

Tutorial Letter 101/3/2014

Optimisation of resources

DSC3702

Semesters 1 and 2

Department of Decision Sciences

IMPORTANT INFORMATION:

This tutorial letter contains important information
about your module.

BAR CODE

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1 INTRODUCTION AND WELCOME

Dear student

Welcome to the Department of Decision Sciences and the module, *Optimisation of resources*. We hope that you will find this module interesting and that you will complete it successfully.

It is essential that you read this Tutorial Letter 101, as well as Tutorial Letter 301, very carefully. Tutorial Letter 101 contains important information on this module. Tutorial Letter 301 contains general information relevant to all undergraduate students in the Department of Decision Sciences.

1.1 Tutorial matter

The Despatch Department should supply you with the following tutorial matter for this module:

- one study guide
- Tutorial Letter 101 (this tutorial letter)
- Tutorial Letter 301
- Tutorial Letter 201 containing the solutions to the assignments (to be sent out later in the semester)
- any other tutorial letters that may be necessary.

Tutorial matter will be posted to you as soon as possible.

The study guide and tutorial letters are also available electronically in pdf format on myUnisa at <https://my.unisa.ac.za>.

2 PURPOSE AND OUTCOMES OF THIS MODULE

2.1 Purpose of this module

To equip the student with the necessary mathematical skills to formulate, solve and interpret linear, goal and integer programming models.

2.2 Outcomes

Learning outcome 1.

Students are able to formulate linear programming (LP) models of practical problems, use a computer package to solve LP problems and interpret the computer solutions.

Assessment criteria:

1. Formulate a given practical problem as an LP model.
2. Use a computer package to solve an LP model.
3. Read and interpret the computer solution to an LP model.
4. Understand and explain the meaning of the status, value, reduced cost and allowable objective function ranges of a variable.
5. Understand and explain the meaning of the status, slack or surplus, dual price and the allowable right-hand side ranges of a constraint.
6. Explain in a written report the LP model and the practical interpretation of its solution.

Learning Outcome 2.

Students understand the simplex method and how it is used to solve LP models, and are able to manually generate the information contained in the computer solution to LP models.

Assessment criteria:

1. Explain concepts relating to LP and the simplex method.
2. Explain the simplex method and how the simplex iterations are performed.
3. Recognise when an initial solution has to be generated artificially and explain the concept of artificial variables.
4. Recognise when there are bounded variables present in a model and explain the concept of bounded variables.
5. Deduce from the final simplex tableau, the value and the reduced cost of a variable, the slack or surplus and the dual price of a constraint, and be able to explain their meanings.
6. Calculate from the final simplex tableau the allowable objective function ranges for a variable within which the current basis remains optimal.
7. Calculate from the final simplex tableau the allowable right-hand side ranges for a constraint within which the current basis remains feasible.

Learning outcome 3.

Students are able to recognise and interpret the different special solutions of LP models and solve problems relating to them.

Assessment criteria:

1. Recognise an infeasible LP model and explain what it means.
2. Recognise alternative optimal solutions to an LP model, explain what they mean and generate alternative solutions.
3. Recognise an unbounded LP model and explain what it means.
4. Recognise degenerate solutions to an LP model and explain what the problems associated with degeneracy are.

Learning outcome 4.

Students are able to evaluate the correctness of LP models, perform a sensitivity analysis of the solutions to LP models and perform a parametric analysis of LP models.

Assessment criteria:

1. Decide, on the basis of the solution, whether the results make sense and whether the model is an accurate representation of the problem.
2. Determine how the solution to the LP model will change if an objective function coefficient or the right-hand side of a constraint of the LP model changes within its allowable range.
3. Perform a parametric analysis to investigate all possible changes in an objective function coefficient or the right-hand side of a constraint.
4. Present the results of a parametric analysis in tabular and graphical form.

Learning outcome 5.

Students are able to formulate integer constrained problems as integer programming (IP) models, and use a computer package to solve IP models.

Assessment criteria:

1. Judge whether rounding off of the LP solution is acceptable or whether the solution method should force the variables to assume integer values.
2. Explain how the branch-and-bound method is used to solve an IP model.
3. Use binary variables to formulate logical conditions and logical constraints.
4. Formulate an IP model for a given problem.
5. Use a computer package to solve an IP model.
6. Explain why the computer output of an IP model cannot be used to do a sensitivity analysis of a parameter.
7. Explain why certain IP models cannot be solved optimally, but require the use of heuristics.

Learning outcome 6.

Students are able to use goal programming to generate solutions to problems with multiple objectives.

Assessment criteria:

1. Explain the differences between the objectives of linear and goal programming models.
2. Explain the differences between the constraints of linear and goal programming models.
3. Use priority values to represent the preferences of the decision maker.
4. Formulate a multi-objective problem as a goal programming model.
5. Use a computer package to solve a goal programming model.
6. Investigate how changes in a decision maker's objectives and preferences influence the solution to a multi-objective problem.

3 LECTURER AND CONTACT DETAILS

3.1 Lecturer

You will find the name of the lecturer responsible for this module in Tutorial Letter 301. We suggest that you write the name and contact details of the lecturer in the space below.

Lecturer

The lecturer will assist you with any problems you may experience with the contents of the study material. Feel free to contact the lecturer, but please phone beforehand to make an appointment if you wish to see the lecturer in person.

3.2 Department

Department of Decision Sciences

Tel: 012 4334682

Fax: 012 4294898

E-mail: dsbk@unisa.ac.za

3.3 University

If you need to contact Unisa about any administrative matters, please consult the booklet *my studies @ Unisa* in this regard.

Always quote your student number whenever you communicate with Unisa.

4 MODULE RELATED RESOURCES

4.1 Prescribed books

The following book has been prescribed for this module and you must purchase it:

WINSTON, W L. 2004.

