also receive model solutions to the assignments that you submitted.

Summative assessment refers to the final mark you receive for this module. In this module, your final mark is calculated from you examination mark (which counts for 80%) and from your semester mark (which counts for 20%). The semester mark is determined by how well you did in your assignments. Details of how this works are given in the following.

The semester mark and the final mark

Your **final mark** will be calculated from your **semester mark** and the **examination mark.**

The **semester mark** is calculated from your assignment results (the percentages you receive for the assignments). The weights of the different assignments differ: Assignment 1 counts for 0%, Assignments 2, 3 and 4 for 15% each, Assignment 5 for 10% and Assignment 6 for 45%. That is, the semester mark is calculated as

$$\text{semester mark} = \frac{1}{100} * (15 \cdot A_2 + 15 \cdot A_3 + 15 \cdot A_4 + 10 \cdot A_5 + 45 \cdot A_6)$$

where A_2 to A_6 are the percentages you received in assignments 2 to 6, respectively. Assignments not submitted, or submitted late, will give you 0%.

- The **examination mark** is the percentage mark you get in the examination.

- The examination mark contributes 80% to the final mark, and the semester mark contributes 20%. That is, your **final mark** is calculated as

$$\text{final mark} = 0.8 * \text{(examination mark)} + 0.2 * \text{(semester mark)}.$$

You pass the module if your final mark is ≥ 50, and you pass it with distinction if your final mark is ≥ 75. There is also a subminimum rule, which says that you must get at least 40% in the examination to pass the module.

IMPORTANT: Please note that a poor semester mark could lower your final mark! It is therefore important that you try to complete all the assignments as well as your can – if your year mark is zero, you must get 63% in the exam to pass the module! Also, you must make sure that you submit all the assignments on time, since if we receive your assignment too late, we have to give you 0% for it.

8.1 Assessment plan

There are six assignments in this module.

- The first assignment is compulsory: You must submit Assignment 01 by its due date, otherwise you will not get examination admission. To make it easier for you to be able to submit it on time, I have made the first assignment shorter than all the other assignments. What is more, the first assignment tests things you should be able to do already – that is, it tests how well you are prepared for this module. You can answer this assignment before you receive you study guide or your text book. If you struggle with any of the questions, you must make sure to revise probability theory from your previous modules! The first assignment does not count for the semester mark. Assignment 01 is a multiple choice assignment, and all students will receive full solutions to this assignment.

- The next three assignments 2, 3 and 4 will help you work through the module and give you an idea on which topics you understand correctly, and where you are struggling. Please do view these assignments as a chance for you to get feedback from your lecturer! These assignments each count for 15% towards the semester mark, so these 3 assignments together make up 45% of the semester mark. To further motivate you to submit your own work for these three assignments, we will send out detailed model solutions to all students who submit these assignments by their due dates. If you don't submit an assignment, you will still get a tutorial letter giving just the final answers to the assignment questions, and comments on common mistakes, but you will not get the detailed model solutions to that assignment! Assignments 02, 03 and 04 are written assignments.

- Assignment number 5 is a survey questionnaire, seeking your opinions on this module. The questionnaire will be sent out to all students later in the semester, and any student who fills it in and submits it gets 10% for the semester mark – which means 2 percentage points in the final mark, absolutely free!

- Assignment number 6 is meant to help you prepare for the examination. It consists of questions from a previous year's examination paper. In order for you to be able to complete Assignment 6 closer to the examination, we have made it a multiple choice assignment. All students will receive detailed model solutions to the examination questions in this assignment. Assignment 06 counts for 45% of the year mark, so it is well worth doing as well as you can!

In conclusion, you should complete all assignments as well as you can: To get admission to the examination; and because of the semester mark system which means that how well you do your assignments will also have a direct effect on the final mark you get for this module; and most importantly, because submitting the assignments gives you a chance to find out how well you have mastered the course contents, and for us to give you feedback on your progress!

Marking of the Assignments

Written assignments (Assignments 02, 03 and 04)
After you have submitted your assignment, we will mark it, give you a percentage mark for it (a number between 0% and 100%), and send it back to you. The percentage mark you received will be indicated in your marked assignment. Your marked assignment will contain detailed feedback on your work. This feedback is very important, so make sure to read through the comments when you receive your assignment back.

Multiple-choice assignments (Assignments 01 and 06)
These assignments must be done on optical mark reading sheets and will be marked by a computer. Instructions on how to submit these assignments appear with the assignment questions. Again, you will get a percentage mark, between 0% and 100%, for these assignments.

The survey questionnaire (Assignment 05)
The survey questionnaire will be sent to you later in the semester; you should submit it in the assignment covers, as usual, and as soon as we receive it, you will get 100% for it.

Feedback to assignments
Comments and the answers to the questions in each assignment will be automatically sent out to all students a few days after the closing date of the assignment, and therefore we have to give 0% for assignments which reach us too late. If you are struggling to meet the closing dates, please contact your lecturer before the closing date!

However, even if we receive your assignment too late, we will mark it and provide you feedback for it. So, it will still be a good idea to submit your assignment even if you know it might be too late for you to receive a percentage mark for it.

If you are genuinely unable to submit an assignment at all, please try to answer the questions in it anyway by yourself, before looking at the solutions. You will learn much more in this way than by simply reading through the correct solutions we send to you.

