11. BASIC LINEAR PROGRAMMING

- Linear programming
- Approach – linear programming
- Uses – linear programming

LINEAR PROGRAMMING (Drury 8th Edition page 656)

Linear programming: A mathematical technique used to determine how to employ limited resources to achieve optimum benefits

Objective function: Quantification of an objective = maximising profits or minimising costs

Assumptions:

- Contribution per unit and the utilization of resources per unit are the same whatever quantity of output is produced and sold
- Units produced and resources allocated are infinitely divisible (i.e. we can produce 12.38 units = interpret 12 units to produce)

APPROACH: LINEAR PROGRAMMING

1. Objective function
2. Contribution/Marginal income per product
3. Find constraints
4. Contribution/Marginal income per limiting factor
5. Ranking
6. Decide what should be manufactured first
7. Use linear programming to decide what should be manufactured second and third

USES OF LINEAR PROGRAMMING (Drury 8th Edition page 665-667)

1. Calculation of relevant costs
2. Selling different products
3. Maximum payment for additional scarce resources
4. Control
5. Managing constraints
6. Capital budgeting
7. Sensitivity analysis

RECAP 1: LINEAR PROGRAMMING

QUESTION 1 (24 marks; 29 minutes)

BGD Enterprises is a manufacturer of three types of products, namely Bee, Gee and Dee.

The three products are manufactured in a mechanised process using two materials.

The standard costs per unit are as follows:

	Bee R	Gee R	Dee R
Material X (@ R45 per kg)	45,00	22,50	90,00
Material Y	30,00	22,50	45,00
Variable overheads (@ R25 per machine hour)	50,00	25,00	37,50
Allocated fixed costs	16,00	16,00	16,00
Total standard costs	**141,00**	**86,00**	**188,50**
Selling price per unit:	185,00	140,00	240,00

RECAP 1: LINEAR PROGRAMMING

Additional information:

1. Material X used in the production of these products are limited to 5 000 kg per annum, while material Y is readily available.
2. The capacity of machine hours is limited to 7 920 hours per annum.
3. The expected potential sales for 2013 are as follows:

Product Bee:	2 000 units
Product Gee	3 000 units
Product Dee	1 000 units

REQUIRED:

Determine the product mix which will maximise the marginal income of the company for 2013. **[24]**

(ACN306 – OCT/NOV 2012)

RECAP 1: LINEAR PROGRAMMING

Objective function:

Let B = Bee, G = Gee and D = Dee

Maximise 60B + 70G + 67,50D

Contribution per product	**Bee** R		**Gee** R		**Dee** R	
Selling price	185.00		140.00		240.00	
Less variable costs	125.00		70.00		172.50	
Material X	45.00	^	22.50	^	90.00	^
Material Y	30.00	^	22.50	^	45.00	^
Overheads	50.00	^	25.00	^	37.50	^
Contribution	**60.00**		**70.00**		**67.50**	

(4½)

RECAP 1: LINEAR PROGRAMMING

Constraints (Limiting factors):

Product	Material X				Machine hours		
	Units	Kg per unit	Total kg	R	Units	Hours per unit	Total hours
Bee	2 000	45/45	2 000^	90 000^	2 000	50/25	4 000,00^
Gee	3 000	22,5/45	1 500^	67 500^	3 000	25/25	3 000,00^
Dee	1 000	90/45	2 000^	90 000^	1 000	37,50/25	1 500,00^
Required			5 500	247 000			8 500,00
Available			5 000^	225 000^			7 920,00^
Shortage			500^	22 500^			580,00^

Material X and machine hours are both limiting factors. ^^ (6)

RECAP 1: LINEAR PROGRAMMING

Marginal income per limiting factor:

		Material X				Machine hours			
	Marginal income	**Kg per unit**	**Marginal income per kg**	**Marginal income per R**	**Rank-ing**	**Marginal income**	**Hours per unit**	**Marginal income per hour**	**Rank-ing**
Bee	60,00	45/45	60,00^	1,333^	2	60,00	50/25	30,00^	3
Gee	70,00	22,5/45	140,00^	3,111^	1^	70,00	25/25	70,00^	1^
Dee	67,50	90/45	33,75^	0,750^	3	67,50	37,50/25	45,00^	2

Product Gee ranks first in terms of both constraints and therefore must be manufactured first.