***Operations Research: applications and algorithms.* 4th edition. Thomson. California.**

We will refer to this book as *Winston* in the remainder of this tutorial letter. *Winston* contains a CD with several computer software packages. You will need the packages **LINDO** and **LINGO** for this module.

Please refer to the list of official booksellers and their addresses in *my Studies @ Unisa* brochure. Prescribed books can be obtained from the University's official booksellers. If you have difficulty locating your books at these booksellers, please contact the Prescribed Books Section at 012 429 4152 or e-mail vospresc@unisa.ac.za.

Sections from Winston to be studied

Most sections in chapters 3 and 4 were studied at the second-level LP module. You must refer back to these chapters when instructed to do so in the study guide. Chapter 3 may also be used for additional problems on formulation.

- Chapter 3 - *Introduction to Linear Programming*: study the entire chapter
- Chapter 4 - *The Simplex Algorithm and Goal Programming*: study sections 4.1 to 4.12 but omit sections 4.13, 4.14, 4.15, 4.17. Section 4.16 will be studied later.
- Chapter 5 - *Sensitivity Analysis: An Applied Approach*: study the entire chapter.
- Chapter 7 - *Transportation, Assignment and Transshipment Problems*: study section 7.1, section 7.5 (Formulation and computer solution only) and section 7.6. Omit sections 7.2, 7.3 and 7.4.
- Chapter 9 - *Integer Programming*: study sections 9.1 and 9.2 (omit *Integer Programming and Piecewise Functions* and *Solving IPs with What's Best*). Also study sections 9.3 and 9.4. Omit sections 9.5, 9.6, 9.7 and 9.8.
- Chapter 4 - *The Simplex Algorithm and Goal Programming*: study only section 4.16, *Multi-attribute Decision Making in the absence of uncertainty: Goal Programming*.

4.2 Recommended books

There are no recommended books for this module.

4.3 Electronic reserves (e-Reserves)

There are no e-Reserves for this module.

4.4 Module practical work and work-integrated learning

There are no practicals for this module.

4.5 Computer software

Access to a computer is a prerequisite for this module. We will use the computer programs included in Winston. All this software are WINDOWS driven. The software is contained on a CD. LINDO and LINGO are also available on the internet at www.lindo.com.

5 STUDENT SUPPORT SERVICES

See the *my Studies @ Unisa* brochure for student support services rendered.

6 MODULE SPECIFIC STUDY PLAN

Use your *my Studies @ Unisa* brochure for general time management and planning skills.

Since this is a semester module your study time is limited. In this limited period of time, you must work through the study material and prepare for the examination. We suggest that you approach the study material as follows:

- Work through a chapter in the study guide (refer to Winston if instructed to do so). Each chapter contains examples, exercises and problems, together with solutions. You are expected to work through all of them.
- Study the corresponding sections in Winston.
- Do the evaluation exercises on the chapter. The solutions to the evaluation exercises are supplied. Use these to evaluate your answers.

Owing to the limited study time you should plan your studies carefully. Plan to finish the final section three to four weeks before the date of the examination so that you will have enough time remaining for revision.

The key to success in this module is consistent practice. You are strongly advised to work through all the exercises given in the study guide and to attempt the evaluation exercises without referring to their solutions. You will gain nothing by peeking at the answers. The only way you will master the study material is by struggling through the exercises on your own. Afterwards you can evaluate yourself by using the model solutions and hopefully clear up any problems or misconceptions you might still have. The same goes for the assignments. By completing these on your own, you will greatly increase your prospects of passing the examinations.

6.1 Information: evaluation exercises

Evaluation exercises are given on each chapter of the study material. They are important for the following reasons:

- Evaluation exercises help you to understand and master the study material and its practical implications. They are therefore an integral part of the study material.

- Evaluation exercises test your knowledge and understanding of the study material. They are a way of evaluating your progress.

How to attempt the evaluation exercises:

Work through the prescribed study material for a section carefully before you start with the evaluation exercises, in fact before you read the questions for the first time. The process of understanding and mastering the study material takes time and you should set aside plenty of time for it.

The evaluation exercises consist of only a few questions. Do not let this fool you into thinking that you can complete these questions quickly. You will need many hours to answer them.

Evaluating your answers:

Compare your answers with those given in the solution set. Study them carefully to find where your knowledge and understanding are still lacking. Review the relevant sections in the study guide until you are satisfied that you understand it well. If, after this, you still have problems understanding the solutions, you should contact your lecturer for help.

7 ASSESSMENTS

7.1 Assessment plan

The Management of the University has introduced compulsory assignments in all modules so that students can benefit fully from Unisa's formative tuition and assessment.

This tutorial letter contains four compulsory assignments, two for the first semester and another two for the second semester.

You must submit the first compulsory assignment for this module on/before the due date to obtain admission to the examination.

Admission will be obtained by **submitting** the **first** compulsory assignment **on time** and **not** by the marks you obtain for it. Please ensure that this assignment reaches Unisa **before** the due dates. **Late submission of this assignment will result in you not being admitted to the examination.**

The two assignments in a semester count towards your semester mark. All assignments must be submitted on time. Late assignments will not be marked, which means that you will obtain a poor semester mark.

Marks will be allocated for Assignment 01 and it will count 50% towards your semester mark. Assignment 01 is based on chapters 1 and 2 in the study guide and the corresponding sections in Winston.

Marks will be allocated for Assignment 02 and it will count 50% towards your semester mark. Assignment 02 is based on chapters 3, 4 and 5 in the study guide and corresponding sections from Winston.

All registered students will receive the solutions to these compulsory assignments after the due dates.

7.2 Unique assignment numbers and due dates

The unique numbers and due dates are as follows:

Assignment	Semester 1	Unique number	Semester 2	Unique number
01	26 March 2014	169755	25 August 2014	171424
02	23 April 2014	169830	22 September 2014	172534

The due date for an assignment is the last date on which the assignment must reach Unisa.