8.2 General assignment numbers

The assignments are numbered 01 to 06. Please remember to give your assignment the correct number in the assignment cover. The assignment questions for Semester 1 and for Semester 2 are listed in Appendix A and Appendix B at the end of this tutorial letter.

8.2.1 Unique assignment numbers

Please note that each assignment has its unique six-digit assignment number which has to be written on the cover of your assignment or on the mark reading sheet upon submission. The unique numbers are given in the table in the next section of this tutorial letter; you will also find them in the heading of each set of assignment questions.

8.2.2 Due dates for assignments

For each assignment there is a **FIXED CLOSING DATE**, which is the date by which the assignment **must reach** the university. The closing dates for submission of the assignments are given in the following table. We also give the contribution of each assignment to the semester mark.

SEMESTER 1				
Assignment no.	**Type**	**Fixed closing date**	Semester mark %	Unique number
01	Multiple choice	17 February 2014	0	876889
02	Written	25 February 2014	15	862177
03	Written	18 March 2014	15	862265
04	Written	8 April 2014	15	862411
05	Written (questionnaire)	22 April 2014	10	862583
06	Multiple choice	22 April 2014	45	862616
SEMESTER 2				
Assignment no.	**Type**	**Fixed closing date**	Semester mark %	Unique number
01	Multiple choice	4 August 2014	0	873654
02	Written	12 August 2014	15	896956
03	Written	2 September 2014	15	354059
04	Written	23 September 2014	15	354730
05	Written (questionnaire)	6 October 2014	10	356957
06	Multiple choice	6 October 2014	45	357298

8.3 Submission of assignments

Enquiries about assignments, such as whether they have been received by the university, what credit you obtained, when they were returned to you, etc., should be addressed to the Assignments section. For detailed information and requirements as far as assignments are concerned, see *My studies @ Unisa*, which you received with your study package.

To submit an assignment **via myUnisa:**

- Go to *myUnisa*.

- Log in with your student number and password.
- Select the module.
- Click on assignments in the left–hand menu.
- Click on the assignment number you want to submit.
- Follow the instructions on the screen.

You can submit mathematics assignments in electronic format, but please note that you must still use all the correct mathematical notation, and include all necessary graphs, diagrams, and so on, just as if you were submitting a hand-written assignment! Your final submission file should be in the PDF format. You can use a word-processing program with an equation editor (e.g. MSWord) or you can use special mathematical typesetting programs such as LaTeX, and convert your assignment to PDF; or you can scan your hand-written assignment into a PDF file.

Please note: Although students may work together when preparing assignments, each student must write and submit his or her own individual assignment. It is unacceptable for students to submit identical assignments on the basis that they worked together. That is copying (a form of plagiarism) and none of these assignments will be marked. Furthermore, you may be penalised or subjected to disciplinary proceedings by the University.

8.4 Assignments

This tutorial letter 101 contains the assignments for both semesters, so select the semester you are enrolled for and do the set of assignments for that semester only. The assignments for Semester 1 are in Appendix A, pages 17 – 28. The assignments for Semester 2 are in Appendix B, pages 29 – 39.

9 OTHER ASSESSMENT METHODS

All the other assessment methods in this module are self-assessment. To find out whether you are on the right track, you can: Do the activities in the study guide and compare your answers with the feedback; do exercises in the text book and compare your answers with the given ones; and take the self-assessment quizzes on myUnisa.

10 EXAMINATION

10.1 Examination Admission

To be admitted to the examination you must submit the compulsory assignment, i.e. Assignment 01 by the due date. Note that admission therefore does not rest with the department and if you do not submit that particular assignment in time, we can do nothing to give you admission. Although you are most probably a part time student with many other responsibilities, work circumstances will not be taken into consideration for exemption from assignments or the eventual admission to the examination.

No concession will be made to students who do not qualify for the examination

10.2 Examination Period

This module is offered in a semester period of fifteen weeks. This means that

- if you are registered for the first semester, you will write the examination in May/June 2014 and should you fail and qualify for a supplementary examination, that supplementary examination will be written in October/November 2014.
- if you are registered for the second semester, you will write the examination in October/November 2014 and should you fail and qualify for a supplementary examination, that supplementary examination will be written in May/June 2015.

The examination section will provide you with information regarding the examination in general, examination venues, examination dates and examination times. Eventually, your results will also be processed by them and sent to you.

10.3 Examination Paper

The examination consists of a two hour paper. You are allowed to use a non-programmable calculator in the examination. Should you have a final mark of less than 50%, it implies that you failed the module. However, should your results be within a specified percentage (from 40% to 49%), you will be given a second chance in the form of a supplementary examination. If you fail the examination with less than 40%, the year mark will not count to help you pass. Please note also that the year mark does not apply in the case of a supplementary examination. The final mark after a supplementary examination is simply the mark you achieved in that examination, expressed as a percentage.

10.4 Previous Examination Papers

Previous examination papers are available to students on myUnisa. In addition, Assignment 6 in each semester is based on a previous year's examination paper, and model solutions to that paper will be sent out to you in a tutorial letter.

10.5 Tutorial Letter with Information on the Examination

To help you in your preparation for the examination, you will receive a tutorial letter that will set out clearly what material you have to study for examination purposes and what the assessment criteria are.

You are automatically admitted to the exam on the submission of Assignment 01 by a specific date – see Section 10.1. Please note that lecturers are not responsible for exam admission, and ALL enquiries about exam admission should be directed by e-mail to exams@unisa.ac.za.