Linear programming should be used to determine the optimal combination between products Bee and Dee. ^^ (5)

RECAP 1: LINEAR PROGRAMMING

Produce 3 000 litres of Gee:

	Material X (kg)	**Machine hours (hours)**
Available	5 000^	7 920
Required for Gee	1 500^	3 000
Available for Bee and Dee	3 500	4 920

RECAP 1: LINEAR PROGRAMMING

Let product Bee = B and product Dee = D

Material X:	1B	+	2 D	≤	3 500	① ^^^
Machine hours:	2 B	+	1,5D	≤	4 920	② ^^^
① x 2	2 B	+	4 D	≤	7 000	③ ^^^
③- ②		+	2,5D	≤	2 080	
			D	≤	832	
Substitute			D	=	832	in ①
	1B	+	2(832)	≤	3 500	^^
	B	+	1 664	≤	3 500	
	B			=	1 836	

Optimal product mix is therefore,

Bee	1 836 units
Gee	3 000 units
Dee	832 units

^^ (conclusion)
(8½)
[24]

12. INTRODUCTION TO TRANSFER PRICING

- Purpose of transfer pricing
- Transfer pricing methods

PURPOSE OF TRANSFER PRICING (Drury 8th Edition page 509-510)

1. To provide information that **motivates divisional managers** to make good economic decisions
2. To provide information that is useful for **evaluating the managerial and economic performance** of the divisions
3. To ensure that divisional **autonomy** is **not undermined**
4. To **intentionally move profits between divisions** and locations

TRANSFER PRICING METHODS (Drury 8th Edition page 511)

1. **Market-based transfer prices**
2. Cost plus a profit mark-up transfer prices
3. Marginal/Variable cost transfer prices
4. Full cost transfer prices
5. Negotiated transfer prices
6. **Marginal/Variable cost plus opportunity cost transfer prices**

(RED – RELEVANT TO MAC3701)

13. LONG-TERM EXTERNAL PRICE SETTING

- Pricing customized products/services
- Pricing non-customized products/services
- Pricing policies
- Product Life Cycle

INTRODUCTION

Price takers: Firms that have little or no influence over the prices of their products or services (i.e. commodity markets – wheat, coffee, gold, silver)

Use cost information to determine profitability and then decide whether to enter or stay in the market at the market- determined price.

Price setters: Firms that have some discretion over setting the selling price of their products or services (i.e. market leaders, customized or differentiated products)

Use cost information to set the price

PRICING CUSTOMIZED PRODUCTS/SERVICES

Because highly customised products or services are likely to be sold to a single customer or a few customers only, it is not that difficult to estimate the sales volume of these products. Direct costs would normally be accumulated with a job-costing system. Prices would also differ between customised products as each product is unique.

- Unique
- No comparable market prices exist
- Sales must cover cost – using **cost-plus pricing** (i.e. Vehicle repairs, accounting services)

PRICING CUSTOMIZED PRODUCTS/SERVICES

Cost base		**Mark up %**	**Cost-plus pricing**
Direct variable cost	R 200	150%	R 500
Direct fixed costs	R 100		
Total direct costs	**R 300**	70%	R 510
Indirect (overhead)	R 80		
Total costs	**R 380**	35%	R 513

PRICING CUSTOMIZED PRODUCTS/SERVICES

The sophistication of the organisation's costing system will determine how much detailed costing information is available. Usually, direct variable costs are easier to determine. These, however, exclude all fixed costs. You therefore need a higher markup percentage to make a contribution towards the direct fixed costs of the production line (or factory) and the general fixed costs. The more costs the accountant can justifiably allocate to the product, the lower the markup percentage needs to be, as the base cost already includes most of the costs. See the table below for further explanation:

	Cost base	Markup %	The markup should cover the following:
1	Direct variable cost	High	Direct fixed costs **and** indirect costs (overheads) **and** profit
2	Total direct cost	Medium	Indirect costs (overheads) **and** profit
3	Total cost (Full/long-term cost)	Low	Profit

PRICING NON-CUSTOMIZED PRODUCTS/SERVICES

DO QUESTION 10.13 IN YOUR TEXTBOOK (Drury 8th edition page 245)

A company manufactures a single product, product Y. It has documented levels of demand at certain selling prices for this product as follows:

Demand (Unit)	Selling price per unit R	Cost per unit R
1 100	48	22
1 200	46	21
1 300	45	20
1 400	42	19

Required:

Using a tabular approach, calculate the marginal revenue and marginal costs for product Y at the different levels of demand, and so determine the selling price at which the company profits are maximised.