Assignment 01 contains multiple-choice questions that must be completed on a mark-reading sheet and submitted through the official channels. You may, however, also send your detailed solutions of multiple-choice questions to your lecturer (before or on the due date) if you want to have personal feedback on your workings, after you have submitted the multiple choice answers through myUnisa.

7.3 Assignment enquiries

The lecturer can assist you with any enquiries relating to the study material, the assignment questions and the solutions to them.

However, note that neither the Department of Decision Sciences nor the School of Economic Sciences will be able to answer your general assignment enquiries. If you wish to find out whether your assignment has been received, registered, processed or returned, please check on *myUnisa* at <https://my.unisa.ac.za>.

For detailed information on and requirements for assignments, see the booklet *My studies @ Unisa*.

Please refer to Tutorial Letter 301 for information on the semester mark and how this will contribute to your final mark.

8 ASSIGNMENTS

ONLY FOR SEMESTER 1 STUDENTS
ASSIGNMENT 01
169755
FIXED CLOSING DATE: 26 MARCH 2014

PROBLEM 1:

A refinery specialises in creating certain alloys of iron. Four mines have offered to sell them ore for creating an alloy for which they have received an order of 1 000 tons from a car manufacturer. They do require, however, that the alloy contains at least 6% carbon and not more than 55% iron. The cost (in 100's of rand) and metal characteristics of the four types of ore are given below:

	Ore 1	Ore 2	Ore 3	Ore 4
Carbon	4	5	-	8
Iron	42	78	50	30
Cost	6	6	5	7

The purchasing manager has formulated the following LP model:

$$\text{Minimise } COST = 6O_1 + 6O_2 + 5O_3 + 7O_4$$

subject to

$$CARBON) \quad 0,04O_1 + 0,05O_2 + \quad \quad \quad 0,08O_4 \geq 60$$

$$IRON) \quad 0,42O_1 + 0,78O_2 + 0,50O_3 + 0,30O_4 \leq 550$$

$$QUANTITY) \quad O_1 + O_2 + O_3 + O_4 = 1000$$

$$\text{and } O_1, O_2, O_3, O_4 \geq 0,$$

where O_i = number of tons of ore type i purchased, $i = 1,2,3,4$.

The optimal solution is given on **PRINTOUT 1**. Use it to answer the following questions.

PRINTOUT 1

```

LP OPTIMUM FOUND AT STEP      3

      OBJECTIVE FUNCTION VALUE

    1)      6376.543

      VARIABLE            VALUE            REDUCED COST
    01              0.000000             0.012346
    02             493.827148             0.000000
    03             64.814812             0.000000
    04             441.358032             0.000000

      ROW  SLACK OR SURPLUS    DUAL PRICES
    1)           0.000000        -23.456791
    2)           0.000000         0.617284
    3)           0.000000        -5.308642
  
```

NO. ITERATIONS = 3

RANGES IN WHICH THE BASIS IS UNCHANGED:

VARIABLE	OBJ COEFFICIENT RANGES		
	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
01	6.000000	INFINITY	0.012346
02	6.000000	0.250000	3.800000
03	5.000000	0.025641	0.666667
04	7.000000	0.026316	0.400000

ROW	RIGHT-HAND SIDE RANGES		
	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
1	60.000000	4.375000	51.071430
2	550.000000	70.000000	200.000000
3	1000.000000	400.000000	44.303795

Question 1

The refinery's optimal purchasing plan (values rounded to one decimal place) and cost are given by which option?

- [1] $O_1 = 6; O_2 = 6; O_3 = 5; O_4 = 7$ and Cost = R637 654,3
- [2] $O_1 = 0; O_2 = 493,8; O_3 = 64,8; O_4 = 441,4$ and Cost = R637 654,3
- [3] $O_1 = 60; O_2 = 550; O_3 = 1\ 000$ and Cost = R637 654,3
- [4] $O_1 = 0; O_2 = 493,8; O_3 = 64,8; O_4 = 441,4$ and Cost = R6 376,5
- [5] None of the above.

Question 2

How good a deal should the purchasing manager get on ore type 1 before he would be willing to buy it for this order?

- [1] Reduced cost of $O_1 = 0,012346$, it will increase cost. Thus he should get more than R1,2346 discount.
- [2] Reduced cost of $O_1 = 0,012346$, it will decrease cost. Thus he could pay R1,2346 more.
- [3] Dual price is of $O_1 = -23,456791$, it will increase cost. Thus he should get more than 23,45679 discount.
- [4] Dual price of $O_1 = 0$, it will have no influence on the cost.
- [5] None of the above.

Question 3

Upon further investigation, the purchasing manager finds that ore type 2 is now being sold at R620 per kg. Will the purchasing plan change?

- [1] No, the increase is within the range that the objective function coefficients may increase.
- [2] No, the increase is within the range that the objective function coefficients may decrease.
- [3] Yes, the increase is not within the range that the variable may increase.
- [4] Yes, the increase is not within the range that the variable may decrease.
- [5] None of the above.

Question 4

Upon further investigation, the purchasing manager finds that ore type 2 is now being sold at R620 per kg. By how much will the cost of purchasing the metals increase (to the nearest rand)?

- [1] R20
- [2] R493
- [3] R9 876
- [4] R306 172
- [5] None of the above.

Question 5

The customer is willing to raise the ceiling on the iron content in order to negotiate a reduction in the price he pays for the order. How should the purchasing manager react to this?