11 FREQUENTLY ASKED QUESTIONS

The my Studies @ Unisa brochure contains an A-Z guide of the most relevant study information. Please refer to this brochure for any other questions.

12 SOURCES CONSULTED

No books were consulted in preparing this tutorial letter.

13 CONCLUSION

We hope that you will enjoy this module and wish you all the best!

Your lecturer,
Dr E Rapoo

ADDENDUM A: FIRST SEMESTER ASSIGNMENTS

The questions for each assignment for Semester 1 follow.

SEMESTER 1
COMPULSORY ASSIGNMENT FOR EXAM ADMISSION

ASSIGNMENT 01
Multiple Choice Assignment
Based on your previous knowledge
Fixed closing date: 17 February 2014
Unique Assignment Number: 876889

This multiple-choice assignment will be marked by computer. Hence the closing date is **fixed** and no extension of time can be granted.

Your answers must be entered on an optical mark reading sheet. But before you attempt that, please study in detail the relevant chapter of the publication *My studies @ Unisa.* Please make sure that you know how to handle the optical mark reading sheets, since sheets which are marked incorrectly and which are rejected by the computer will not be marked by hand and students will not receive marks for such assignments.

The unique number appearing in the box above links your assignment to the corresponding set of answers in the computer. It must therefore be filled in correctly on the optical mark reading sheet.

Note that your assignment will not be returned to you. Please keep a record of your answers so that you can compare them with the correct answers.

In each of the following four questions, mark the number of the answer that you think is correct. Each correct answer gives you 25%, adding up to a total of 100%.

Question 1: Which one of the following statements does NOT hold true for all random variables X, Y?
1. $E(X-Y) = E(X) - E(Y)$
2. $Var(X-Y) = Var(X) + Var(Y)$
3. $Var(-X) = Var(X)$
4. $Var(2+X) = Var(X)$

Question 2: Assume that X and Y are independent. Which of the following statements is NOT true?
1. $Var(X-Y) = Var(X) + Var(Y)$
2. $E(XY) = E(X)E(Y)$
3. $Cov(X,Y) = 0$
4. $E(XY) = 0$

Question 3: Which one of the following statements is true for all random variables X, Y?

1. $P(X + Y = 2) = P(X = 1) + P(Y = 1)$
2. $\{X + Y = 2\} \subset \{X = 1\}$
3. $P(X = 1) \leq P(X = 2)$
4. $P(X = 1) \leq P(X \geq 1)$

Question 4: Which one of the following statements is true:

1. If two random variables are from the same distribution then they cannot be independent.
2. If X and Y are from the same distribution then always $X = Y$.
3. A random variable from the Poisson distribution can never have negative values.
4. If $E(X) < E(Y)$ then $X < Y$.

SEMESTER 1

ASSIGNMENT 02
Written Assignment
Based on Unit 2
Fixed closing date: 25 February 2014
Unique Assignment Number: 862177

Question 1

A raffle ticket wins a prize with probability 0.3.

(a) Calculate the probability that the first time I win a prize is with the fourth ticket.

(b) Calculate the probability that the second prize I win is with the fifth ticket

(c) Calculate the probability that the first time I win a prize is with the fourth ticket AND the second prize I win is with the fifth ticket.

Question 2

Suppose the random variable X has the following density function:

$$f_X(x) = \begin{cases} \frac{3}{64}x^2 & \text{for} \quad 0 \leq x \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Determine the distribution function of X.

(b) Find $P(-1.5 < X < 1.5)$.

(c) Find $P(X > 1)$.

(d) Find the density function of the random variable $Y = 10 - 2X$,

(i) Using the transformation method.

(ii) By first finding the distribution function of Y.

Question 3

If the random variable X has the following cumulative distribution function

$$F_X(x) = \begin{cases} 0 & \text{for } x < -1 \\ \dfrac{x+1}{2} & \text{for } -1 \leq x \leq 1 \\ 1 & \text{for } x > 1 \end{cases}$$

find the distribution function and the probability density function for $Z = X^4$.

Question 4

Suppose that the random variable X has an exponential distribution with parameter $\lambda = 5$.

(a) Calculate the probability $P(X > 5)$.

(b) Determine the probability $P(Y > 50)$ for $Y = X^5$.

Question 5

The density function of a random variable Y is given by

$$f_Y(y) = \begin{cases} \frac{3}{8}(7-y)^2 & \text{for} \quad 5 \leq y \leq 7 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find $P(Y < 6)$.

(b) Find the median of the distribution.

SEMESTER 1

ASSIGNMENT 03
Written Assignment
Based on Unit 3
Fixed closing date: 18 March 2014
Unique Assignment Number: 862265

Question 1

The joint distribution function for the random variables X and Y for the area $0 < x < 2, 0 < y < 4$ is

$$F_{X.Y}(x, y) = \frac{1}{48}\left(12xy - x^2y - xy^2\right).$$

(a) Find the joint density function.

(b) Calculate $P(X > 2Y - 1)$.

(c) Find the marginal distribution function $F_X(x)$ of X.

(d) Find the condition density function $f_{X/Y}(x|y)$.

Question 2

Let $f_{X/Y}(x \mid y) = \begin{cases} \dfrac{c_1 x}{y^2}, & 0 < x < y,\ 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$

and

$$f_Y(y) = \begin{cases} c_2 y^2, & 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Determine the constants c_1 and c_2.

(b) Find the joint probability density function of X and Y.