PRICING NON-CUSTOMIZED PRODUCTS/SERVICES

Demand in units	Selling price per unit	Total revenue	Marginal revenue	Cost per unit	Total cost	Marginal cost	Total profit	Marginal profit
(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
1 100	48	52 800		22	24 200		28 600	
1 200	46	55 200	2 400	21	25 200	1 000	30 000	1 400
1 300	**45**	58 500	3 300	20	26 000	800	**32 500**	2 500
1 400	42	58 800	300	19	26 600	600	32 200	-300

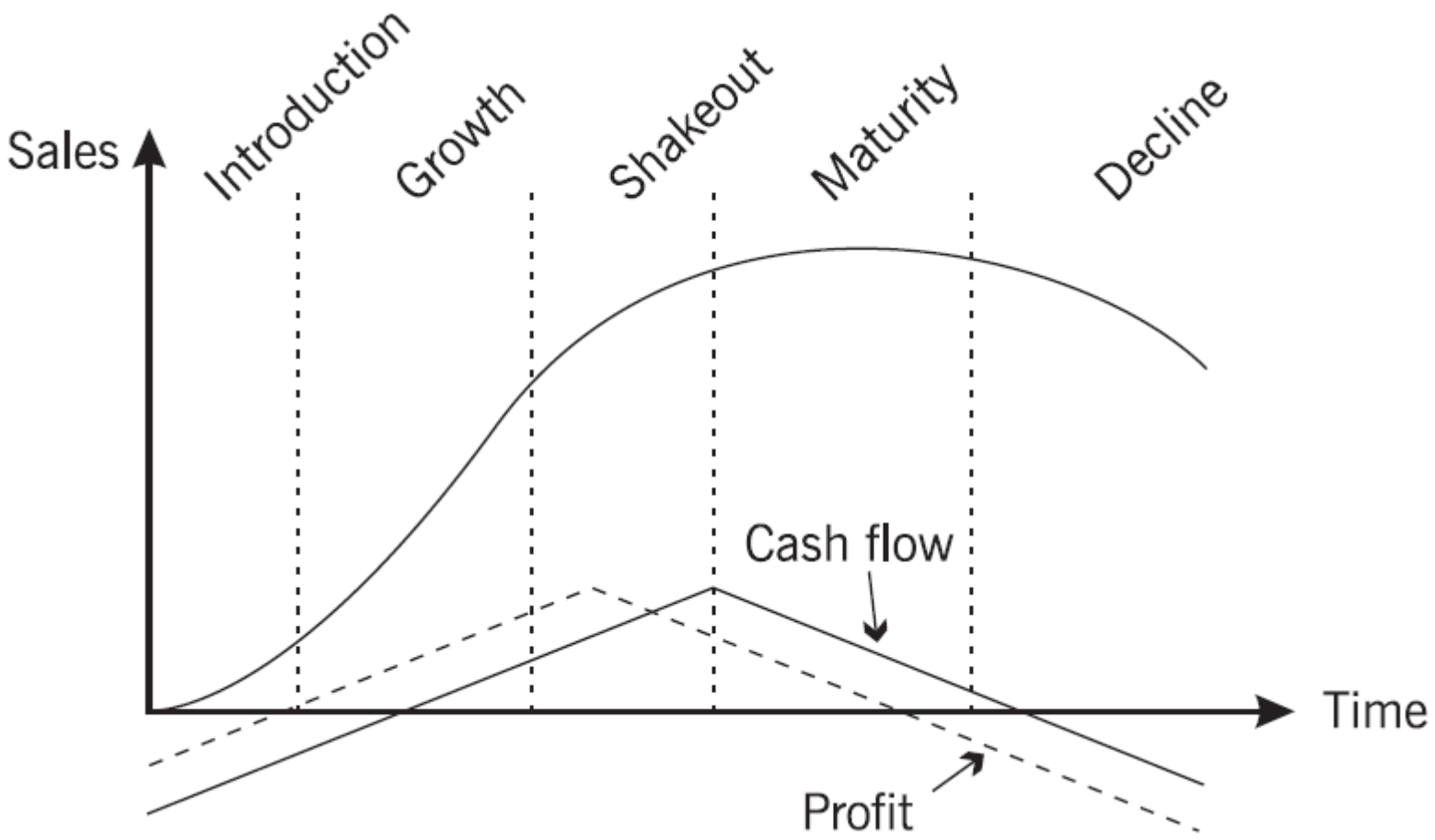
Note:

- The **marginal** revenue/cost/profit is calculated by deducting the **total** revenue/cost/profit at a specific demand level from the total revenue/cost/profit at the **previous** level.
- Total profits will increase as long as marginal revenue exceeds marginal cost. The optimum selling price is where marginal revenue = marginal cost (or the last price for which marginal revenue still exceeds marginal cost).
- Setting prices where marginal revenue is ≥ marginal cost is an important concept when decision making involves different levels of pricing and costs. Make sure you understand this concept.

PRICING POLICIES

1. **Price-skimming**
   - Approach that charge high initial price to exploit sections of market that are relatively insensitive to price changes (i.e. new cell phones, new technology)
2. **Penetration pricing policy**
   - Charging low prices, initially with intention of gaining rapid acceptance of product/service (i.e. launch of new laundry brand)

PRODUCT LIFE CYCLE



CIMA E1 textbook

PRODUCT LIFE CYCLE

Product life cycle: The period from initial expenditure on research and development to the withdrawal of support to customers

- **Introductory** – Minimal awareness and acceptance of product
- **Growth** – Sales rapidly expand due to introductory promotions and greater customer awareness
- **Maturity** – Sales taper off as potential new customers are exhausted
- **Decline** – Sales diminish as product is gradually replaced with new and better versions

14. ADVANCED SENSITIVITY ANALYSIS

- Multi-product CVP analysis (common fixed cost)
- Operating leverage

MULTI-PRODUCT CVP ANALYSIS (COMMON FIXED COSTS)

When an organisation sells more than one product, we can still use break-even analysis to identify the number of units the organisation needs to sell of each product in the sales mix to break even. Organisations with a wider product range usually have fixed overhead costs that are not directly related to individual products but are incurred to support the organisation as a whole. These **common fixed costs** should also be covered by the contribution generated by all the products.

When we calculate the break-even point in a multi-product organisation, we follow the same pattern as in a single product organisation. The numerator will be the combined fixed costs, while the denominator will be the weighted average contribution margin. The contributions are "weighted" to give an average contribution for the products.

MULTI-PRODUCT CVP ANALYSIS (COMMON FIXED COSTS): [Drury 8th Edition Example 8.2 page 179]

The Super Bright Company sell two types of washing machines – a deluxe model and a standard model. The financial controller has prepared the following information based on the sales forecast for the period:

Sales volume	**Deluxe**	**Standard**	**TOTAL**
Selling price	R 300	R 200	
Variable cost	R 150	R 110	
Contribution	R 150	R 90	
Total sales revenue	R 360 000	R 120 000	R480 000
Less total variable cost	(R180 000)	(R66 000)	(R 246 000)
Contr. to direct & common fix cost*	R 180 000	R 54 000	R 234 000
Less direct avoidable fixed costs	(R 90 000)	(R 27 000)	(R 117 000)
Contr. to common fixed costs	R 90 000	R 27 000	R 117 000
Less common(indirect) fixed costs			(R 39 000)
Operating profit			**R 78 000**

** Relates to cost of common facilities and can only be avoided if neither of the products are sold. The MD is concerned that sales may be less than forecast and has requested information relating to break-even point for the activities for the period.*

MULTI-PRODUCT CVP ANALYSIS (COMMON FIXED COSTS): [Drury 8th Edition Example 8.2 page 179]

Breakeven	**Deluxe**	**Standard**
=Direct fixed costs	R 90 000	R 27 000
Unit contribution	R 150	R 90
BEP:	600 units	300 units

- Only cover direct fixed costs!