- [1] Iron content is a binding constraint. Lowering it by up to 200 will increase cost by R61,72.
- [2] Iron content is a non-binding constraint. Lowering it by up to 200 will not influence the cost.
- [3] Iron content is a non-binding constraint. Lowering it by up to 200 will increase cost by R61,72.
- [4] Iron content is a binding constraint. Raising it by up to 70 will increase cost by R61,72.
- [5] None of the above.

Question 6

The customer now specifies that the alloy must contain at least 9% carbon. Can the purchasing manager comply with this new specification?

- [1] Yes, it is less than the allowable increase for constraint 1.
- [2] A new computer run is not needed to answer this question.
- [3] No, it is more than the allowable increase for constraint 1.
- [4] No, it is more than the allowable increase for variable 1.
- [5] None of the above.

Question 7

What will happen to the cost if 1 more ton of carbon may be included?

- [1] Carbon is a binding constraint with a dual price of R−23,456791. The cost will be lower by −23,456791.
- [2] Carbon is a non-binding constraint with a reduced cost of R−23,456791. The cost will be higher by R−23,456791.
- [3] Carbon is a non-binding constraint with a dual price of R0,012346. The cost will be lower by R0,012346.
- [4] Carbon is a binding constraint with a dual price of R−23,456791. The cost will be R23,456791 higher.
- [5] None of the above.

PROBLEM 2:

Imagine a furniture company that makes tables, chairs and cabinets. The profit on a table is R2170 and a chair R600.

The owner of the firm designs a new cabinet. Suppose that the company can sell a cabinet for R2 200.

Variables:

Let the number of tables be denoted by T and the number of chairs by C .

Let the number of cabinets be denoted by B .

The objective function and constraints are:

$$\text{Maximise profit} = 2170T + 600C + 2200B$$

subject to

$$0,14T + 0,02C + 0,24B \leq 95 \quad (\text{Wood})$$

$$5T + 2C + 90B \leq 6000 \quad (\text{Labour})$$

$$2T + 2,6C + 5B \leq 3000 \quad (\text{Machine - hours.})$$

MAX 2170T+600C+2200B

subject to

$$.14t+.02c+.24b<95$$

$$5t+2c+90b<6000$$

$$2t+2.6c+5b<3000$$

PRINTOUT 1

THE FINAL TABLEAU

THE TABLEAU

ROW	(BASIS)	T	C	B	SLK	2	SLK	3
1	ART	0.000	0.000	1716.914	13709.877		0.000	
2	T	1.000	0.000	1.617	8.025		0.000	
3	SLK	3	0.000	80.556	-27.778		1.000	
4	C	0.000	1.000	0.679	-6.173		0.000	

ROW	SLK	4
1	130	1678364
2	-0.062	577.161
3	-0.556	1694.444
4	0.432	709.877

Question 8

Which resource/s is/are scarce?

- [1] Wood
- [2] Labour
- [3] Machine-hours
- [4] Wood and machine-hours
- [5] None of the above.

Question 9

What would happen if the profit on chairs decreased? Currently, the profit is R600.

- [1] If the profit decreased by more than R290 a new run would be needed.
- [2] If the profit increased by more than R290 a new run would be needed.
- [3] They would make more profit on chairs.
- [4] The reduced cost on chairs is zero. It would have no influence.
- [5] None of the above.

Question 10

Give the shadow cost of B .

- [1] The shadow cost of B is 1 716,91; each cabinet produced would increase the profit with R1 716,91.
- [2] The shadow cost of B is 1 716,91; each cabinet produced would decrease the profit with R1 716,91.
- [3] The shadow cost cannot be found from this tableau.
- [4] The shadow cost of B is 80,56; each cabinet produced would increase the profit with R80,56.
- [5] None of the above.

Question 11

To determine the range within which the per unit profit for tables can vary without affecting the optimality of the current basis we have to assume that

- [1] the objective function coefficient changes to $577 + d$.
- [2] the right-hand side of the constraint changes to $6\,000 + d$.
- [3] the objective function coefficient changes to $2\,170 + d$.
- [4] the number of tables to produce is 577.
- [5] none of the above.

Question 12

Determine the range within which the available labour time (rounded to the nearest integer) can vary without affecting the feasibility of the current optimal solution.

- [1] $6\,000 - 2d \geq 0$; $d \leq 1\,694$
- [2] $6\,000 \leq d \leq 7\,694$
- [3] $[4\,306; \infty)$
- [4] $1\,694 \leq d \leq 6\,000$
- [5] None of the above.

Question 13

The optimal solution (rounded to nearest integer) is

- [1] $T = 2\,170$; $C = 600$; $B = 2\,200$; Profit=R1 678 364.
- [2] $T = 577$; $C = 710$; $B = 0$; Profit=R1 678 364.
- [3] $T = 577$; $C = 1\,694$; $B = 710$; Profit=R1 678 364.
- [4] $T = 0$; $C = 0$; $B = 1\,717$; Profit=13 710.
- [5] none of the above.

Question 14

Explain what is meant by the reduced cost of a non-basic variable.

- [1] The amount by which the objective function value would weaken or strengthen if 1 unit more is produced.
- [2] The amount by which the variable would strengthen or weaken if 1 more unit is available.
- [3] The amount by which the constraint value would weaken if the right-hand side of the constraint would increase by 1 unit.
- [4] The amount by which the objective function value would strengthen or weaken if the right-hand side of the constraint is increased by 1 unit.
- [5] None of the above.

Question 15

What values can the shadow price of a binding \leq (smaller than) constraint be?

- [1] > 0
- [2] < 0
- [3] $\neq 0$
- [4] The value of the right-hand side of the constraint.
- [5] None of the above.