(c) Calculate $P\left(X < \frac{1}{4} \mid Y = \frac{1}{2}\right)$.

Question 3

Let

$$f_{X,Y}(x,y) = \begin{cases} cx^2y^3 & 2-y < x < 2,\ 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}.$$

(a) Draw a sketch of the region A where the joint density function of the variables is non-zero.

(b) Find the value of c.

(c) Find the joint distribution function $F_{X,Y}(x,y)$ in the region $x > 2,\ 0 < y < 1$.

SEMESTER 1

ASSIGNMENT 04
Written Assignment
Based on Units 4, 5, 6
Fixed closing date: 8 April 2014
Unique Assignment Number: 862411

Question 1

Suppose the random variable X has the following density function:

$$f_X(x) = \begin{cases} \frac{3}{64}x^2 & \text{for} \quad 0 \leq x \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Determine $E(X)$, $E\left(X^2\right)$ and $Var(X)$.

(b) Determine the moment generating function $M_X(t)$ of X.

Question 2

Suppose that the number of insurance claims N, filed during a period of one year, is Poisson distributed with $E(N) = 10000$.use the normal approximation to the Poisson distribution to approximate $P(N < 9700)$.

Question 3

Suppose that X_1, X_2, X_3, X_4 are a random sample from a $N(1; 4)$ distribution. Let

$$U = X_1 + X_2 + X_3.$$

(a) Given the names of the following distributions as well as the values of their parameters

(i) $\frac{X_4 - 4}{4}$

(ii) $\frac{U}{3} \frac{U^2}{12}$

(iii) $\frac{(U-3)^2}{12}$

(b) Compute $E(U^2)$.

(c) Explain how you can construct a random variable which has the t-distribution with degree of freedom 3 from the random variables X_1, X_2, X_3, X_4.

SEMESTER 1

ASSIGNMENT 05
Written Assignment
Questionnaire
Fixed closing date: 22 April 2014
Unique Assignment Number: 862583

The questionnaire will be sent out to you later in the semester. Fill it in, and submit it in the assignment covers by the due date, to get 100% for this assignment.

SEMESTER 1

ASSIGNMENT 06
Multiple Choice Assignment
Based on the whole module
Fixed closing date: 22 April 2014
Unique Assignment Number: 862616

This multiple-choice assignment will be marked by computer. Hence the closing date is **fixed** and no extension of time can be granted.

Your answers must be entered on an optical mark reading sheet. But before you attempt that, please study in detail the relevant chapter of the publication *My studies @ Unisa*. Please make sure that you know how to handle the optical mark reading sheets, since sheets which are marked incorrectly and which are rejected by the computer will not be marked by hand and students will not receive marks for such assignments.

The unique number appearing in the box above links your assignment to the corresponding set of answers in the computer. It must therefore be filled in correctly on the optical mark reading sheet.

In the following we give questions from a previous year's examination paper. Answer first the examination questions, working through the questions on paper. Then answer the multiple choice questions based on your work. In each of the multiple choice questions, up to four possible answers are given. In each case, mark the number of the answer that you think is correct. Note that you will be sent full model solutions to the examination questions later on, so do keep your written work where you answered the examination questions so that you can compare it against the model solutions later on! Make also a note of your choices in the multiple choice questions, so that you can see where you went wrong.

Each multiple choice question counts for 5%, with a total of 100% from the twenty questions.

Examination question 1

Two discrete random variables X and Y have a joint probability mass function given in the table below.

$p_{X,Y}(x,y) =$

		x = 1	x = 2	x = 3
y	1	$\frac{1}{8}$	$\frac{1}{4}$	0
	2	$\frac{1}{8}$	$\frac{1}{8}$	0
	3	$\frac{1}{4}$	0	$\frac{1}{8}$

1.1 Calculate the marginal probability mass functions $p_X(x)$ and $p_Y(y)$ of X and Y.

1.2 Are the X and Y independent? Justify your answer!

1.3 Calculate the following:

(a) $P(X \leq 2 \mid Y = 2)$

(b) $E(Y \mid X = 2)$

(c) $P(X \geq 2, \quad Y \geq 1)$

Question 1: The answer to 1.2 is:

1. Yes 2. No

Question 2: Which one of the following is a correct answer to 1.2?

1. No, because $p_X(2) \neq p_Y(2)$.
2. No, because $p_{X,Y}(3,1) \neq p_X(3) \cdot p_Y(1)$.
3. Yes, because the marginal probability mass functions of both distributions sum up to 1.
4. Yes, because $p_{X,Y}(x,y) = p_X(x) \cdot p_Y(x) = 1 \cdot 1 = 1$.

Question 3: The answer to 1.3 (a) is:

1. $1/8$ 2. $1/4$

3. $7/12$ 4. 1

Question 4: The answer in 1.3 (b) is:

1. $1/2$ 2. $3/8$

3. $3/4$ 4. $4/3$

Question 5: The answer in 1.3 (c) is:

1. $(1/2, 1)$ 2. $1/4$

3. $1/2$ 4. $5/8$

Examination question 2
Let

$$f_{Y|X}(y \mid x) = \begin{cases} \dfrac{4x+2y}{4x+1}, & 0 < x < 1,\ 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

and

$$f_X(x) = \begin{cases} \dfrac{4x+1}{3}, & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

2.1 Find the marginal density function of Y.

2.2 Calculate $P\left(\frac{1}{2} < X < 1 \mid Y = 1\right)$.

2.3 Determine $P(Y < X)$.

Question 6: The marginal density function in 2.1 is found by evaluating the following:

1. $\frac{f_{Y|X}(y|x)}{f_X(x)}$ 2. $\int_{-\infty}^{\infty} \frac{f_{Y|X}(y|x)}{f_X(x)} dx$

3. $\int_{-\infty}^{\infty} f_{Y|X}(y \mid x) dx$ 4. $\int_{-\infty}^{\infty} f_{Y|X}(y \mid x) f_X(x) dx$

Question 7: The answer to 2.2 is:

1. $5/8$ 2. $2/3$

3. $5/6$ 4. $4/3$

Question 8: The answer to 2.3 is found by evaluating the following:

1. $1 - \int_0^Y f_X(x)\, dx$ 2. $\int_0^1 \int_0^x f_{X,Y}(x, y)\, dy dx$

3. $\int_0^1 \int_0^1 (f_X(x) - f_Y(y))\, dx dy$ 4. $\int_0^X f_{Y|X}(y \mid x) dy$

Examination Question 3
Suppose X and Y are two independent random variables of the exponential distribution with parameter $\lambda = 2$.

3.1 Find the distribution function of the random variable $U = X + Y$.

3.2 Find the moment generating function of X.

3.3 Calculate $E\left(X^5\right)$ by differentiating the moment generating function of X.

3.4 Find the moment generating function of the random variable $W = 2XY$. You may leave your solution as an integral expression.

Question 9: The distribution function of $U = X + Y$ in 3.1 is

1. $e^{2x} + e^{2y},\ x, y \geq 0$ 2. $2e^{-2x} + 2e^{-2y},\ x, y \geq 0$

3. $2e^{2xy},\ x, y \geq 0$ 4. $1 - e^{-2u} - 2ue^{-2u},\ u \geq 0$

Question 10: The answer to 3.3 is given by

1. $M'(5)$ 2. $M^{(v)}(t)$

3. $M^{(v)}(0)$ 4. setting $M^{(v)}(t) = 0$ and solving for t.

Examination Question 4

4.1 Let X_1 and X_2 be two independent random variables so that the variances of X_1 and X_2 are $\sigma_1^2 = k$ and $\sigma_2^2 = 2$, respectively. Given that the variance of $Y = 3X_1 - 2X_2$ is 35, find the value of k.

4.2 Find the mean and the variance of the sum Y of the items of a random sample of size 5 from the distribution with probability density function

$$f_X(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Question 11: The answer to 4.1 is:

1. 3 2. 13 3. 31/3 4. 43/9

Question 12: The variance of Y in 4.2 is:

1. 1/20 2. 1/4 3. 1/2 4. 5/4

Examination Question 5

5.1 Suppose that the number of insurance claims (N) filed during a period of one year is Poisson distributed with $E(N) = 10000$. Use the normal approximation to the Poisson distribution to approximate $P(|N - E(N)| > 500)$.

5.2 State if each of the following statements is true or false. If the statement is false, explain why it is false.

(a) If two random variables come from the same distribution with the same parameters, then they cannot be independent.

(b) Two discrete random variables are independent if their marginal probability mass functions both sum up to 1.

(c) If $f_Y(y) = f_{X|Y}(x \mid y)$ for all possible values x and y then X and Y are independent.

(d) Only continuous random variables have moment generating functions.

(e) If the joint distribution function $F_{X,Y}(x, y)$ is known then the marginal distribution of X can be found by integrating $F_{X,Y}$ over all values of Y.

Question 13: The answer 5.1 is given by

1. $P(Z > 5)$ where Z is a standard normal random variable.
2. $P(|Z| > 5)$ where Z is a standard normal random variable.

Question 14: The answer to 5.2 (a) is
1. True. 2. False.

Question 15: The answer to 5.2 (b) is
1. True. 2. False.

Question 16: The answer to 5.2 (c) is
1. True. 2. False.

Question 17: The answer to 5.2 (d) is
1. True. 2. False.

Question 18: The answer to 5.2 (e) is
1. True. 2. False.

Examination Question 6

Let $X_1, X_2, \cdots, X_{10}$ be a sample from a $N(2, 25)$ distribution, independent of the sample $Y_1, Y_2, \cdots, Y_{20}$ coming from a $N(2, 100)$ distribution. Let

$$S_X^2 = \frac{1}{9}\sum_{i=1}^{10}(X_i - \overline{X})^2 \qquad U = \frac{1}{19}\sum_{j=1}^{20}(Y_j - 2)^2$$

$$\hat{\sigma}_X^2 = \frac{1}{10}\sum_{i=1}^{10}(X_i - 2)^2 \qquad \hat{\sigma}_Y^2 = \frac{1}{20}\sum_{j=1}^{20}(Y_j - 2)^2$$

6.1 Explain why $Q = \sum_{j=1}^{6}\left(\frac{Y_j - 2}{10}\right)^2$ does not follow a $\chi^2(10)$ distribution.

6.2 Write down the name of the distribution of $U = \frac{19U}{10^2}$ and give the value(s) of the parameter(s).

6.3 Define $V = A\frac{\hat{\sigma}_X^2}{\hat{\sigma}_Y^2}$. For what value of the constant A does V follow an F distribution? What are the degrees of freedom of this F distribution? Justify your answer!