What about the Common (Indirect) fixed cost of R39 000?

- Common cost cannot be specifically identified to products
- Can only be avoided if **both** products are not sold

Solution: Convert individual products into standard batches of products based on planned sales mix!

MULTI-PRODUCT CVP ANALYSIS (COMMON FIXED COSTS): [Drury 8th Edition Example 8.2 page 179]

Sales mix	Deluxe		Standard
Total sales/Unit selling price	1 200 units		600 units
Sales mix:	**2**	**:**	**1**

Standard batch of products compromise 2 Deluxe and 1 Standard machine.

Contribution per batch: 2 x R150 + 1 x R90

R 390

$$\text{Breakeven} = \frac{\text{Total fixed costs}}{\text{Contribution per batch}}$$

$$\frac{\text{R 90 000 + R 27 000 + R 39 000}}{\text{R 390}}$$

Breakeven = 400 batches (800 units of Deluxe & 400 units of Standard)

MULTI-PRODUCT CVP ANALYSIS (COMMON FIXED COSTS): [Drury 8th Edition Example 8.2 page 179]

With multi-product CVP analysis, we must make an assumption regarding the sales mix. Should the sales mix change, the calculated amounts (weighted contribution) would also change. Please work through example 8.2 carefully. In this example, the optimal product mix is 800 units of the deluxe machine and 400 units of the standard machine. The break-even sales **value** would be as follows

Break-even sales value = (800 units x R300) + (400 units x R200)
= R240 000 + R80 000
= R320 000

However, if you are required to calculate the break-even sales value only, you need not determine the sales mix first. The calculation would then be as follows:

Break-even sales value = Fixed costs/ contribution ratio
= (R117 000 + R39 000)/ (R234 000/R480 000)
= R156 000/ 0,4875
= R320 000

Total contribution:
R234 000
Total revenue:
R480 000

OPERATING LEVERAGE

Operating leverage is the ratio of an organisation's fixed costs to its variable costs. Organisations with a higher proportion of fixed costs and a lower proportion of variable costs have a high operating leverage, while organisations with a lower proportion fixed costs and a higher proportion variable costs have a low operating leverage.

Degree of operating leverage = Contribution margin / Profit

Work through Exhibit 8.1 (Drury 8th Edition page 182)

- *Higher degree of operating leverage can provide significantly greater profits when sales are increasing but higher percentage decreases will also occur when sales are declining!*

OPERATING LEVERAGE – ACTIVITY 14.8 (STUDY GUIDE PAGE 242)

Activity 14.8

You have the following information for two organisations:

	ABC Ltd	XYZ Ltd
Contribution	8 000	8 000
Profit	2 000	6 500

REQUIRED

Which organisation is more vulnerable to a downturn in the economy?

Solution to Activity 14.8

	ABC Ltd	XYZ Ltd
Operating leverage	8 000/2 000	8 000/6 500
	= 4	= 1,2308

ABC Ltd has a higher operating leverage due to its high fixed costs (8 000 – 2 000 = R6 000). They would need to sell a lot more units in order to break even and therefore are more vulnerable to changes in their sales volumes.

RECAP 1: CVP & OTHER SENSITIVITY

UNISA REVISION PACK 2014:

QUESTION 2 (16 marks;19 minutes)

Travelfrenzy Ltd manufactures suitcases. The company values inventory based on the standard **absorption** costing method. Materials are recorded at actual costs. There were no raw materials, work-in-process or finished goods inventory at the beginning or end of 2014.