ONLY FOR SEMESTER 1 STUDENTS
ASSIGNMENT 02
169830
FIXED CLOSING DATE: 23 APRIL 2014

Question 1

Jonson, a pharmaceutical manufacturing company, developed a drug against hypertension which was a huge success. The patent for the hypertension drug has now expired. Jonson is therefore looking to invest significant amounts of money in research and development this year to begin the search for a new breakthrough drug. As the head of research and development at Jonson, you are responsible for choosing potential projects and assigning project directors to lead each of the projects. You have decided that your department will pursue five separate projects, which are listed below:

Project Up	Develop an antidepressant that does not cause serious mood swings
Project Stable	Develop a drug that will treat manic depression
Project Choice	Develop a less intrusive birth-control method for women
Project Hope	Develop a vaccine to prevent HIV infection
Project Release	Develop a more effective drug to lower blood pressure

You have appointed five senior scientists to lead the five projects. To ensure that the senior scientists are assigned to projects they will find motivating, you have established a bidding system for the projects. You have given each of the five scientists 1 000 bid points. They assign bids to each project, giving a higher number of bid points to projects that they most prefer to lead. The following table provides the bids from the five individual senior scientists for the five individual projects:

Project	Botha [A]	Rangoaga [B]	Mulaudzi [C]	Modise [D]	Jones [E]
Up [1]	100	0	100	267	100
Stable [2]	400	200	100	153	33
Choice [3]	200	800	100	99	33
Hope [4]	200	0	100	451	34
Release [5]	100	0	600	30	800

- Given the bids, formulate the problem as an LP model to decide which scientist should be assigned to which project. [Which scientists should lead which projects to maximise preferences?] Each doctor can and must be assigned one project.
- One of the doctors is looking for other opportunities. If she leaves, the least popular project would be dropped. Which project would that be?
- If Dr Jones leaves, Dr Rangoaga or Dr Modise can head two projects. Give the new formulation of the model. Only four doctors are available.
- Considering the first scenario again, you think that some scientists should not lead certain projects. Dr Modise [D] cannot lead Project Hope [4]; he is also not a good candidate for Project Stable [2]. Dr Botha cannot lead projects Hope [4] and Release [5]. Dr Jones cannot lead Project Up. Formulate the model again taking into account this new information. Write down only the new constraints.

Question 2

The Tshwane Advertising Agency, TAA, is trying to determine a television advertising schedule for Mzansi Auto Company, MAC. MAC has set itself the following goals listed in order of priority from most to least important:

- Its advertisements should be seen by at least 4 million high-income men, HIM.
- Its advertisements should be seen by at least 6 million low-income people, LIP.
- Its advertisements should be seen by at least 3,5 million high-income women, HIW.

TAA can buy two types of television advertisements: those advertisements shown during football games and those shown during soap operas. TAA can spend at most R6 000 000 on such advertisements.

The advertising costs and potential viewers of a one-minute advertisement of each type are shown in the following table:

Group	HIM	LIP	HIW	Cost (rand)
Football advertisements	700 000	1 million	500 000	100 000
Soap opera advertisements	300 000	500 000	400 000	60 000

This problem can be solved by means of a goal programming model.

Do not formulate this goal programming model, but do the following:

- (a) Define the decision variables needed to determine the advertising time allocated to each type of television advertisement.
- (b) Define the deviation variables associated with MAC's most important goal.
- (c) Formulate and write down the objective function of the goal programming model.
- (d) Formulate and write down the goal (soft) constraint corresponding to MAC's most important goal.
- (e) Formulate and write down the system (hard) constraint needed to restrict TAA to the advertising budget.

Question 3

The Dewright Company is considering three new products to replace current models that are being discontinued. Their OR department has been assigned the task of determining which mix of these products should be produced.

Management wants primary consideration to be given to three factors that would be required for new equipment: long-run profit, stability in the workforce, and the level of capital investment.

In particular, management has established the goals of

1. achieving a long-run profit (net present value) of at least R125 million from these products
2. maintaining the current employment level of 4 000 employees
3. holding the capital investment to less than R55 million.

However, management realises that it probably will not be possible to attain all these goals simultaneously, so it has discussed priorities with the OR department.

This discussion has led to setting penalty weights:

- 5 for missing the profit goal (per R1 million under)
- 2 for going over the employment goal of 4 000 employees (per 100 employees)
- 4 for employing less than the 4 000 employees (per 100 employees)
- 3 for exceeding the capital investment goal (per R1 million over).

Each new product's contribution to profit, employment level and capital investment level is proportional to the rate of production. These contributions per unit rate of production are shown in the table.

Product	Long-run profit (in million rand)	Employment level ('00 employees)	Capital investment (in million rand)
A	12	5	5
B	9	3	7
C	15	4	8

Formulate the problem but do not solve it.

ONLY FOR SEMESTER 2 STUDENTS
ASSIGNMENT 01
171424
FIXED CLOSING DATE: 25 August 2014

PROBLEM 1

A manufacturer makes three kinds of products. The quantity of each type manufactured is symbolised by x_1 , x_2 and x_3 . The profit is given in hundreds rand. The process is subject to restrictions as elaborated in the following model:

$$\text{Maximise Profit} = 12x_1 + 18x_2 + 15x_3$$

subject to

$$5x_1 + 4x_2 + 3x_3 \leq 160 \text{ (machine time in minutes)}$$

$$4x_1 + 10x_2 + 4x_3 \leq 288 \text{ (labour hours)}$$

$$2x_1 + 2x_2 + 4x_3 \leq 200 \text{ (kilograms of raw material)}$$

$$x_2 \leq 16 \text{ (units manufactured.)}$$

$$x_1; x_2; x_3 \geq 0$$

Refer to **PRINTOUT 2** to answer the following questions:

PRINTOUT 2

Max $12x_1+18x_2+15x_3$

subject to

$$5x_1+4x_2+3x_3 < 160$$

$$4x_1+10x_2+4x_3 < 288$$

$$2x_1+2x_2+4x_3 < 200$$

$$x_2 < 16$$

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 792.0000

VARIABLE	VALUE	REDUCED COST
X1	0.000000	10.200000
X2	4.000000	0.000000
X3	48.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	4.200000
3)	56.000000	0.000000
4)	0.000000	0.600000
5)	12.000000	0.000000