Question 19: The distribution in question 6.2 is

1. $N(0, 1)$ 2. $N(0, 100)$ 3. $\chi^2(19)$ 4. $\chi^2(20)$

Question 20: The value of A question 6.2 is

1. $1/2$ 2. 1 3. 2 4. 4

ADDENDUM B: SECOND SEMESTER ASSIGNMENTS

The questions for each assignment for Semester 2 follow. .

SEMESTER 2
COMPULSORY ASSIGNMENT FOR EXAM ADMISSION

ASSIGNMENT 01
Multiple Choice Assignment
Based on your previous knowledge
Fixed closing date: 4 August 2014
Unique Assignment Number: 873654

This multiple-choice assignment will be marked by computer. Hence the closing date is **fixed** and no extension of time can be granted.

Your answers must be entered on an optical mark reading sheet. But before you attempt that, please study in detail the relevant chapter of the publication *My studies @ Unisa.* Please make sure that you know how to handle the optical mark reading sheets, since sheets which are marked incorrectly and which are rejected by the computer will not be marked by hand and students will not receive marks for such assignments.

The unique number appearing in the box above links your assignment to the corresponding set of answers in the computer. It must therefore be filled in correctly on the optical mark reading sheet.

Note that your assignment will not be returned to you. Please keep a record of your answers so that you can compare them with the correct answers.

In each of the following four questions, mark the number of the answer that you think is correct. Each correct answer gives you 25%, adding up to a total of 100%.

Question 1: Which one of the following statements does NOT hold true for all random variables X, Y?
1. $E(X-Y) = E(X) - E(Y)$
2. $Var(X-Y) = Var(X) + Var(Y)$
3. $Var(-X) = Var(X)$
4. $Var(2+X) = Var(X)$

Question 2: Assume that X and Y are independent. Which of the following statements is NOT true?
1. $Var(X-Y) = Var(X) + Var(Y)$
2. $E(XY) = E(X)E(Y)$
3. $Cov(X,Y) = 0$
4. $E(XY) = 0$

Question 3: Which one of the following statements is true for all random variables X, Y?

1. $P(X+Y=2) = P(X=1) + P(Y=1)$
2. $\{X+Y=2\} \subset \{X=1\}$
3. $P(X=1) \leq P(X=2)$
4. $P(X=1) \leq P(X \geq 1)$

Question 4: Which one of the following statements is true:

1. If two random variables are from the same distribution then they cannot be independent.
2. If X and Y are from the same distribution then always $X = Y$.
3. A random variable from the Poisson distribution can never have negative values.
4. If $E(X) < E(Y)$ then $X < Y$.

SEMESTER 2

ASSIGNMENT 02
Written Assignment
Based on Unit 2
Fixed closing date: 12 August 2014
Unique Assignment Number: 896956

Question 1

A raffle ticket wins a prize with probability 0.3.

(a) If I buy 10 tickets, calculate the probability that I win more than 4 prizes.

(b) If I buy 10 tickets, calculate the probability that the first 5 win me one prize and the second 5 win me two prizes.

(c) Calculate the probability that I buy 20 tickets without winning anything.

Question 2

Suppose the random variable X has the following cumulative distribution function:

$$F_X(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ \frac{1}{2}x & \text{for} \quad 0 \leq x < 1 \\ \frac{1}{4}x + \frac{1}{4} & \text{for} \quad 1 \leq x < 3 \\ 1 & \text{for} \quad x \geq 3 \end{cases}$$

(a) Determine the density function of X.

(b) Find $P(-1.5 < X < 1.5)$.

(c) Find the density function of the random variable $Y = X^5$,

(i) Using the transformation method.

(ii) By first calculating the distribution function of Y.

Question 3

If the random variable X has the following cumulative distribution function

$$F_X(x) = \begin{cases} 0 & \text{for } x < -1 \\ \dfrac{x+1}{2} & \text{for } -1 \leq x \leq 1 \\ 1 & \text{for } x > 1 \end{cases}$$

find the distribution function and the probability density function for $Z = X^2$.

Question 4

Suppose that the random variable X has the Poisson distribution with $E(X) = 5$. Determine the probability $P(Y > a)$ for $a > 0$ for $Y = X + 2$.

Question 5

Let X have the probability density function

$$f_X(x) = \begin{cases} 1, & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Show that the random variable $Y = -2\ln X$ has a chi-square distribution with two degrees of freedom.

SEMESTER 2

ASSIGNMENT 03
Written Assignment
Based on Unit 3
Fixed closing date: 2 September 2014
Unique Assignment Number: 354059

Question 1

Let X and Y be two random variables with the following joint density function:

$$f_{X,Y}(x,y) = \begin{cases} a\left(x^3 + 2xy^2\right) & \text{for} \quad 0 < x < 2, \;\; 0 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find the value of a.

(b) Find the marginal density of Y.

(c) Find the conditional density of X given $Y = y$.

(d) Compute $P\left(Y > 1 \mid X = \frac{1}{4}\right)$.

(e) Compute $P\left(Y > 1\right)$.

(f) Compute $P\left(Y > X + 1\right)$.

(g) Compute $P\left(X > 1, Y > 1\right)$.

Question 2

Let

$$f_{X,Y}(x,y) = \begin{cases} cx^2y^2 & 2 - x < y < 2, \;\; 0 < x < 2 \\ 0 & \text{elsewhere} \end{cases}.$$

(a) Draw a sketch of the region A where the joint density function of the variables is non-zero.

(b) Find the value of c.

(c) Find the joint distribution function $F_{X,Y}(x,y)$ in the region $0 < x < 2$, $y > 2$.