The following actual results are available for the 2014 year:

Number of suitcases manufactured		80 000
		R
Material purchased	83 000kg	19 090 000
Direct labour	119 500 hours	12 350 000
Variable overheads		6 000 000
Fixed overheads		1 500 250

Additional information:

1. Material issued to production = 81 050 kg.
2. The actual sales for 2014 amounted to R88 000 000 (80 000 suitcases).

RECAP 1: CVP & OTHER SENSITIVITY

UNISA REVISION PACK 2014:

Management of Travelfrenzy Ltd is considering investing in a new machine that could perhaps boost their manufacturing capacity. The estimated annual number of suitcases and probabilities based on different output levels are as follows:

Nr of suitcases	Probability
61 000	0,10
73 000	0,15
74 500	0,18
80 000	0,24
87 000	0,22
95 000	0,11

REQUIRED

(a) Calculate the actual breakeven sales volume in units for 2014. (5)
(b) Calculate the actual breakeven sales value for 2014. (1)
(c) Calculate the units to be sold to obtain a R 2 000 000 profit for 2014. The effects of taxation can be ignored (2)
(d) Calculate the expected number of suitcases the new machine will manufacture and advise management whether the new machine should be bought (4)
(e) Explain the meaning of the terms standard deviation and coefficient of variation as measures of risk (4)

RECAP 1: CVP & OTHER SENSITIVITY

UNISA REVISION PACK 2014:

(a) Calculation of actual breakeven sales units for 2014

Breakeven sales = $\frac{\text{Fixed costs}}{\text{Contribution per unit}}$ ^

= $\frac{\text{R1 500 250}}{\text{R637,60①}}$

① Calculation of contribution per unit

	R	
Selling price (R88 000 000 ÷ 80 000 suitcases)	1 100,00	^
Less: Variable costs		
Material R19 090 000 ÷ 83 000kg = R230 per kg R230 x 81 050kg = R18 641 500 ÷ 80 000 suitcases = R233,02	233,02	√√
Labour R12 350 000 ÷ 80 000 suitcases = R154,38	154,38	^
Variable overheads (R6 000 000 ÷ 80 000 suitcases = R75 per suitcase)	75,00	^
= Contribution	**637,60**	

RECAP 1: CVP & OTHER SENSITIVITY

UNISA REVISION PACK 2014:

b. Calculation of actual breakeven sales units for 2014

Breakeven sales units = 2 353 x selling price R1 100/suitcase = R2 588 300 √

c. Calculation of the units to be sold to obtain a R 2 000 000 profit for 2014

$$\text{Units sold for the target profit} = \frac{\text{Fixed costs + target profit}}{\text{Contribution per unit}}$$

$$= \frac{\text{R1 500 250 + R2 000 000}}{\text{R637,60}①}$$

$$= 5\,489{,}73$$

$$\approx 5\,490 \text{ units } √ √$$

RECAP 1: CVP & OTHER SENSITIVITY

UNISA REVISION PACK 2014:

d. The expected value of suitcases the new machine will manufacture and advice to management

Nr of suitcases	Probability	Weighted nr of suitcases	
61 000	0,10	6 100	^
73 000	0,15	10 950	^
74 500	0,18	13 410	^
80 000	0,24	19 200	^
87 000	0,22	19 140	^
95 000	0,11	10 450	^
		79 250	or √√√

Management should not purchase the new machine as the expected output of suitcases will be less than the current 80 000 being manufactured without the machine. √

RECAP 1: CVP & OTHER SENSITIVITY

UNISA REVISION PACK 2014:

e. Explain the meaning of the terms standard deviation and coefficient of variation as measures of risk

Standard deviation is the square root of the mean of the squared deviations from the expected value (Drury 2012:291). √

Coefficient of variation is a ratio measure of dispersion derived by dividing the standard deviation divided by the expected value (Drury 2012:291). √

or

Standard deviation measures the dispersion of the possible outcomes. It is an absolute measure. In contrast, the coefficient of variation is a relative measure derived from dividing the standard deviation by the expected value (Drury 2012:290) √√

Both measures attempt to summarise the risk associated with a probability distribution. They assume that risk is measured in terms of the spread of possible outcomes (Drury 2012:290). √√

Remember that for MAC3701 you do not need to know how to calculate standard deviation or the coefficient of variation, but you must be able to interpret them when making decisions regarding uncertain future profits.

FINAL WORD…

"…We shall fight on the beaches
We shall fight on the landing grounds
We shall fight in the fields and in the streets
We shall fight in the hills
We shall never surrender…"

~Winston Churchill

Speech to the House of Commons
4 June 1940
Following the retreat of Dunkirk

REFERENCES

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- DRURY 8TH EDITION
- CIMA P1 VARIOUS EXAM PAPERS
- BBP P1 TEXTBOOK