NO. ITERATIONS= 2

RANGES IN WHICH THE BASIS IS UNCHANGED:

VARIABLE	CURRENT COEF	OBJ COEFFICIENT RANGES	
		ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	12.000000	10.199999	INFINITY
X2	18.000000	2.000000	7.285714
X3	15.000000	20.999998	1.500000

ROW	CURRENT RHS	RIGHTHAND SIDE RANGES	
		ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	160.000000	17.500000	10.000000
3	288.000000	INFINITY	56.000000
4	200.000000	13.333333	40.000000
5	16.000000	INFINITY	12.000000

Question 1

The OR department obtained an optimal solution and generated the additional output provided for performing post-optimality analysis. What is the predicted profit from the sales of these products?

- [1] R4 500.
- [2] R5 600.
- [3] R1 200
- [4] R79 200.
- [5] None of the above.

Question 2

What is the optimal number of x_1 products to manufacture?

- [1] 0
- [2] 4
- [3] 10
- [4] 48
- [5] None of the above.

Question 3

How much extra machine time is available?

- [1] 0
- [2] 4,2
- [3] 56
- [4] 12
- [5] None of the above.

Question 4

How many labour hours are used?

- [1] 344
- [2] 56
- [3] 232
- [4] 288
- [5] None of the above.

Question 5

Should they consider obtaining extra machine time?

- [1] Yes, the machine time constraint is binding.
- [2] No, the machine time constraint is binding.
- [3] No, it would decrease the profit.
- [4] It would increase the profit by 10,2.
- [5] None of the above.

Question 6

What should they be prepared to pay per kilogram of extra raw material?

- [1] Nothing, they should not get extra raw material.
- [2] R60 or less for up to 13,3 kg.
- [3] R60 or less for up to 40 kg.
- [4] R4,20 for up to 13,33 kg.
- [5] A new run of the model is necessary to answer this question.

Question 7

They can hire extra labour hours. What is the allowable range of labour hours in order for the current solution to stay optimal?

- [1] $[288; \infty)$
- [2] $[232; \infty)$
- [3] $[56; \infty)$
- [4] No slack, dual price is 60.
- [5] A new run of the model is necessary to answer this question.

Question 8

How would the problem's objective function value vary if the profit from one unit of x_3 increase to R22?

- [1] The objective function value would increase by R336.
- [2] The objective function value would increase by R1 056.
- [3] The objective function value would increase by R330.
- [4] No change.
- [5] None of the above.

Question 9

Within what range (rounded to 1 decimal) can the objective function value for x_1 change so that the optimal basis stays unchanged?

- [1] Increase to 10,2; decrease by ∞ .
- [2] Increase to 22,2; decrease by ∞ .
- [3] Increase by 22,2; decrease by 1,5.
- [4] Increase to ∞ ; decrease by 10,2.
- [5] None of the above.

Problem 2

A manufacturer uses metal alloys to make three kinds of products. The amount of each type manufactured is symbolised by A , B and C . The process is subject to restrictions on material, welding, finishing and inspection, as elaborated in the following model:

$$\begin{aligned} 2A + 3B + 2C &\leq 120 \text{ (MAT)} \\ 4A + 3B + C &\leq 160 \text{ (WELD)} \\ 3A + 2B + 4C &\leq 100 \text{ (FINISH)} \\ 0,25A + 0,25B + 0,25C &\leq 12 \text{ (INSPECT)} \end{aligned}$$

Product A contributes R4 000 to the profit, Product B contributes R3 000, and Product C contributes R4 500. Workers are paid R50 an hour during normal working hours.

Refer to **PRINTOUT 2** to answer the following questions:

PRINTOUT 2

THE TABLEAU

ROW	(BASIS)	A	B	C	SLK	2	SLK	3	SLK	4
1	ART	0.000	0.000	700.000	200.000		0.000	1200.000		
2	B	0.000	1.000	-0.400	0.600		0.000	-0.400		
3	SLK	3	0.000	0.000	-4.200	-0.200	1.000	-1.200		
4	A	1.000	0.000	1.600	-0.400		0.000	0.600		
5	SLK	5	0.000	0.000	-0.050	-0.050	0.000	-0.050		

ROW	SLK	5
1	0.00E+00	0.14E+06
2	0.000	32.000
3	0.000	16.000
4	0.000	12.000
5	1.000	1.000

Answer the following questions.

Question 10

In which department is time scarce?

- [1] Welding department
- [2] Finishing department
- [3] Material and finishing departments; both constraints are binding
- [4] Inspection department
- [5] None of the above.

Question 11

Explain the meaning of the shadow cost of C .

- [1] The shadow cost of C is 700; each product C manufactured would decrease the profit with R700.
- [2] The shadow cost of C is 700; each product C manufactured would increase the profit with R700.
- [3] The shadow cost cannot be found from this tableau.
- [4] $C = 1,6$: number of products C that should be manufactured is two.
- [5] None of the above.

Question 12

To determine the range within which the per unit profit for product C can vary without affecting the optimality of the current basis we have to assume that

- [1] the objective function coefficient changes to $4\,500 + d$.
- [2] the objective function coefficient changes to $3\,000 + d$.
- [3] the right-hand side of the constraint changes to $100 + d$.
- [4] the number of product C to manufacture is 500.
- [5] none of the above.

Question 13

Determine the range within which the available time in the welding department can vary without affecting the feasibility of the current optimal solution.

- [1] $16 \geq 0; d \leq \infty$
- [2] $[144; \infty)$
- [3] $[16; \infty)$
- [4] $[176; \infty)$
- [5] None of the above.