SEMESTER 2

ASSIGNMENT 04
Written Assignment
Based on Units 4, 5, 6
Fixed closing date: 23 September 2014
Unique Assignment Number: 354730

Question 1

Suppose the random variable X has the following cumulative distribution function:

$$F_X(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ \frac{1}{2}x & \text{for} \quad 0 \leq x < 1 \\ \frac{1}{4}x + \frac{1}{4} & \text{for} \quad 1 \leq x < 3 \\ 1 & \text{for} \quad x \geq 3 \end{cases}$$

(a) Determine $E(X)$, $E(X^2)$ and $Var(X)$.

(b) Determine the moment generating function $M_X(t)$ of X.

(c) Determine the moment generating function $M_Y(t)$ of $Y = 2X^2$.

Question 2

Let the random variable Z_n have a Poisson distribution with parameter $\lambda = n$. Show that the limiting distribution of the random variable $Y_n = \dfrac{Z_n - n}{\sqrt{n}}$ is normal with mean zero and variance one.

Question 3

Let $X_1, X_2, \cdots, X_{10}$ be a random sample from a normal distribution with mean zero and variance four and $Y_1, Y_2, \cdots, Y_{12}$ an independent random sample from a normal distribution with mean three and variance nine.

Let $\hat{\sigma}^2_X = \frac{1}{10}\sum_{i=1}^{10} X_i^2$ and $\hat{\sigma}^2_Y = \frac{1}{12}\sum_{j=1}^{12}\left(Y_j - 3\right)^2$. Give the distributions and degrees of freedom of

(a) $U = \frac{5}{2}\hat{\sigma}^2_X$

(b) $V = \frac{4}{3}\hat{\sigma}^2_Y$

(c) $W = \frac{\hat{\sigma}^2_X}{4} \Big/ \frac{\hat{\sigma}^2_Y}{9}$.

(d) Explain how you can construct a $F_{6,6}$ random variable from the random variables $Y_1, Y_2, \cdots, Y_{12}$.

SEMESTER 2

ASSIGNMENT 05
Written Assignment
Questionnaire
Fixed closing date: 6 October 2014
Unique Assignment Number: 356957

The questionnaire will be sent out to you later in the semester. Fill it in, and submit it in the assignment covers by the due date, to get 100% for this assignment.

SEMESTER 2

ASSIGNMENT 06
Multiple Choice Assignment
Based on the whole module
Fixed closing date: 6 October 2014
Unique Assignment Number: 357298

This multiple-choice assignment will be marked by computer. Hence the closing date is **fixed** and no extension of time can be granted.

Your answers must be entered on an optical mark reading sheet. But before you attempt that, please study in detail the relevant chapter of the publication *My studies @ Unisa.* Please make sure that you know how to handle the optical mark reading sheets, since sheets which are marked incorrectly and which are rejected by the computer will not be marked by hand and students will not receive marks for such assignments.

The unique number appearing in the box above links your assignment to the corresponding set of answers in the computer. It must therefore be filled in correctly on the optical mark reading sheet.

In the following we give questions from a previous year's examination paper. Answer first the examination questions, working through the questions on paper. Then answer the multiple choice questions based on your work. In each of the multiple choice questions, up to four possible answers are given. In each case, mark the number of the answer that you think is correct. Note that you will be sent full model solutions to the examination questions later on, so do keep your written work where you answered the examination questions so that you can compare it against the model solutions later on! Make also a note of your choices in the multiple choice questions, so that you can see where you went wrong.

Each multiple choice question counts for 5%, with a total of 100% from the twenty questions.

Examination question 1

Two discrete random variables X and Y have a joint probability mass function given in the table below.

$p_{X,Y}(x,y) =$

		x		
		1	**2**	**3**
	1	0.1	0	0
y	**2**	0	0	0.2
	3	0	0.2	0.5

1.1 Calculate the marginal probability mass functions $p_X(x)$ and $p_Y(y)$ of X and Y.

1.2 Are the X and Y independent? Justify your answer!

1.3 Calculate $P(X = 1 \mid Y \geq 2)$.

1.4 Calculate $E(X \mid Y = 2)$.

1.5 Calculate $E(XY)$.

1.6 Find $M_X(t)$, the moment generating function of X.

Question 1: The answer to 1.2 is:

1. Yes. 2. No.

Question 2: The answer to 1.3 is:

1. 0 2. 0.1 3. 1/3 4. 0.5

Question 3: The answer to 1.4 is:

1. 0.6 2. 0.9 3. 3 4. 13

Question 4: The answer to 1.5 is:

1. 0.54 2. 2 3. 6.76 4. 7

Question 5: The answer to 1.6 is:

1. e^t 2. $1 + 0.1t + \frac{1}{2}(0.2t)^2 + \frac{1}{6}(0.7t)^3$

3. $0.1e^{0.1t} + 0.2e^{0.2t} + 0.7e^{0.7t}$ 4. $0.1e^t + 0.2e^{2t} + 0.7e^{3t}$

Examination question 2
Assume that the cumulative distribution function of a continuous random variable X is given by

$$F_X(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{2}x, & 0 \leq x < a \\ \frac{1}{6}x + \frac{1}{3}, & a \leq x < 4 \\ 1, & x \geq 4 \end{cases}$$