Question 14

Determine the range within which the available time in the finishing department can vary without affecting the feasibility of the current optimal solution.

- [1] $20 \geq 0; d \leq 120$
- [2] $-20 \leq d \leq 113,33$
- [3] $[80; 113,33]$
- [4] $113 \leq d \leq 180$
- [5] None of the above.

Question 15

The optimal solution is

- [1] A=12; B=32; C=0; Profit=1 440 000.
- [2] A=12; B=32; C=0; Profit=144 000.
- [3] A=32; B=16; C=12; Profit=1 440 000.
- [4] A=0; B=0; C=500; Profit=1 440 000.
- [5] none of the above.

ONLY FOR SEMESTER 2 STUDENTS
ASSIGNMENT 02
172534
FIXED CLOSING DATE: 22 SEPTEMBER 2014

Question 1

An advertising agency is considering six possible advertising campaigns. Information on these campaigns is given in the following table:

Projects	Initial outlay (R000)	New staff needed	Profit generated (R000)
1	100	3	495
2	175	5	953
3	77	2	410
4	25	1	125
5	265	8	1105
6	170	3	745
Maximum available	500	12	

The agency must decide which campaigns to choose. The following conditions must be met:

- At most four campaigns may be selected.
- Campaigns 1 and 2 are for competing brands and hence may not both be selected.
- If projects 3 and 4 are selected, project 5 must also be selected to follow them up.

Answer the following questions:

- (a) Define the decision variables needed to model this problem.
- (b) Write down the feasibility restrictions, if any.
- (c) Write down the integer programming model for this problem.

Question 2

The Southern Communal Farming Confederation (SCFC) is a group of three communal farms in South Africa. Overall planning for the three farms is currently being done for the coming year.

The agricultural output of each farm is limited by both the amount of irrigable land and the quantity of water allocated to them. These data are given in the table.

Farm	Irrigable land	Water allocation in litres
1	400	600
2	600	800
3	300	375

Because of the limited quantity of water, SCFC would not be able to use all its land for planting crops this year. To ensure equity it has agreed that each farm will plant the same proportion of its available land. (For example, if farm 1 plants 200 of its 400 hectares of land, farm 2 may plant 300 of its 600 hectares.) However, any combination of crops may be planted on any farm.

Crop	Maximum hectares	Water consumption (litres/hectare)	Net return (rand hectare)
Sugar beet	600	3	1 000
Cotton	500	2	750
Sorghum	325	1	250

The coordinating office must decide how many hectares to devote to each crop on each farm while satisfying the restrictions.

The objective is to maximise the total net return to the SCFC as a whole. Formulate an LP model for this problem, but do not solve the model.

Question 3

A company produces six products in the following fashion. Each unit of raw material purchased yields four units of product 1, two units of product 2, and one unit of product 3.

Up to 1 200 units of product 1 can be sold, and up to 300 units of product 2 can be sold.

Each unit of product 1 can be sold or processed further.

Each unit of product 1 that is processed yields a unit of product 4.

Each unit of product 2 can be sold or processed further.

Each unit of product 2 that is processed further yields 0,8 units of product 5 and 0,3 units of product 6.

Demand for products 3 and 4 is unlimited. Up to 1 000 units of product 5 can be sold, and up to 800 units of product 6 can be sold.

Up to 3 000 units of raw material can be purchased at R6 per unit.

Leftover units of products 5 and 6 must be destroyed. It costs R4 to destroy each leftover unit of product 5 and R3 to destroy each leftover unit of product 6.

The per-unit sales price and the production cost for each product are shown in the table below. Formulate an LP the solution of which will yield a profit-maximising production plan. Do not solve the problem.

Product	Selling price (R)	Production cost(R)
1	7	4
2	6	4
3	4	2
4	3	1
5	20	5
6	35	5

Question 4

A machine tool company conducts a job-training programme for machinists. Trained machinists are used as teachers in the programme at a ratio of one for every ten trainees. The training programme lasts for one month. From past experience it has been found that out of ten trainees hired, only seven complete the programme successfully (the unsuccessful trainees are released). Trained machinists are also needed for machining and the company's requirements for the next three months are as follows:

- January - 100
- February - 150
- March - 200

In addition, the company requires 250 trained machinists by April. There are 130 trained machinists

available at the beginning of the year. Salary costs per month are:

- Each trainee earns R4 000.
- Each trained machinist (machining or teaching) earns R7 000.
- Each trained machinist idle (union forbids firing them!) earns R5 000.

Formulate the linear programming problem that will produce minimum cost hiring and training schedule and meet the company's requirements. Do not solve the problem.

9 EXAMINATION

Use your *my Studies @ Unisa* brochure for general examination guidelines and examination preparation guidelines.

To be admitted to the examination you must submit the compulsory assignments to reach Unisa before the due dates.

You **must** write the examination in **May/June** if you are registered for the **first** semester, and in **October/November** if you are registered for the **second** semester.

If for some reason you cannot write the examination, you will have to re-register (and pay again!) for the next semester.

The duration of the examination is **two** hours.

You will be allowed to use a programmable pocket calculator in the examination, but a non-programmable calculator will be more than sufficient.

You may take only your writing materials and your pocket calculator into the examination hall.

Please make sure that you know the correct **date**, **time** and **venue** of your examination, and plan your studies accordingly. **We will not accept any mistakes regarding the date, time, or venue, as an excuse for a supplementary examination at the end of the next semester.**

To pass this module, you must obtain a final mark of at least 50%.

10 OTHER ASSESSMENT METHODS

No other assessment methods exists for this module.

11 FREQUENTLY ASKED QUESTIONS

The *my Studies @ Unisa* brochure contains an A-Z guide of the most relevant study information.

1. MATTERS PERTAINING TO REGISTRATION

- What should I do if any of my personal details (name, address, etc) have been incorrectly captured during the registration process?

Please use *myUnisa* (<https://my.unisa.ac.za>) to rectify any mistakes. Alternatively send an e-mail to econ@unisa.ac.za. Alternatively mail us a letter addressed to The Registrar, PO Box 392, Unisa, 0003. Please remember to quote your student number in all correspondence.

- What do I need to do to obtain items that were out of stock when I registered?

Please use the internet facility at <https://my.unisa.ac.za> in order to download copies of any

of the study material. Alternatively send us an e-mail to despatch@unisa.ac.za. Please remember to quote your student number in all correspondence. Please allow 4 weeks for delivery of any outstanding items before contacting Unisa about out-of-stock items.

2. ASSIGNMENTS

- My assignment is late because/ Can I submit it at a later date?

It is your responsibility to ensure that your assignment is submitted at myUnisa or reaches the Unisa Main Campus on or before the closing date. Please do not call us to request an extension for the submission of an assignment. Once you have received the tutorial letter containing the solutions you will know that you definitely can no longer submit the assignment.

- Are the assignments of this module compulsory?

The assignments are compulsory. The intention is that you should use the assignments to prepare yourself for the examination.

- Do the marks obtained for assignments count towards the final mark?

The assignment marks do count towards the final marks, as explained in this tutorial letter.

- Who do I contact to find out if Unisa received my assignment(s) that I sent by mail?

Preferably submit your assignments via myUnisa. If you do not have access to the Internet and mailed Unisa your assignments, visit <https://my.unisa.ac.za> in order to determine if Unisa received your assignment(s).

- What do I do if my assignment goes missing in the mail?

Preferably submit your assignments via myUnisa. Always make a copy of your assignment and keep it in a safe place in case you need to submit a duplicate copy.

- Can I submit my assignments by e-mail?

Assignments should be submitted via the myUnisa facility. Please do not e-mail assignments to the centre via e-mail.

- Can I send my assignments by e-mail directly to the relevant lecturer?

Please do not send your assignments directly to the lecturers. Please use the myUnisa facility.

- How long will I have to wait before I can expect my assignment(s) results?

We aim to have assignments marked within three (3) weeks after each due date. Visit <https://my.unisa.ac.za> to determine what mark you achieved. Allow an additional week or two for the post office to deliver the results to your postal address. The assignment results will be returned to the postal address that you indicated. Please do not call us to make enquiries about your assignment before four (4) weeks after the due date have elapsed.

- Can I expect my assignment to be returned sooner if I submit it well before the due date?

The marking of assignments normally starts on the due dates indicated in the relevant tutorial letters 101. Assignments are not marked or returned prior to the due dates.

3. EXAMINATION

- What happens if I am unable to write the examination because of ill health, work commitments or any other crisis?

Any requests for aegrotat examinations need to be done in writing and directed to the Head: Examinations, Box 392, Unisa, 0003 or exams@unisa.ac.za. A fee is payable for such requests. The amount payable is determined by the Examination Section.

- What do I need to bring to the examination venue?

Bring your identity document (ID), student card, black pen, ruler, eraser, pencil, sharpener and calculator.

- Will it be an open-book examination?

No.

- Could you give me an idea of the scope of the examination paper?

The examination paper is based on your prescribed book and the learning outcomes provided in your study guide. Each learning outcome statement and assessment criterion may be used to set examination questions. Please do not call us to enquire about the "scope" of the examination paper.

4. RESULTS

- How will the results be announced?

The results are normally mailed to students. They are also published at myUnisa <https://my.unisa.ac.za> and the MTN results facility, cellular number 083 1234.

5. STUDY GUIDE AND TUTORIAL LETTERS

- I have lost my study guide and tutorial letters. Could you please mail or fax me a copy?

Please download a copy from myUnisa. Register as a user at <https://my.unisa.ac.za>. The lecturers do not mail or fax copies of study guides to students. You may also order replacement copies of study guides from Unisa's Despatch Department. Please send an e-mail to despatch@unisa.ac.za or a letter to The Head, Despatch, PO Box 392, Unisa, 0003.

6. ACCOUNTS

- What do I do if I do not agree with my account statement?

Please send a letter addressed to The Head, Student Accounts, Box 392, Unisa, 0003. Please provide a copy of your receipt(s), deposit slips or electronic funds transfer (EFT) as proof of payment.

- Will the payment immediately appear on my account statement?

Please note that your payment will only be reflected on your next account statement.

SECTION B: ACADEMIC MATTERS

1. CALCULATORS

- Am I allowed to use a calculator? Yes.

2. SOFTWARE

- There was no CD in my prescribed book. Where can I get LINDO or LINGO?

Download the software from the internet at <http://www.lindo.com>.

- Where can I get the Maxima software?

Download the software **Maxima** at <http://maxima.sourceforge.net/download.html>.

- The LINDO CD is not compatible with MS Office 2007.

Look on myUnisa under additional resources, how to manage the problem.

3. EXAMINATION

- Are any old examination papers available?

Previous examination papers are published at myUnisa, but no memoranda are made available to students. Old questions will not necessarily be repeated in subsequent examination papers and students are warned not to rely on old examination papers in order to pass the module.

- What will the format of the examination paper be like?

The format of the examination paper will be confirmed in one of the tutorial letters (normally tutorial letter 202) which will be sent to you during the year.

- If I study only the assignment questions, will I be adequately prepared for the examination?

No. It is unlikely that you will be adequately prepared by studying only the assignment questions. The assignment questions represent a sample of the work and cannot be regarded as being representative of all the study material.

- Can you give me the "scope" of or any "hints" for the exam?

No "hints" are provided to students. Please do not call the lecturers about the "scope" of or "hints" for the exam. Base your preparation for the examination on the learning outcome statements.