2.1 Prove that this defines a cumulative distribution function only if $a = 1$.

2.2 Calculate $P(X < 1 \mid X < 2)$ and $P(X < 2 \mid X < 1)$.

2.3 Calculate $E(X)$.

2.4 Find the distribution function $F_U(u)$ of $U = X - 2$.

Question 6: The answers to 2.2 are:

1.	$\frac{1}{2}, 0$	2.	$\frac{1}{2}, \frac{1}{6}$
3.	$\frac{3}{4}, 1$	4.	$1, 1$

Question 7: The answer to 2.3 is:

1.	$2/3$	2.	$5/6$
3.	3	4.	$13/12$

Examination question 3

Assume that the joint density function of continuous random variables X and Y is given by

$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{8}(x + y + 2) & -1 < x < 1,\ -1 < y < 1 \\ 0 & \text{elsewhere} \end{cases}.$$

3.1 Find the marginal density function of X.

3.2 Calculate $P\left(\frac{1}{2} < Y < 1 \mid X = 0\right)$.

3.3 Determine $P(X + Y > 1)$.

Question 8: The answer to 3.1 is:

1.	$\frac{1}{8},\ -1 < x < 1$	2.	$\frac{1}{8}(x + 2),\ -1 < x < 1$
3.	$\frac{1}{4}(x + 2),\ -1 < x < 1$	4.	$\frac{1}{2},\ -1 < x < 1$

Question 9: The answer to 3.2 is:

1. 1/8
2. 11/16
3. 11/32
4. 59/32

Question 10: The answer to 3.3 is found by evaluating the following:

1. $1 - \int_0^{1-Y} f_X(x)\, dx$
2. $\int_0^1 \int_{1-x}^1 f_{X,Y}(x, y)\, dydx$
3. $\int_{-1}^1 \int_{1-x}^1 f_{X,Y}(x, y)\, dydx$
4. $\int_{1-X}^1 f_Y(y)dy$

Examination question 4
Assume that the joint density function of continuous random variables X and Y is given by

$$f_{X.Y}(x, y) = \begin{cases} 24xy, & 0 < x < 1,\ 1-x < y < 1 \\ 0 & \text{elsewhere} \end{cases}.$$

4.1 Calculate $E(XY)$.

4.2 Find the joint cumulative distribution function in the region $0 < x < 1,\ 1 - x < y < 1$.

Question 11: The area where $f_{XY}(x, y) > 0$ is

1. A rectangle.
2. A triangle.
3. Neither a rectangle nor a triangle.

Question 12: True or false: The area where $f_{XY}(x, y) > 0$ can also be expressed as $0 < y < 1$, $1 - y < x < 1$.

1. True.
2. False.

Examination question 5

5.1 Suppose that the moment generating function of a random variable X is given by

$$M_X(t) = t^2 - 1, \quad \text{for all } t \in R.$$

Find the moment generating function of the random variable Y where

$$Y = \frac{1-X}{6}.$$

5.2 State if each of the following statements is true or false. If the statement is false, explain why it is false. In the following, X and Y are two continuous random variables.

(a) If the marginal distributions of random variables X and Y are the same, then X and Y are independent.

(b) If $f_X(x) f_Y(y) = f_{X|Y}(x \mid y)$ for all possible values x and y then X and Y are independent.

(c) The exponential distribution with parameter λ has the density function $f(x) = \lambda e^{-\lambda x}$, for all $x \in R$.

(d) The joint cumulative distribution function $F_{X,Y}$ is found by integrating the joint density function $f_{X,Y}(x, y)$ over all possible values of x and y.

5.3 Explain how we can find the marginal distribution function $F_X(x)$ directly from the joint cumulative distribution function $F_{X,Y}(x, y)$ without first finding the joint density function.

Question 13: The answer to 5.1 is:

1. $-\frac{1}{6}t^2$ 2. e^{tx}

3. $e^{t/6}\left(\frac{t^2}{36} - 1\right)$ 4. $\left(\frac{1-t}{6}\right)^2 - 1$

Question 14: The answer to 5.2(a) is:

1. True 2. False

Question 15: The answer to 5.2(b) is:

1. True 2. False

Question 16: The answer to 5.2(c) is:

1. True 2. False

Question 17: The answer to 5.2(d) is:

1. True 2. False

Examination question 6

Let $X_1, X_2, \cdots, X_{20}$ be a random sample from a $N(1, 25)$ distribution, independent of the random sample $Y_1, Y_2, \cdots, Y_{20}$ coming from a $N(5, 100)$ distribution.

6.1 Give the name and parameters of the distribution of the random variable

$$U = \left(\frac{\sum_{i=1}^{9} X_i}{15} - \frac{2}{3}\right)^2.$$

Justify your answer!

6.2 Find the value of A for which the random variable

$$V = A\frac{\sum_{i=1}^{6}(X_i - 1)^2}{\sum_{i=1}^{3}(Y_i - 5)^2}$$

has the F-distribution with some degrees of freedom m and n. Give also the values of m and n. Justify your answer!

6.3 Give a definition of a random variable W, constructed only from the random variables $X_1, X_2, \cdots, X_{20}$, such that W has the F–distribution with degrees of freedom $m = 10$, $n = 10$.

Question 18: The answer to 6.1 is:

1. $N(0,1)$ 2. $N(0,3)$

3. χ_1^2 4. χ_9^2

Question 19: The value of A in 6.2 is:

1. 1/2
2. 1
3. 2
4. 4

Question 20: The random variable W in 6.3:

1. can be constructed in only one way.
2. can be constructed in several different ways.